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# THE COISE OF CONVENTION FOR AFFIXING REFERENT COORDINATE SYSTEMS IN KINEMATIC AND DINAMIC ANALYSIS OF ROBOTIC MANIPULATORS

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## 1. INTRODUCTION

One of the most relevant issues in the analysis of the motion of some body is the choise of referal coordinate systems. In kinematics and also in dinamics of robotic manipulators different authors use different referal coordinate systems, and this choise significantly affect simplification and programming of robot work. In this paper will be done the detail analysis of most used choises of convention for affixing of coordinate systems during computation of kinematics and dinamics of robotic manipulator. For this reason it will be analysed use of coordinate systems by insighting in most important books of world literature from robotics field, as: J.J. Craig «Introduction to Robotics» [4], L. Sciavicco, B. Siciliano «Modeling and Control of Robot Manipulators» [2], M.W. Spong «Robot Modeling and Control» [1], and P.J. McKerrow «Introduction to Robotics» [3], and will be done also the insight in announced articles which are published in temporarily most actual world magazines: Robotics and Autonomous Systems [11], IEEE Robotics and Automation Magazine [12], and International Journal of Robotics and Automation [13]. Additionaly, it will be said a word about which coise of convention for affixing the coordinate systems are most appropriate for kinematic analysis and sintesys, and which are appropriate for dinamic analysis and determination of forces between consecutive links of robotic manipulator. All of this will be accomplished also through the influence of choosen coordinate system on the transformations in robotic kinematics.
The description of the position and orientation of important points of our interest of open kinematic chain of robotic manipulator is used to obtaining composition of coordinate transformation between consecutive link, ie.relative location of consecutive links. The definition of mechanisms by means of four quantities (which describe link itself and link's connection to a neighboring link) is a convention which is ussualy called the Denavit - Hartenberg notation.
Depending on the way of the description of robotic manipulator, assigninig of coordinate frames, and also manipulator's joints, we destinguish DH (Denavit - Hartenberg) convention and modified DH convention. In this paper we will, on the example of calculation of direct kinematics for robotic manipulator with six rotational degrees of freedom, show that both convention lead to the same transformation matrix.

## 2. MODIFIED DH CONVENTION (J.J. Craig)

Links of robotic manipulator are assigned starting from fixed base of manipulator, that is called "link 0". The first moving body is link *1* etc, out to the free end of the arm, which is link *n*.
Axes of the joints are defined by lines in space. Axis of joint *i* is defined by line in space or by vector about which link *i* rotates relative to link *i-1*. For any two axes in 3-space, there exists a well-defined measure of distance between them. This distance is measured along line that is mutually perpendicular to both axes, is denoted by $a_{i-1}$ and is called **link lenght**. The second parametar needed to define the relative location of the two axes is called the **link twist** $\alpha_{i-1}$. If we imagine a plane whose normal is

link length, we can project axes $i$-$1$ and $i$ onto this plane and measure the angle between them. This angle is measured from axis $i$-$1$ to axis $i$ in the right-hand sense about $a_{i-1}$. Neighboring links have a common joint axis between them. One parameter of interconnection has to do with the distance along this common axis from one link to the next. This parameter is called the **link offset**. The offset at joint axis $i$ is called $d_i$. The second parameter describes the amount of rotation about this common axis between one link and its neighbor and is called **joint angle**, $\theta_i$. When assigning coordinate frames, we point $z_i$ axis along the $i$-th joint axis, and $x_i$ axis pointing along the common perpendicular.

**3. DH KONVENCIJA (Mark W. Spong)**

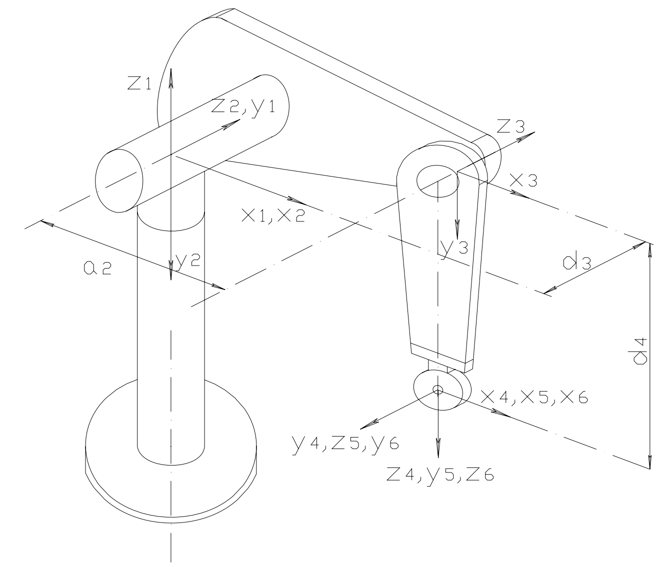
Robotic manipulator with $n$ joints has $n+1$ links, taking into account that each joint connects two consecutive links. We number the joints from $1$ to $n$, and we number the links from $0$ to $n$, starting from the base. By this convention, joint $i$ connects link $i$-$1$ to link $i$. When joint $i$ is actuated, link $i$ moves. So, link $0$ is fixed and does not move when the joints are actuated. With the $i$-th joint, we associate a joint variable, denoted by $q_i$. In the case of a revolute joint, $q_i$ is the angle of rotation. To perform the kinematic analysis, we attach a coordinate frame rigidly to each link. Coordinate frame $O_ix_iy_iz_i$ is attached to link $i$. This means that, whatever motion the robot executes, the coordinates of every point on link $i$ are constant when expressed in the $i$-th coordinate frame. Furthermore, when $i$-th joint is actuated, link $i$ and its attached frame, $O_ix_iy_iz_i$ , experience a resulting motion. The coordinate frame $O_0x_0y_0z_0$, which is attached to robot base, is referred to as the inertial frame. Specificly, we attach $z_i$ as actuation axis (rotation) for joint $i+1$. So, axis $z_0$ is actuation axis for joint $1$, $z_1$ is actuation axis for joint $2$, etc.

