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DESIGN OF STEPHENSON-I TYPE OF SIX-BAR MECHANISM USING BURMESTER CURVES AND INVERSION METHOD

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ABSTRACT
A lot has been done so far in the use of Burmester curves in designing four-bar mechanisms that prescribe three and four positions but not so much for five positions and even more in design of six-bar mechanisms prescribing four and five positions. This paper briefly describes a way to utilize Burmester curves in design of Stephenson-I type of six-bar planar mechanism prescribing five precision points. This will be achieved via combining inversion method with Burmester theory (a graphical theory combined with a numerical one) all in an analytical/numerical way. Since the Stephenson-I six-bar mechanism consists of a main four-bar and a dyad, first we'll invert/transform the mechanism in to two four bar mechanisms (where the output of the first four-bar is the input of the second four-bar). Furthermore, in this paper we would like to present an application of this combination of theories and design methods in the design of Stephenson-I type of a six-bar planar mechanism that prescribes five positions.
Keywords: Burmester, Stephenson, six-bar

1. INTRODUCTION
Looking at the world around, approximately every few seconds one encounters a mechanism of some kind without being aware of its presence. Although the wheel is one of the greatest inventions of the man, manipulating with it was just as well challenging. For any object that's needs moving from one position to another a mechanism of some sort is required. In other words, mechanisms ore nothing more than a matter combined with a simple geometry. Although mechanisms can be tree dimensional, most of them are limited in two dimensions.
The objectives of this paper are to utilize the Burmester Theory in combination with Inversion Method in a design of a Stephenson-I type of six-bar mechanisms that will prescribe five precision points.

2. BURMESTER THEORY
When a body moves through two and three prescribed positions there are three respectively two infinities of solutions in the design of the mechanism; however, when the body passes through four and five prescribed positions then there is one-infinity respectively few possible designs of a mechanisms. Burmester found the solutions to problem of four prescribed positions of the plane using complex numbers [3], Figure 1.
Notice that the vectors defined above form a closed loop, including the first and j-th positions:

$$We^{i\beta j}+Ze^{i\alpha j}-\delta_j-Z-W=0 \qquad (1)$$

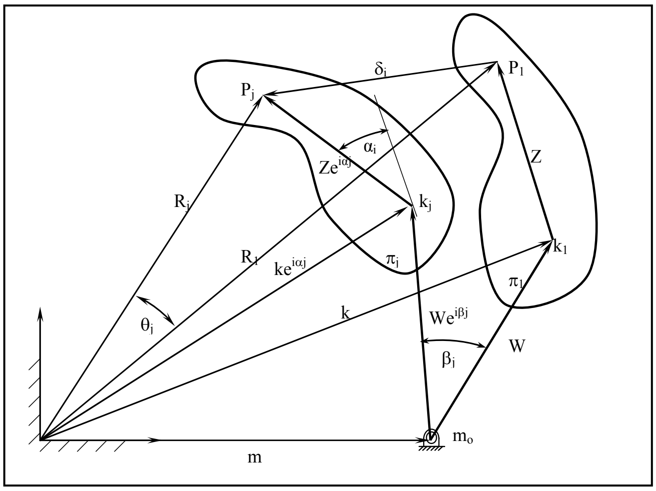

Figure 1: The unknown dyad's W,Z which can guide the moving plane π from the first to the j-th position

After the compatible sets of β_2 are obtained, then any method for solving simultaneous complex equations can be used to find Z and W. Then the circle point k and the center point m are given by the following expressions (Figure 1).

$$k = R_1 - Z \quad \text{and} \quad m = k - W \qquad (2)$$

The above pair of these two points is called Burmester Point Pair (B.P.P.), and the locus of these pairs for different values of β_2 form the Burmester Curves.

3. APPLICATION OF BURMESTER THEORY IN STEPHENSON-I TYPE OF SIX-BAR MECHANISMS

Chan utilized Burmester Theory with a specially written computer program [1]. It calculated circle-point and centre-point circles (specific for Burmester Theory) for four positions. This program was later modified initially by Noussas D. and finally by the co-author of this paper, Pira B., for mechanisms that prescribe five precision points [2].

