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TOOTH-ROOT STRESS DISTRIBUTION RELATED TO GEAR RIM AND WEB THICKNESS

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ABSTRACT
The paper deals with the research into the effects of rim and web thickness on the stress distribution along a spur tooth facewidth in the root area. Under the assumption of machining errors and assembly misalignments absence, the contribution of certain gear structure to non-uniform stress distribution has been analysed. The investigation has been performed by means of the 3D FEM and the developed model of pinion-wheel system. The chosen values of rim thickness cover the corresponding ones for thin rim, and the values of web thickness are in accordance with the recommendations in practice. Two different gear structures have been taken into consideration i.e. the cases of middle and offset web position.
Keywords: spur gear, tooth-root stress, rim, web

1. INTRODUCTION

A good prediction of gear stresses by taking into account actual teeth support, has to be fulfilled for the achievement of gear weight reduction. Additionally, the gear teeth design has been well elaborated and covered by the recommendations in the standards, unlike the design of gear blank.

The structure of thin-rimmed gear contributes to the resulting tooth deformation and effects the load and stress distribution along the tooth facewidth. It means that the shape of teeth foundation itself, causes non-uniform load distribution, regardless of possible gear pair errors.

Thin-rimmed gears have found their application by offering weight and space reduction. The issue of stress calculation and strength evaluation related to these gears have been actual and it is not adequately covered in the proposal of standards.

The insight into a tooth-root stress state of thin-rimmed gear has been accomplished gradually, and the research results have pointed to the differences in comparison with a solid gear state of stress. Numerous studies started with 2D stress calculation and by use of simple one - tooth models of gear without web [1, 2, 3].

The application of 3D numerical calculation methods enabled more complete stress analysis of thin-rimmed gear considering various gear structures. The attention has been devoted to develop and improve numerical models for the proper simulation of actual stress state. More complex single gear models [4, 5, 6, 7] , a gear pair models [8, 9, 10] and a gear system models [11], were established.

In spite of the gear contact zone through which the load is applied on the wheel, it has been often assumed concentrated or artificially distributed load when developing the gear models [4, 6]. An additional effort has been done in order to take this fact into account, which resulted with different gear models related to the treatment of contact between pinion and wheel [8, 9, 11].

In this paper thin-rimmed stress behaviour in the root area is determined and analysed considering actual geometry relations of gear blank i.e. the rim, web and hub. The effect of thin rim thickness and web thickness of differently positioned web, upon the stress distribution along the facewidth i.e. the edge effect, have been the subject of analysis. As the edge effect varies in relation to the facewidth value, the gear facewidth value is chosen corresponding to maximum edge effect at the considered engagement position, in accordance with [8].

2. GEAR STRUCTURE AND 3D FEM MODEL

The chosen structures of thin-rimmed gear blank with middle web position and offset web position adopted through the calculations belong to the wheel that is always mating with a solid gear.
Both mating pinion and wheel are standard involute ones with the same tooth geometrical parameters, as shown in Tab. 1.

Table 1. Data of spur gear pairs.

Number of teeth	z_1=z_2	20			
Module	m	10 mm			
Pressure angle	α	20°	Material properties	Module of elasticity E	$2{,}1 \cdot 10^5$ N/mm^2
Profile shift coefficient	x	0		Poisson`s ratio ν	0,3
Facewidth	b	100 mm	Loading torque	T	376 Nm
Rim thickness of wheel	s_R	30; 20; 15; 10 mm			
Web thickness	s_w	10, 20, 30, 40 mm			

For the rim thickness are adopted values of s_R =3*m*; 2*m*; 1,5*m* and 1*m* (*m* – module) that are corresponding to thin rim, as recommended by ISO [12]. The gear facewidth value in all cases is *b*=10*m*. The web thickness takes the values of s_w = 0,1*b*; 0,2*b*; 0,3*b* and 0,4*b*, regardless of the web case position. The offset web plane of symmetry is always at the same position along the tooth facewidth.
The models of pinion and wheel are established at the outer point of single pair tooth contact. A detailed description of the applied 3D FEM numerical model (mesh, constraints, loading, contact simulation) can be found in [13], while the material properties and value of the loading torque are presented in Tab. 1.