Homogeneous transformation matrix that expresses the position and orientation of coordinate frame $O_jx_jy_jz_j$ with respect to frame $O_ix_iy_iz_i$ is called transformation matrix and is denoted by ${}^iT_j$.

$${}^iT_j = A_{i+1}A_{i+2}A_{i+3} \ldots A_{j-1}A_j \qquad \text{for } i<j$$

**4. KINEMATICS OF PUMA 560 WITH MODIFIED DH CONVENTION (J.J. CRAIG)**

Puma 560 is a robot with 6dof. It is shown on Picture 1 with assigned coordinate systems in posture which corespond to 0 values for all joint angles. Coordinate system $0$ is colinear with system $1$ when $\theta_i$ =0. Axes of joints 4,5 and 6 intersect in one point, in which are placed also origins of coordinate systems 4,5 and 6. Axes of joints 4,5 and 6 are mutualy orthogonal. Link parameters which correspond to this robot posture are given in Table 1.




*Picture 1: Robotic manipulator PUMA560 with assigned link parameters according to J.J. Craig*

| i | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0° | 0 | 0 | $\theta_1$ |
| 2 | -90° | 0 | 0 | $\theta_2$ |
| 3 | 0° | $a_2$ | $d_3$ | $\theta_3$ |
| 4 | -90° | $a_3$ | $d_4$ | $\theta_4$ |
| 5 | 90° | 0 | 0 | $\theta_5$ |
| 6 | -90° | 0 | 0 | $\theta_6$ |

*Table 1: Link parameters for PUMA 560 robotic manipulator*

Transformations for every link are as follows:

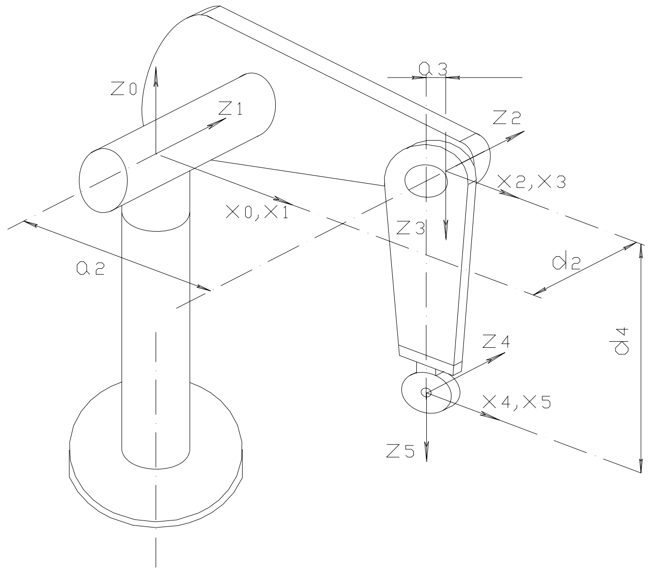
$$\mathbf{A}_1^0=\begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_2^1=\begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -s_2 & -c_2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_3^2=\begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$\mathbf{A}_4^3=\begin{bmatrix} c_4 & -s_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_5^4=\begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_6^5=\begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -s_6 & -c_6 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Elements of homogeneous transformation matrix are:

