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MOTIONS AND DISPLACEMENTS IN THE EUCLIDEAN SPACE

Suad Hasanbegović
University of Sarajevo,
Mechanical Engineering Faculty
Vilsonovo šetalište 9, 71000 Sarajevo
Bosnia and Herzegovina

Ahmet Hadžipašić, Amra Talić-Čikmiš
University of Zenica, Mechanical
Engineering Faculty
Fakultetska 1, 72000 Zenica
Bosnia and Herzegovina

ABSTRACT
The measure of strain the uniform deformed zone in the process is defined with the change of the distance between anyone two points of this zone. The reduction the measure of strain to the unit of the initial distance of these points is obtained the deformation of the one - dimensional element of this zone. Accordingly, the deformation of some zone can be defined in the space which has defined metric, i.e. in the metric space.
On the example the extension of the test bar is demonstrated that the change of the distance between the points is the function only the initial distance of these points. Therefore, the change the vector position between the points can be described with linear vector functions.
In this paper, with the observation the change of the vector position between anyone two points, is defined strain of the uniform strained zone. For strain defined in this manner one the fixed point is necessary and she is independent of coordinate system.
Key words: Euclidean space, uniform strain, strain vector.

1. INTRODUCTION

In the vector space of the oriented segments (the vectors) it is materialized the connection between the geometrically structure and the vector space which is the algebraic structure. This connection is very much in the application near an analysis stress and strain whether in elasticity or plasticity region. The vector space, the algebraic structure, is convenient for the calculation, and common three-dimensional space, the geometrically structure, is convenient for the interpretation geometrically and physical values.
In affine space the position of a point is complete definite with one vector if it is knower the initial point (pole) from which the vectors are deposited. Such vectors which define the position of the point to relation on some point (pole) are called the position vectors. The elementary coordinate vectors in this vector space can have the different measure introduced the arbitrary length as the unit, i.e. the units in the coordinates of the elementary coordinate vectors must not be the equal length.
The Euclidean space is the metric space, i.e. such space in which for the length of the coordinate vectors can be taken the equal units. This means that from everybody point in everybody directions can be measured with the equal units, such what it is take that is case in the simple perceptible space of the three-dimensional. In such space on the base of Pythagoras' theorem and the theorem of cosine can be defined the distance between arbitrary two points. Then are, certainly, and the corners between the elementary coordinate systems define, and if it is take that are right then this leads to Cartesian' coordinate system in this space. In such space is defined the vector of deformation.

2. THE VECTOR SPACE OF THE ORIENTED SEGMENTS

In the elementary course of the algebra and the analytical geometry, using the stereometry of the simple three-dimensional space E^3, it is constructed the vector space $\mathbf{X}_0 = \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ of the oriented segments, which is intimate engaged for the space E^3. Besides the following conditions are satisfied [1]:

1° the every pair points $P,\ Q \in E^3$ is affixed one and only one the vector $\mathbf{x} \in \mathbf{X}_0$. For all that is the length of vector equal to the distance between the points P and Q.

2° if are $P,\ Q,\ R$ three points from E^3 and the vectors: the $\mathbf{x}$ vector held to the oriented segment $\overrightarrow{PQ}$, the $\mathbf{y}$ vector held to the oriented segment $\overrightarrow{QR}$, the $\mathbf{z}$ vector held to the oriented segment $\overrightarrow{PR}$, then is

$$\mathbf{z} = \mathbf{x} + \mathbf{y}. \tag{1}$$

This is result the addition of the oriented segments ″ according to rule of triangle″, i.e. the consequence of equal

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}. \tag{2}$$

3° for arbitrary point $P \in E^3$ and arbitrary vector $\mathbf{x} \in \mathbf{X}_0$ exists one and only one point $Q \in E^3$ with the property that is the vector $\mathbf{x}$ annexed the pair of the points $\overrightarrow{PQ}$, i. e. that the $\mathbf{x}$ is the class of the equivalence generalized with the oriented segment $\overrightarrow{PQ}$.

3. AFFINE SPACE

If by $\mathbf{X}_0 = \{\mathbf{x},\ \mathbf{y},\ \mathbf{z}\}$ is denoted the three-dimensional real vector space, by E non empty set which elements are called points and by $E \times E$ set everything ordered pairs $(P,\ Q)$ of points $P, Q \in E$. In this case the two pairs $(P_1,\ Q_1)$ and $(P_2,\ Q_2)$ are considered equal, if is $P_1 = P_2$ and Q1 = Q_2.

