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TOOL PATH GENERATION FOR FREE FORM SURFACES WITH B-SPLINE CURVES

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ABSTRACT
Free form parts used in the design and manufacture of molds, dies and aerodynamic shapes …etc., are machined on 3-axis or 5-axis milling machines. In the classical approach of tool path generation for free form surfaces, the tool movements are expressed in linear interpolation. For this method of interpolation, the theoretical curves of tool positions are approximated with a polygonal line. Consequently, the tool movements are discontinue in tangency and in curvature which create vibrations and consequently affects the finish of the surface. With the development of CNC interpolators that accept B-Spline curve, it is more advantageous to use them. In our work, from the NURBS surfaces machined with parallel plane machining strategy on 3-axis CNC milling machines, we automate the generation of the tool path in terms of B-Spline curves by the determination of the minimum number of control points that give the best approximation of the different tool positions.
Key words : Parallel Plane Strategy, Approximation, B-Spline, Free Form Surface.

1. INTRODUCTION
Free form parts are used in the design and manufacture of molds, dies …etc, and are machined on 3-axis or 5-axis milling machines. For machining free form surfaces, many aspects must be considered: surface definition, machining parameters …etc, with the objective to obtain a good surface finish. Different problems related to the machining of free form surfaces have been considered by researchers [1, 7]. Among the fields of research is the quality of the finished surfaces. For machining these surfaces, different machining strategies are used: isoparametric, Z-constant, parallel plane …etc. The parallel plane machining strategy permits the machining of multiple adjoining surfaces and guarantees the continuity of the machining. In the classical approach of tool path generation, the tool movements are expressed in linear interpolations. For this method of interpolation, a polygonal line is used to approximate the theoretical curves of tool positions. Consequently, the tool movements are discontinue in tangency and in curvature because at each point there is an abrupt change in the direction that creates vibrations and consequently affects the finish of the surface. With the development of CNC interpolators that accept B-Spline curve, it is more advantageous to use these interpolators because they minimize vibrations, permit the obtaining of good surface finish and permit the reduction of the size of the machining program. In our work, from the NURBS surfaces machined with parallel plane machining strategy on 3-axis CNC milling machines, we automate the generation of the tool path with B-Spline curves by the determination of the minimum number of control points that give the best approximation of the different tool positions with a given accuracy.

2. DEFINITION OF NURBS CURVES AND SURFACES

A NURBS curve is defined by (n+1) control points P_i with their weight W_i, ($0 \le i \le n$), knot vector U and the degrees p. If all the weights are equal, a NURBS curve becomes a B-Spline curve. A NURBS curve is given by [8, 9]:

$$p(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i p_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \tag{1}$$

A NURBS surface is defined by a network of (m+1)×(n+1) control points $P_{i,j}$ with their weight $W_{i,j}$ ($0 \le i \le m$ and $0 \le j \le n$), two knot vectors U and V, the degrees p and q in the u and v directions. The NURBS surface is given by [8, 9]:

$$P(u,v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u).N_{j,q}(v).P_{i,j}.W_{i,j}}{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u).N_{j,q}(v).W_{i,j}} \tag{2}$$

Where $N_{i,p}(u)$ and $N_{j,q}(v)$ are the B-Spline basis functions of degree p and q respectively. To the coordinates, we need to calculate the unit normal vector $\vec{n}$ and the principal curvatures k_1 and k_2 [9].

3. TOOL PATH GENERATION FOR FREE FORM SURFACE MACHINING

The following sections describe the necessary steps for generating tool path with B-Spline curves.

3.1. Interference and tool position relative to the surface

For machining free form surface, the tool must be tangent to this surface and generally, for 3-axes CNC machining, the ball end mill tool is used and its positions are given by [3] (figure 1):

$$\overrightarrow{OC_E} = \overrightarrow{OC_C} + r\vec{n} \quad ; \quad \overrightarrow{OC_L} = \overrightarrow{OC_C} + r\vec{n} - r\vec{u} \tag{3}$$

With: C_C: cutter contact, C_E: tool center, C_L: cutter location, $\vec{u}$ the tool orientation vector and r the tool radius. To avoid the problem of interferences, the radius of the used tool must be less or equal than the small principal radius of curvature for all surfaces [1].

