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ABOUT THE ANALYTICAL CALCULUS OF LONG CYLINDRIC THIN WALL TUBE WITH AN UNIFORM INTERIOR LOAD BY TRANSFER-MATRIX METHOD

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ABSTRACT
We present the analytical calculus for a thin wall tube embebded at the right end, with an axisymmetrical uniform interior load using the Transfer-Matrix Method in according with the hypothese: the normally continue to remain invariable after deformation. The method consist in discretizing the tube in ring elements, each element have a state vector. With a general formula we determine the deformations in all sections of the thin wall tube. This is a simple formulation for a soft for an optimization program.
Key words: thin wall tube, the Transfer-Matrix Method, state vector, ring element, Dirac's fonction, Heaviside's fonction.

1. INTRODUCTION
The study of a cylindric and thinly tube is made with an hypothese: the normally continue to remain invariable after deformation [1]. For a cylindric long tube, the stresses and the deformations have a revolution's symetry.

2. THE BASIC EQUATIONS FOR A CYLINDRIC TUBE
We have a cylindric tube with a thin wall, the thickness is h and the radius R. We have an axisymerical uniform load at all interior surface of the tube q(x) (Figure 1) [1].

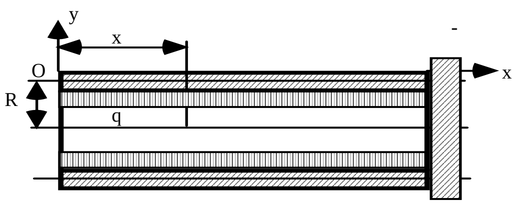

Figure 1.

The radius deplacement is v(x) and an angular deformation is ω(x) (Figure 2. a. and b.).

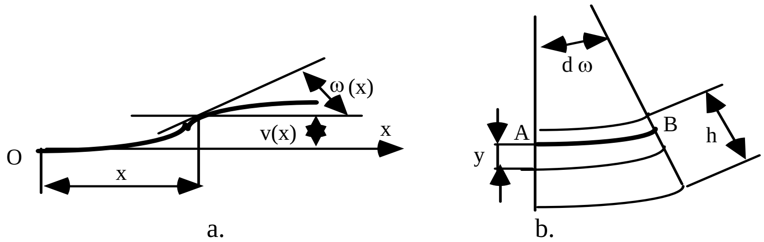

Figure 2.

We can write for the deformation ω(x):

$$\omega(x) = \frac{dv}{dx} \tag{1}$$

The orthogonal three-axial reference system is fixed: the axis Ox is after the cylindric tube's generator, the axis Oy is after a radius and the axis Oz is oriented as the system Oxyz will be a direct system. The line AB is at the distance y given the midle fiber (midle surface) of the tube's wall. The deformation of the line AB is:

$$\varepsilon_x = \varepsilon_x^{'} - y\frac{d\omega}{dx} \tag{2}$$

when ε_x is the deformation of the midle fiber. The circomferential deformation is:

$$\varepsilon_z = \frac{2\pi v(x)}{2\pi R} = \frac{v(x)}{R} \tag{3}$$

The stresses will be:

$$\begin{cases} \sigma_x = \dfrac{E}{1-\nu^2}\left[\varepsilon_x^{'} - y\dfrac{d\omega}{dx} + \nu\dfrac{v(x)}{R}\right] \\ \sigma_z = \dfrac{E}{1-\nu^2}\left[\dfrac{v(x)}{R} + \nu\left(\varepsilon_x^{'} - y\dfrac{d\omega}{dx}\right)\right] \end{cases} \tag{4}$$

We take an element of the cylindric tube: dx is the length and ds is the breadth (Figure 3.).

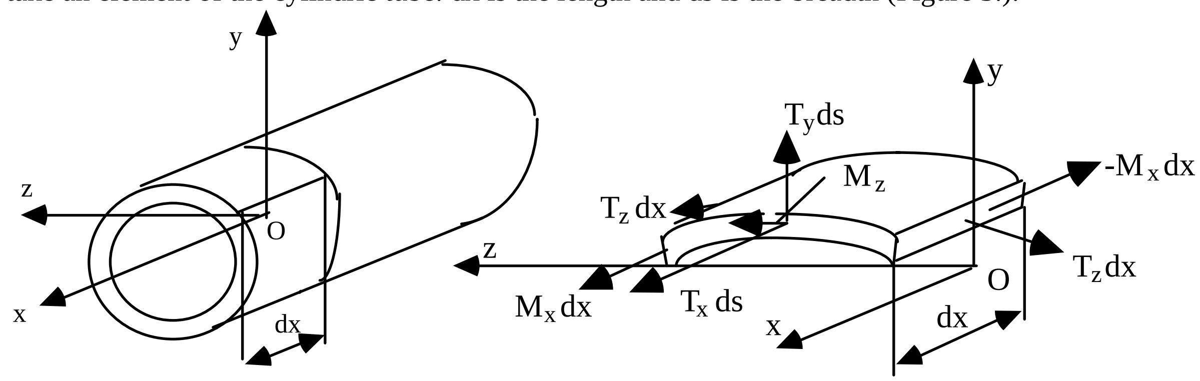

Figure 3.