3. RESULTS ANALYSIS, DISCUSSION AND CONCLUSIONS

The values of tooth-root tangential stress component σ_{1max} i.e. principal stress on tensile side of a wheel-loaded tooth, are separated approximately along the tooth profile position where maximum stress occurs.
For the sake of completeness and comparison, the tooth-root stress distribution along the facewidth is presented for a solid and thin-rimmed gear without web, too. Fig. 1 presents this stress distribution in the form of contour map. The tooth-root stress distribution along the facewidth shows the increment and movement of maximum stress towards the both tooth edges, as the rim thickness decreases.
In Fig. 2 a, b and Fig. 3 a, b, the contour maps are developed along the tooth facewidth for the chosen range of rim thickness and certain web thickness, and for offset and middle web position, respectively.
In relation to thin-rimmed gear with web, if the rim thickness decreases, maximum tooth-root stress increases, regardless of web thickness and its position along the facewidth.
The longitudinal stress distribution for the case of end web position shows maximum stress position near the tooth end where web is placed (Fig. 2), unlike longitudinal stress distribution for the cases of a solid gear, thin-rimmed gear without web (Fig. 1) and with middle web position (Fig. 3). As the rim thickness decreases, maximum stress position moves towards the both tooth ends, thus increasing maximum stress area.
The deviation of maximum tooth-root stress of the thickest rim under consideration (s_R/m=3) from the corresponding stress of the thinnest rim (s_R/m=1), is strongly influenced by the web thickness. In the case of offset web position, this deviation is about 20% for the web thickness value of s_W=0,1 *b*, and decreases for the web thickness of s_W=0,4 *b* to about 7%.
For middle web position, actual web thickness more contributes to the rigidity of teeth support, and maximum tooth-root stress is more effected by it than in the case of offset web. When the web thickness is equal s_W=0,4 *b* maximum stress of the thinnest rim (s_R/m=1) increases for about 13%, and for s_W=0,1 *b* for about 28%, related to the thickest rim (s_R/m=3) under consideration.

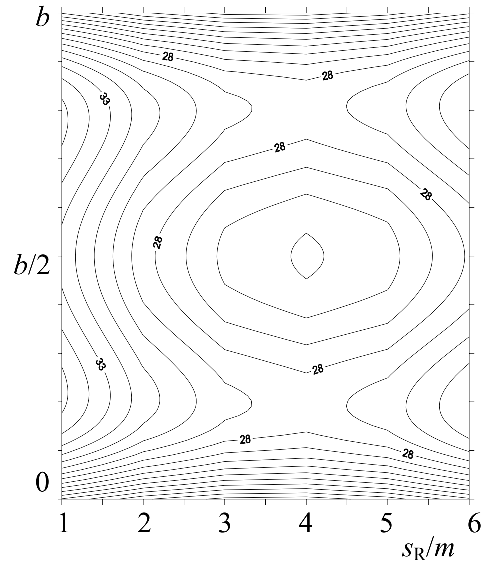
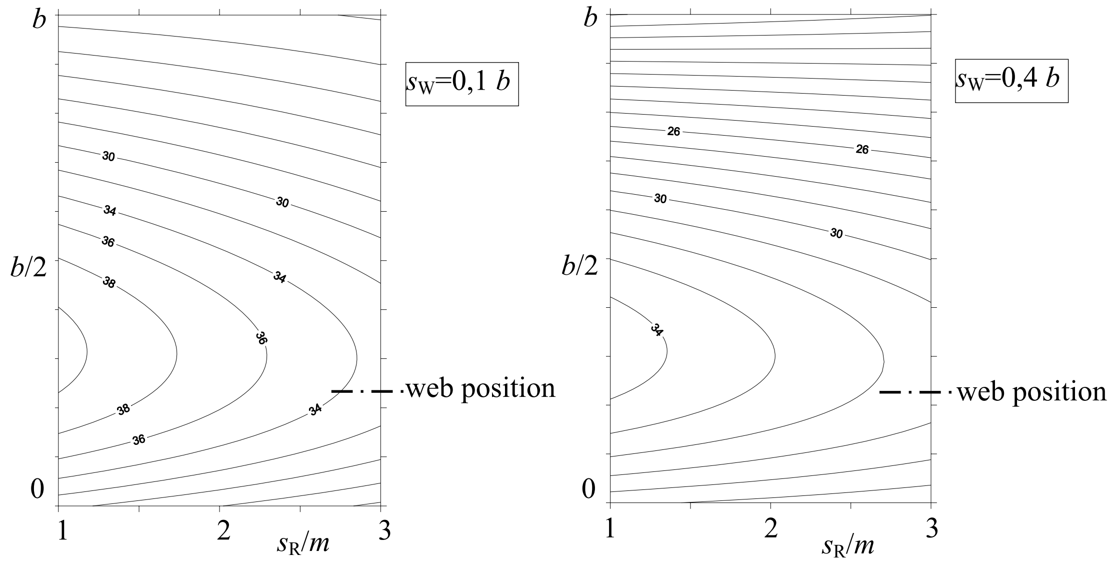