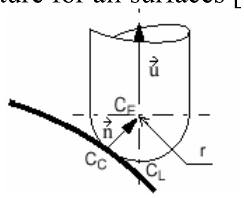


Figure 1. Ball end mill position.

3.2. Parallel machining method

Different machining strategies are used for machining free form surfaces: isoparametric, Z-constant, parallel plane …etc. The parallel plane machining strategy permits the machining of multiple adjoining surfaces, and guarantees the continuity of the machining. For this strategy, the tool path is determined from the intersection points of a vertical plane, having any orientation, with the surfaces to manufacture. In our work, we have considered the One-Way and Zig-Zag sweeping strategies. To machine the surfaces, we need to specify the deviation error and the distance between two consecutive planes. The surfaces are triangulated in order to simplify the calculation of the intersection points.

3.3. Approximation with B-Spline curves

Generally, once the points of intersection are calculated, the linear interpolation is used to express the tool movements. For this kind of interpolation, the movements are discontinue in tangency and in curvature which creates vibrations and affects the surface finish. To obtain a good surface finish and to reduce the size of the program, the tool path is generated in the terms of B-Spline curves. In our work, the approximation is used because it is more flexible and decreases the size of the program. We suppose that the curve must pass by the first and the last point of the data points for each curve.

Given (m+1) points Q_k ($0 \le k \le m$), the approximation consists to fixing the number of control points (n+1) and the degree p ($1 \le p \le n$) and then to calculate the coordinates of the control points of the curve by applying the least square method by minimizing the following function [8]:

$$F(P_i) = \sum_{k=0}^{m} \left| \sum_{i=0}^{n} N_{i,p}(u_k) P_i - Q_k \right|^2 \qquad (4)$$

To obtain a linear system of equations and to calculate the coordinates of the control points, the values of parameters u_k are calculated using the uniformly spaced method, the centripetal method or the chordal length method and the knot vector is calculated using the uniform method or the average method [8]. To qualify the quality of the generated curve, two criteria of accuracy are used [10]:

- The superior error C_1 : measures the gap on the less approached point:

$$C_1 = \underset{i=0,\ldots m}{Max} \left| Q_i - C(u_i) \right| \qquad (5)$$

- The average quadratic error C_2 : measures the average error committed at a point.

$$C_2 = \frac{\sqrt{\sum_{i=0}^{m} (Q_i - C(u_i))^2}}{m+1} \qquad (6)$$

The criteria of precision C_1 and C_2 are necessary to judge the precision of the obtained curve, but these criteria do not permit to validate the curve between data points.

4. SOFTWARE DEVELOPMENT AND RESULTS

To automate the generation of the tool path with B-Spline curves and the determination of the minimum number of control points that give the best approximation with a given accuracy, we have developed an MS Windows based object oriented software using the C++ Builder and the graphics library OpenGL[11]. In the developed software, we have two stages (figure 2). In the first stage, the user selects the surfaces to machine and introduces the following parameters: tools dimensions, feed rate, distance between two planes, the triangulation method, the sweeping strategy, the orientation of the vertical plane, the points to approximate with B-Spline curves (cutter contact, tool center or cutter location), the degree of the curve and the accuracy of the approximation. After this, the software calculates the points of intersection between all positions of the vertical planes and the triangles. In the second stage, the software selects automatically for each B-Spline curve the methods that calculate the nodal values for each point of intersection and the knot vector that give the best approximation with a minimum number of control points. Finally, the obtained curves are use in the generation of the machining program.