when T_X, T_Y and T_Z are the cutting forces, M_X and M_Z the flexion moments.
We have:

$$\begin{cases} T_x = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \sigma_x dy \\ T_z = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \sigma_z dy \end{cases} \text{(6),} \quad \begin{cases} M_x = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \sigma_z y dy \\ M_z = -\int_{-\frac{h}{2}}^{+\frac{h}{2}} \sigma_x y dy \end{cases} \text{(7),} \quad \begin{cases} T_x = \dfrac{Eh}{1-\nu^2}\left(\varepsilon_x^{'} + \nu\dfrac{v}{R}\right) \\ T_z = \dfrac{Eh}{1-\nu^2}\left(\dfrac{v}{R} + \nu\varepsilon_x^{'}\right) \end{cases} \text{(8),} \quad \begin{cases} M_x = -\nu D\dfrac{d^2v}{dx^2} \\ M_z = D\dfrac{d^2v}{dx^2} \end{cases} \text{(9)}$$

when:

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{10}$$

After writing the balance equations of the element dx we have the deformations equation of the cylindric long tube:

$$\frac{d^4v(x)}{dx^4} + 4\alpha^4 v(x) = \frac{q(x)}{D} - \nu\frac{T_x}{RD} \tag{11}$$

where:

$$4\alpha^4 = \frac{Eh^3}{R^2 D} \qquad (12)$$

The general solution of the equation (11) is:

$$v(x) = A\cos\alpha x ch\alpha x + B\sin\alpha x ch\alpha x + C\cos\alpha x sh\alpha x + D\sin\alpha x sh\alpha x + v^*(x) \qquad (13)$$

where v*(x) is the particular solution for the equation with second member.
The basic diferential equations are:

$$\begin{cases} \dfrac{d^4 v(x)}{dx^4} + 4\alpha^4 v(x) = \dfrac{q(x)}{D} - \nu\dfrac{T_x}{RD} \\ \omega(x) = \dfrac{dv(x)}{dx} \\ M_z = D\dfrac{d^2 v(x)}{dx^2} \\ M_x = -\nu D\dfrac{d^2 v(x)}{dx^2} = -\nu M_z \\ T_y = -\dfrac{dM_z}{dx} \end{cases} \qquad (14)$$

3. THE TRANSFER-MATRIX OF THE CYLINDRIC TUBE WITH AN UNIFORM LOAD AT ALL THE INTERIOR SURFACE

We consider a long cylindric tube, embebded at the right end and free in the origin, challanged with an uniform axisymetrical load at all its interior surface (Figure 1.).
The state vector at the point x is:

$$\{U(x)\} = \{v(x), \omega(x), M_z(x), T_y(x)\}^T \qquad (15)$$

The passage between the state vector at the origin and the state vector at the point x is made by a Transfer-Matrix, we can write in the point x:

$$\begin{Bmatrix} v(x) \\ \omega(x) \\ M_z(x) \\ T_y(x) \end{Bmatrix} = \begin{bmatrix} a_1 & \dfrac{a_2 + a_3}{2\alpha} & \dfrac{a_4}{2\alpha^2 EI} & -\dfrac{a_2 - a_3}{4\alpha^3 EI} \\ \alpha(a_3 - a_2) & a_1 & \dfrac{a_2 + a_3}{2\alpha EI} & -\dfrac{a_4}{2\alpha^2 EI} \\ -2\alpha^2 EIa_4 & \alpha EI(a_3 - a_2) & a_1 & -\dfrac{a_3 + a_2}{2\alpha} \\ 2\alpha^3 EI(a_3 + a_2) & \alpha^2 EIa_4 & -\alpha(a_3 - a_2) & a_1 \end{bmatrix} \begin{Bmatrix} v_0 \\ \omega_0 \\ M_{z0} \\ T_{y0} \end{Bmatrix} +$$

$$+ \begin{Bmatrix} \dfrac{1}{4\alpha^3 EI}\displaystyle\int_0^x (a_2 - a_3)[x - t]q(t)dt \\ \dfrac{1}{2\alpha^2 EI}\displaystyle\int_0^x a_4[x - t]q(t)dt \\ \dfrac{1}{2\alpha}\displaystyle\int_0^x (a_2 + a_3)[x - t]q(t)dt \\ -\displaystyle\int_0^x a_1[x - t]q(t)dt \end{Bmatrix} \qquad (16)$$