$r_{11}=c_1(c_{23}(c_4c_5c_6-s_4s_6)-s_{23}s_5c_6)+s_1(s_4c_5c_6+c_4s_6)$,
$r_{21}=s_1(c_{23}(c_4c_5c_6-s_4s_6)-s_{23}s_5c_6)-c_1(s_4c_5c_6+c_4s_6)$,
$r_{31}=-s_{23}(c_4c_5c_6-s_4s_6)-c_{23}s_5c_6$,
$r_{13}=-c_1(c_{23}c_4s_5-s_{23}c_5)-s_1s_4s_5$,
$r_{23}=-s_1(c_{23}c_4s_5-s_{23}c_5)+c_1s_4s_5$,
$r_{33}=s_{23}c_4s_5-c_{23}c_5$,
$r_{12}=c_1(c_{23}(-c_4c_5s_6-s_4c_6)+s_{23}s_5s_6)+s_1(c_4c_6-s_4c_5s_6)$,
$r_{22}=s_1(c_{23}(-c_4c_5s_6-s_4c_6)+s_{23}s_5s_6)-c_1(c_4c_6-s_4c_5s_6)$,
$r_{32}=-s_{23}(-c_4c_5s_6-s_4c_6)+c_{23}s_5s_6$,
$p_x=c_1(a_2c_2+a_3c_{23}-d_4s_{23})-d_3s_1$,
$p_y=s_1(a_2c_2+a_3c_{23}-d_4s_{23})+d_3c_1$,
$p_z=-a_3s_{23}-a_2c_2-d_4c_{23}$,

**5. KINEMATICS OF PUMA 560 WITH DH CONVENTION (Mark W. Spong)**




*Picture 2: Robot manipulator PUMA560 with assigned link parameters according to Spong*

| i | $\alpha_i$ | $a_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | -90° | 0 | 0 | $\theta_1$ |
| 2 | 0° | $a_2$ | $d_2$ | $\theta_2$ |
| 3 | -90° | $a_3$ | 0 | $\theta_3$ |
| 4 | 90° | 0 | $d_4$ | $\theta_4$ |
| 5 | -90° | 0 | 0 | $\theta_5$ |
| 6 | 0° | 0 | 0 | $\theta_6$ |

*Table 2: Link parameters for robot manipulator PUMA 560 acc. to Spong*

Transformations for every link are as follows:

$$\mathbf{A}_1^0=\begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_2^1=\begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_3^2=\begin{bmatrix} c_3 & 0 & -s_3 & a_3c_3 \\ s_3 & 0 & c_3 & a_3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$\mathbf{A}_4^3=\begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_5^4=\begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \mathbf{A}_6^5=\begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Elements of homogeneous transformation matrix are the same as in case of defining link parameters, coordinate systems and assigning of joints according to J.J. Craig (modified DH parameters):

$r_{11}=c_1(c_{23}(c_4c_5c_6-s_4s_6)-s_{23}s_5c_6)+s_1(s_4c_5c_6+c_4s_6)$,
$r_{21}=s_1(c_{23}(c_4c_5c_6-s_4s_6)-s_{23}s_5c_6)-c_1(s_4c_5c_6+c_4s_6)$,
$r_{31}=-s_{23}(c_4c_5c_6-s_4s_6)-c_{23}s_5c_6$,
$r_{13}=-c_1(c_{23}c_4s_5-s_{23}c_5)-s_1s_4s_5$,
$r_{23}=-s_1(c_{23}c_4s_5-s_{23}c_5)+c_1s_4s_5$,
$r_{33}=s_{23}c_4s_5-c_{23}c_5$,
$r_{12}=c_1(c_{23}(-c_4c_5s_6-s_4c_6)+s_{23}s_5s_6)+s_1(c_4c_6-s_4c_5s_6)$,
$r_{22}=s_1(c_{23}(-c_4c_5s_6-s_4c_6)+s_{23}s_5s_6)-c_1(c_4c_6-s_4c_5s_6)$,
$r_{32}=-s_{23}(-c_4c_5s_6-s_4c_6)+c_{23}s_5s_6$,
$p_x=c_1(a_2c_2+a_3c_{23}-d_4s_{23})-d_2s_1$,
$p_y=s_1(a_2c_2+a_3c_{23}-d_4s_{23})+d_2c_1$,
$p_z=-a_3s_{23}-a_2c_2-d_4c_{23}$,

## 6. CONCLUSION

It was shown that we basically distinguish two basic conventions for assigning the coordinate frames: DH convention and modified DH convention, that use J. J. Craig and Wolovich. The difference is in assignment of the coordinate frames and in affixing the coordinate frames to certain links, which results in different transformation matricies that describe the position and orientation of certain coordinate frames with respect to preceding coordinate frame. DH convention assignes *n+1* coordinate frames for manipulator with *n* degrees of freedom. Modified DH convention assignes *i*-th coordinate frame on proximal end of the link *i*, which results in a different homogeneous transformational matrix of neighboring coordinate frames. No metter which of these two conventions we use during description of the kinematics of the manipulator, homogeneous transformation matrix that describe location of en effector with respect to referal coordinate frame is the same – untill we keep adopted convention. Both conventions, if we keep the correctly, can give complete description of the manipulator kinematics; Nevertheless, modified DH convention seems to be more simple and natural choise for problems of industrial serial manipulators, from the aspects of simplicity of link assignment, joints, and axes, and afterwords also in determination of link parameters of the manipulator, which affects the speed of basic principles of problem solving.

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