The set E is called the three-dimensional affine space, the elements of him points of affine space, if exists function $\tau : E \times E \rightarrow \mathbf{X}_0$ by following two characteristics [1]:

i) $\tau(P,\ Q) + \tau(Q,\ R) = \tau(P,\ R)$ $(P,\ Q,\ R \in E)$ and

ii) for all point $P \in E$ and the vector $\mathbf{x} \in \mathbf{X}_0$ exists one and only one point $Q \in E$ by characteristics that is $\mathbf{x} = \tau(P,\ Q)$.

Affine space is, therefore, trio of the set E, the vector space $\mathbf{X}_0$ and the mapping τ.

4. EUCLIDEAN SPACE

The special case of affine space is Euclidean space, which is obtained up the assumption that is the $\mathbf{X}_0$ vector Euclidean space, i.e. the real unitary space [1]. Consequently, the tree dimensional Euclidean space do the set E, the unitary tree dimensional real space $\mathbf{X}_0$ and mapping $\tau\colon E\ x\ E \rightarrow \mathbf{X}_0$ by the properties

i) $\tau(P,Q) + \tau(Q,R) = \tau(P,R)$ $(P,Q,R \in E)$ and

ii) for every point P $\in E$ and the vector $\mathbf{x} \in \mathbf{X}_0$ exists one and only one point $Q \in E$ with properties that is

$$\mathbf{x} = \tau(P,\ Q).$$

5. METRIC SPACE

The non empty S is called metric space if is all ordered pair $P,\ Q \in S$ affixed the number $d\,(P,\ Q)$, d is a distanced function, and if this affix has properties [1]:

1° $d\,(P,Q) \geq 0$ for every $P,Q \in E$; $d\,(P,Q) = 0$ next and only next if is $P = Q$.

2° $d\,(P,\ Q) = d\,(Q,\ P)$.

3° $d\,(P,R) \leq d\,(P,Q) + d\,(Q,R)$, for every $P,Q,R \in E$.

In this way is conclude that Euclidean space is metric space, and distanced function d in a rectangular coordinate system is given by

$$d\,(P,\ Q) = \left[\sum_{k=1}^{3}(x_k - y_k)^2\right]^{1/2}. \tag{3}$$

The fact that E is metric space enabled in his convergence of sequence points and a concept of continuous function defines on E [1].

6. VELECITY AND DISPLACEMENT OF POINT IN EUCLIDEN SPACE

The path line of a material point which is displaced with time in space is a curveΓ [Figure 1]. Let by t denoted time and by the **r** (t) position vector of this point at time t. For a time Δt a point to passes from the position P_0 in the position P_1. The vector

$$\mathbf{v}_s = \frac{\vec{r}(t_0 + \Delta t) - \vec{r}(t_0)}{\Delta t} \tag{4}$$

is called average or mean velocity of motion point at time Δt. For $\Delta t > 0$ vector (11) has sense from P_0 to P_1, i.e. to sense future position of a material point. The vector

$$\overrightarrow{P_0P_1} = \mathbf{u} = \mathbf{v}_s\,\Delta t = \mathbf{r}\,(t_0 + \Delta t) - \mathbf{r}\,(t_0) \tag{5}$$

is called the displacement of a material point at time Δt. With this vector (direction) is approximated a portion of the curve Γ between the points P_0 and P1.

The limit value (limiting) of the men velocity when $\Delta t \to 0$ is called the velocity of the material point at the time t_0. According to definition the vector

$$\mathbf{v}\,(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} \tag{6}$$

is the velocity of a material point at moment t. When Δt tends zero, then the point P_1 (Figure 1) tends to the points P_0 according to curve Γ, and the direction which goes along the points P_0 and P_1 rotates about the point P_0 and on the frontier crosses to the tangent on the curve Γ at the point P_0. Since this tangent crosses across the point $P_0 = \mathbf{r}\,(t_0)$ and that in to she the vector **r**' (t_0) lies this tangent is given by [3]

$$t \mapsto \mathbf{r}\,(t_0) + (t - t_0)\,\mathbf{r}'\,(t_0). \tag{7}$$

In fact, according to definition of the direction (7) is called tangent on the curve Γ at the point $P = \mathbf{r}\,(t_0)$. The vectorr

$$\mathbf{u} = \mathbf{v}\,(t)\,dt = \mathbf{r}\,(t_0 + dot) - \mathbf{r}\,(t_0) \tag{8}$$

is called the displacement of a material point across to tangent on the path line of she for an infinite little interval of time, It is said frequent that is this increment displacement of a material points at the moment t_0.