For the Figure 1., we have the load density, with Dirac's and Heaviside's fonctions:

$$q(x) = qY(x) \qquad (17)$$

For x=L, in the right edge, we can write:

$$\begin{Bmatrix} v(L) \\ \omega(L) \\ M_z(L) \\ T_y(L) \end{Bmatrix} = \begin{bmatrix} a_1[L] & \frac{(a_2+a_3)[L]}{2\alpha} & \frac{a_4[L]}{2\alpha^2 EI} & -\frac{(a_2-a_3)[L]}{4\alpha^3 EI} \\ \alpha(a_3-a_2)[L] & a_1[L] & \frac{(a_2+a_3)[L]}{2\alpha EI} & -\frac{a_4[L]}{2\alpha^2 EI} \\ -2\alpha^2 EIa_4[L] & \alpha EI(a_3-a_2)[L] & a_1[L] & -\frac{(a_3+a_2)[L]}{2\alpha} \\ 2\alpha^3 EI(a_3+a_2)[L] & \alpha^2 EIa_4[L] & -\alpha(a_3-a_2)[L] & a_1[L] \end{bmatrix} \begin{Bmatrix} v_0 \\ \omega_0 \\ M_{z0} \\ T_{y0} \end{Bmatrix} +$$
$$+ \begin{Bmatrix} -\frac{q}{4\alpha^4 EI}(a_1[L]-1) \\ \frac{q}{4\alpha^3 EI}(a_2-a_3)[L] \\ \frac{q}{2\alpha^2}a_4[L] \\ -\frac{q}{2\alpha}(a_2+a_3)[L] \end{Bmatrix} \tag{18}$$

We have the edges's conditions:

$$M_{x0} = 0, \quad T_{y0} = 0, \quad v(L) = 0, \quad \omega(0) = 0 \tag{19}$$

The relation (18), with (19), is:

$$\begin{Bmatrix} 0 \\ 0 \\ M_z(L) \\ T_y(L) \end{Bmatrix} = \begin{bmatrix} a_1[L] & \frac{(a_2+a_3)[L]}{2\alpha} & \frac{a_4[L]}{2\alpha^2 EI} & -\frac{(a_2-a_3)[L]}{4\alpha^3 EI} \\ \alpha(a_3-a_2)[L] & a_1[L] & \frac{(a_2+a_3)[L]}{2\alpha EI} & -\frac{a_4[L]}{2\alpha^2 EI} \\ -2\alpha^2 EIa_4[L] & \alpha EI(a_3-a_2)[L] & a_1[L] & -\frac{(a_3+a_2)[L]}{2\alpha} \\ 2\alpha^3 EI(a_3+a_2)[L] & \alpha^2 EIa_4[L] & -\alpha(a_3-a_2)[L] & a_1[L] \end{bmatrix} \begin{Bmatrix} v_0 \\ \omega_0 \\ 0 \\ 0 \end{Bmatrix} +$$
$$+ \begin{Bmatrix} -\frac{q}{4\alpha^4 EI}(a_1[L]-1) \\ \frac{q}{4\alpha^3 EI}(a_2-a_3)[L] \\ \frac{q}{2\alpha^2}a_4[L] \\ -\frac{q}{2\alpha}(a_2+a_3)[L] \end{Bmatrix}$$

(20)T

he equations's system for the calculus of the state vector in the origin is:

$$\begin{cases} a_1[L]v_0 + \frac{(a_2-a_3)[L]}{2\alpha}\omega_0 = \frac{q}{4\alpha^4 EI}(a_2[L]-1) \\ \alpha(a_3-a_2)[L]v_0 + a_1[L]\omega_0 = \frac{q}{4\alpha^3 EI}(a_2-a_3)[L] \end{cases} \tag{21}$$

or:

$$\begin{cases} a_{11}v_0 + a_{12}\omega_0 = b_1 q \\ a_{21}v_0 + a_{11}\omega_0 = b_2 q \end{cases} \tag{22}$$

with the solution:

$$\begin{cases} v_0 = q\frac{a_{12}b_2 - a_{11}b_1}{a_{12}a_{21} - a_{11}^2} \\ \omega_0 = q\frac{a_{21}b_1 - a_{11}b_2}{a_{12}a_{21} - a_{11}^2} \end{cases} \tag{23}$$

and thus, we have all the elements of the origin's state vector and after, we can calculate all the state vectors for all the sections of the cylindric long tube.

4. CONCLUSIONS

The Transfer-Matrix Method is a simple formulation for a soft for an optimization program, which can be used in a lot of industrial domain and this method give the liceance to calculate the deformations and the stresses using the Dirac's and Heaviside's fonctions and operators.

5. REFERENCES

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