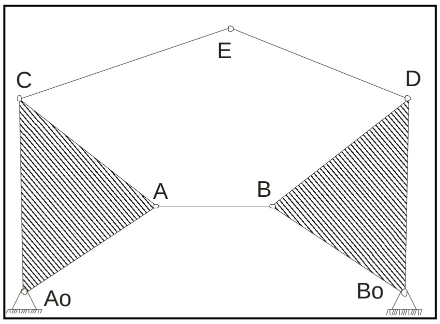

Figure 2: The Stephenson-I six-bar mechanism

In the design of Stephenson-III and Watt-II type of six-bar mechanism the use of the Burmester theory is strait forward since it has three ground points and as it is it can be treated as two four-bar mechanisms. However, the Stephenson-I (Figure 2) type of six-bar mechanism has only two ground points and as such it is very difficult and at times impossible to use Burmester theory on its own in its design. A possible way to utilize Burmester theory in Stephenson-I mechanism is by inverting/transforming [4] it in to two four-bar mechanisms (Figure 3) and then using Burmester Theory in designing these two four-bar mechanisms: Ao-C-E-O and Bo-D-E-O.

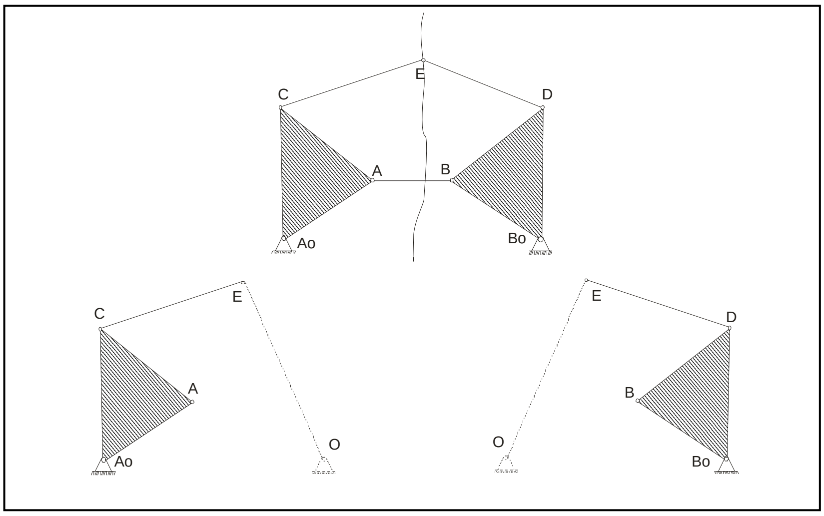

Figure 3: The Stephenson-I six-bar mechanism inverted/transformed in to two four-bar mechanisms

Once the four-bars are constructed then they are joined again on the common point E to form back the Stephenson-I six bar mechanism. Additionally, the points A and B of the mechanism which in itself form a four-bar can also be found during the inversion/transformation. Let’s take a specific case study of a mechanism that prescribe 5 precision points (Table 1).

Table 1. The coordinates of a precision point P for the first and second coupler

Number	*Precision points* *X*	*Y*	*First coupler angle* *θ°*	*Second coupler angle* *θ°*
1	*0.0*	*0.0*	*0*	*0*
2	*-0.625*	*-0.22*	*-10*	*10*
3	*-1.0*	*-0.625*	*-20*	*20*
4	*-1.25*	*-1.25*	*-32*	*37*
5	*-1.0625*	*-1.50*	*-45*	*58*

With the execution of the Burmester curves two sets of curves will be generated: the first set of curves which derives by entering the five precision points and the first coupler angle and the second set of Burmester curves resulting from the five precision points and the second coupler angle. From both sets of curves, at the intersection of centre lines of each set of curves there is a possible point for the ground point while at the intersection of circle lines on each set of curves there are possible points for moving points of the mechanism.

Table 2. Coordinates of potential positions for the ground and moving points of the six-bar mechanism

Potential positions	*First set of curves* *Ground Points*	*Moving Points*	*Second set of curves* *Ground Points*	*Moving Points*
1	*0.32 ; -3.94*	*-1.55 ; 0.83*	*0.31 ; -2.20*	*0.33 ; -1.84*
2	*-1.80 ; 0.51*	*0.04 ; -1.30*	*1.70 ; -2.67*	*1.01 ; -1.11*
3	*-2.54 ; 2.17*	*-2.81 ; 1.47*	*1.50 ; -2.57*	*5.91 ; -1.83*
4	*-3.13 ; 1.72*	*-2.27 ; 2.52*		

The Table 2 shows seven potential solutions for the ground and moving points of the mechanism. However, not all of these points give solutions to the mechanism. For the given problem, by means of trial and error (designing the mechanism and animating it) the best solution was selected or rather identified. By superimposing the main dyads of both four-bars that are generated by the both sets of Burmester curves it can be clearly seen how would the each part of the Stephenson six-bar mechanism will look like and that in its initial position.