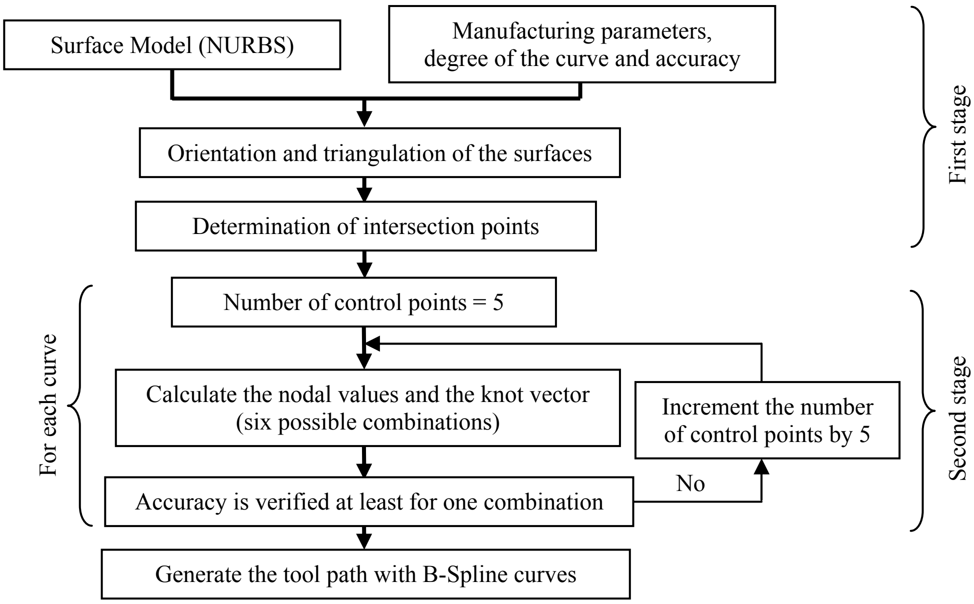
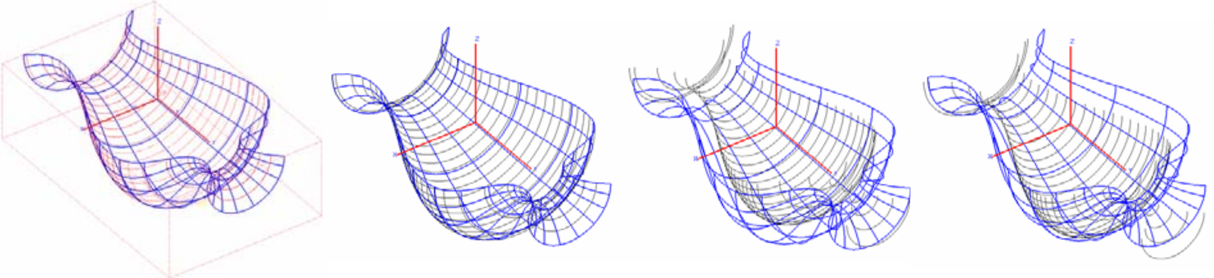


Figure 2. Flow chart of tool path generation with B-Spline curves.

To demonstrate the automatic generation of tool path in terms of B-Spline curves, we have considered the surface represented by figure (3.a) with the following parameters: tool radius = 10 mm, feed rate = 50 mm/min, distance between two planes = 5 mm, number of triangles in each direction = 200, orientation of the vertical plane = 0°, accuracy = 0.1 mm, degree = 3. These data give a total of 13033 intersection points and 33 curves. The obtained results for cutter contact points, tool center points and cutter location points are given in table1 and the figure 3 represents the generated curves. For this example, the maximum number of control points is equal to 10 and the results show the good approximation of the points of intersection with B-Spline curves and the important reduction in the number of points and consequently the reduction of the size of the machining program.
For the generation of the B-Spline curves, the software takes a time that depends on surface complexity, dimensions of surfaces, distance between two planes, number of curves, the imposed accuracy and the CPU of the computer.

Table 1. Results of the approximation with B-Spline curves.

	Points of intersection	Number of control points	Percentage	Error C1	Error C2
C_C points	13033	330	2.53	0.095	0.0026
C_E points	13033	320	2.45	0.1	0.003
C_L points	13033	320	2.45	0.1	0.003



a- Intersection points. b- Cutter contact curves. c- Tool center curves. d- Cutter location curves.

Figure 3. The considered surface and the generated B-Spline curves.

5. CONCLUSION

In this paper, we have presented a method that permits, from CAD models of surfaces to machine with parallel plane machining strategy on 3-axes CNC milling machines, the automation of the generation of the tool path with B-Spline curves in order to obtain a good surface finish and to reduce the size of the machining program. The presented method can be used for other machining strategies with a few modifications.

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