Figure 1. Maximum tooth-root stress distribution along the facewidth for a solid gear and thin-rimmed gear without web.



a) b)

Figure 2. Maximum tooth-root stress distribution along the facewidth for thin-rimmed gear with offset web position and different web thickness, a) s_W=0,1 *b,* b) s_W=0,4 *b*

In central area of tooth, axial rigidity determined by actual gear structure, i.e. the rim and web thickness and web position along the facewidth, causes the modus of load distribution and the corresponding edge effect on the stress.

The tooth edge effect is followed through the values of ratio $\sigma_{1max} / \sigma_{1e}$ established between maximum tooth-root stress σ_{1max} and the stress σ_{1e} at the tooth edge. For the case of offset web position two values of this ratio are determined, embracing the tooth-root stress at the edge far from and near the web.

The obtained values of the ratio shows that the influence of gear rim thickness on the edge effect is predominant, compared with the web thickness one, as the ratio values differ slightly for the web thickness under consideration, regardless of web position. Maximum edge effect increment going to the thinnest rim, is about 20% for the case of middle web position and web thickness s_W=0,4 *b*.

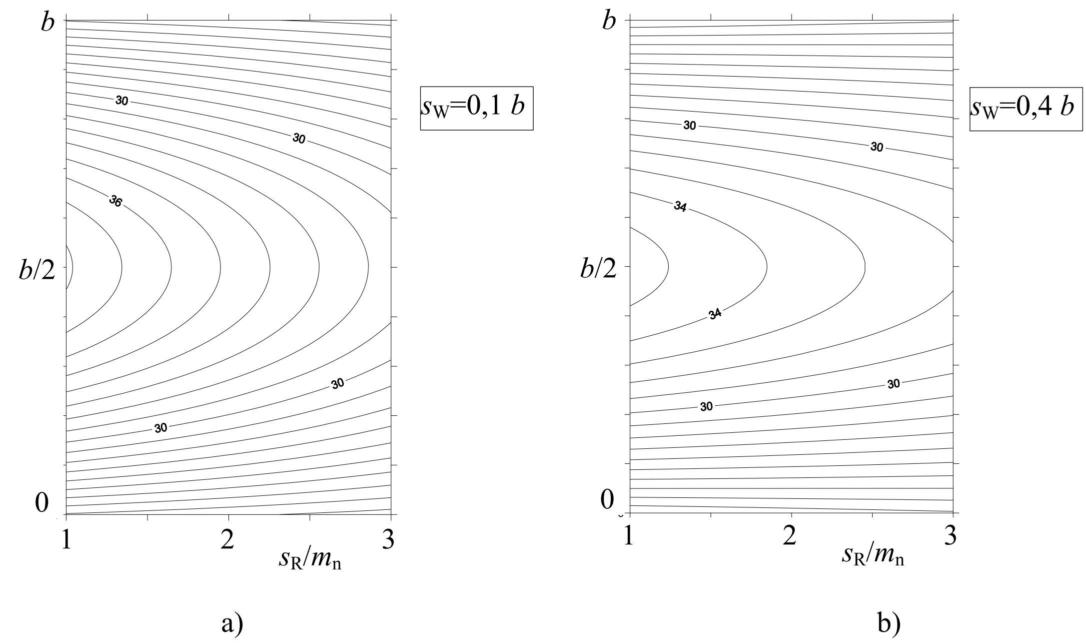


Figure 2. Maximum tooth-root stress distribution along the facewidth for thin-rimmed gear with middle web position and different web thickness, a) s_W=0,1 *b,* b) s_W=0,4 *b*

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