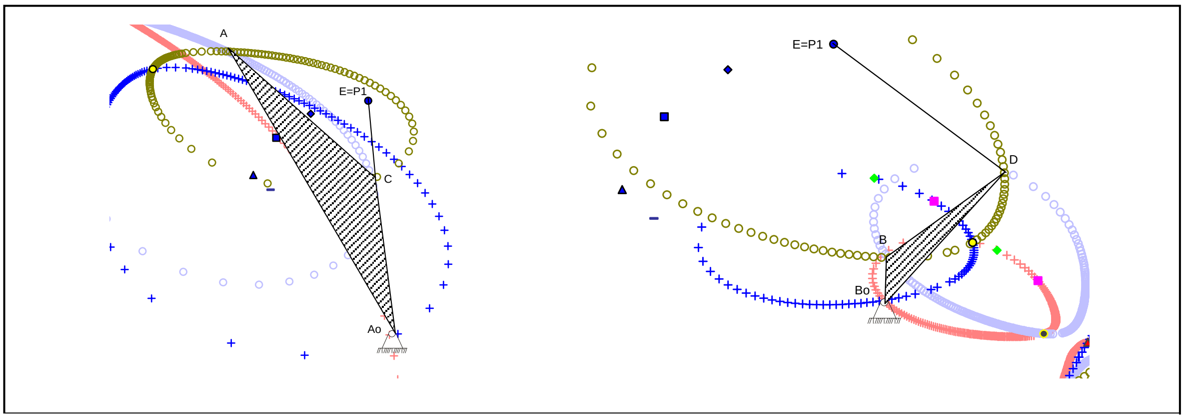

Figure 4: The superimposed main dyads of both sets of four-bars in its initial position over both sets of Burmester curves

By joining the both dyads of the four-bars well have a Stephenson-I- six-bar mechanism in its first position (Figure 5a). Using Demec computer program we can animate the mechanism and clearly see that it prescribes all five positions as required.

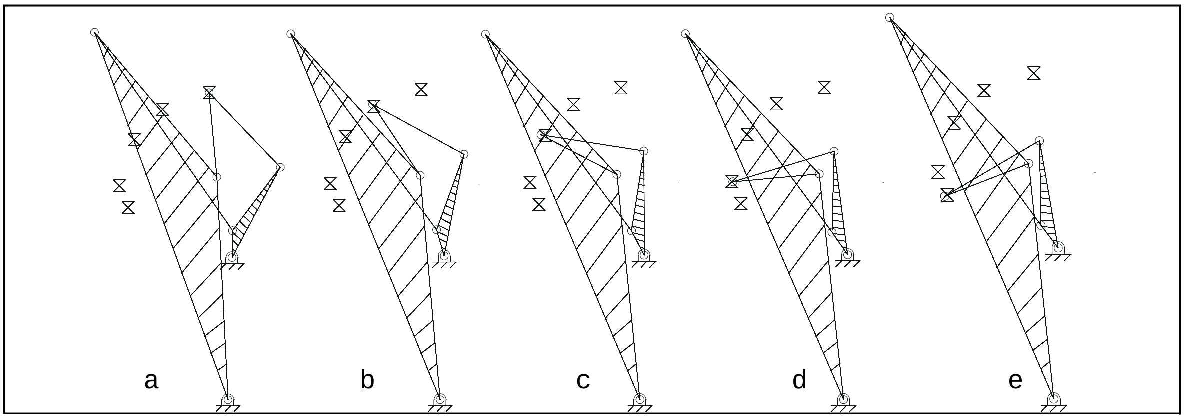

Figure 4: The Stephenson-I six-bar mechanism perscribing all five preciossion points as required

4. CONCLUSION

Although it looks as if it is very strait forward to design Stephenson-I six-bar mechanisms using Burmester curves and inversion method, this method so far was applicable for individual case only. Our future attempts will be geared towards finding a generalized solution of other types of six-bar mechanisms.

5. REFERENCES

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