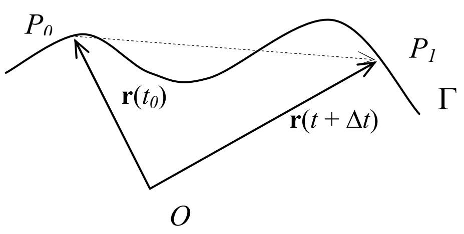

Figure 1. Velocity motion of a material point

7. DEFORMATION VECTOR IN EUCLEDEN SPACE

With a motion and a deformation of deformable body, or a part of this body, under the uniform deformation, a material point Q of its, from country the material point P, at momentum t_0 matches to the geometrical point Q_0, and after an interval of time to be in a geometrical point Q_t. If is looked every points from the region E' respectively the space E in the given time t, than by [4]

$$Q_0 \to Q_t = a_t\,(Q_0)$$

a transformation a_t is given of space in only myself. She arbitrary point Q_0 transports to a point Q_t.

For a fixed point O, is

$$\tau\,(O,Q_t) = \mathbf{u}\,(t) + A_1\,(t)\;\tau\,(O,Q_0), \quad \mathbf{u}\,(t) = \tau\,(O,O') \tag{9}$$

for every $Q_0 \in E$ ($Q_0 \in E'$), where the vector **u** (t) and lineal operator A_1 (t) are uniform determine with motion, uniform deformation and the fix point O. This made for one t, can be made for every t. Then are obtained the functions

$$t \to \mathbf{u}\,(t) \;\text{ and }\; t \to A_1(t).$$

For that the wish that is at initial moment $Q_0 = Q_t$ for every $Q_0 \in E$ leads to this that is $a_0\,(Q_0) = Q_0$ for every Q_0, i.e. a_0 is identity. Then is $\mathbf{u}\,(0) = \mathbf{0}$ and $A_1\,(0) = I$.

How is $t \to Q_t$ the continuous mapping for every point $Q \equiv Q_0$, this $t \to t_s$ pulls that Q_t convergences to point Q_s. Then and $\tau\,(O,Q_t) \to \tau\,(O,Q_s)$, because is

$$|\,\tau\,(O,Q_t) - \tau\,(O,Q_s)\,| = |\,\tau\,(Q_s,Q_t)\,| = d\,(Q_s,Q_t)$$

and $d(Q_s, Q_t) \to 0$ when $t \to s$. Accordingly is the function $t \to \tau(O,Q_t)$ continuous. For the material point $P \equiv O$ is obtained continuous of the function $t \to \mathbf{u}(t)$, and from here and from

$$A_1(t)\ \tau(O,Q_0) = \tau(O,Q_t) - \mathbf{u}(t)$$

flowing continuous of the function $t \to A_1(t)$. Accordingly, to motion and deformation of space belongs the one- parametric family $t \to a(t)$ by properties that is a_0 identity, i.e. the continuous one-parametric family for which is $t \to a(t)$, a_0 is identity, inductive motion and deformation of space. This motion and deformation is given by

$$\tau(O, Q_t) = \mathbf{u}(t) + A_1(t)\ \tau(O, Q_0) \quad (Q_0 \in E),$$

where $O \in E$ is fix point.

In this way a motion and uniform deformation, with selection fix point $O \in E$, is described by the displacement vector $\mathbf{u} \in \mathbf{X}_0$ and lineal operator A_1: $\mathbf{X}_0 \to \mathbf{X}_0$ by the expression (Figure 2)

$$\tau(O,Q_1) = \mathbf{u} + A_1\ \tau(O,Q_0) \quad (10)$$

for every $Q_0 \in E$ at interval $0 \leq t \leq 1$.

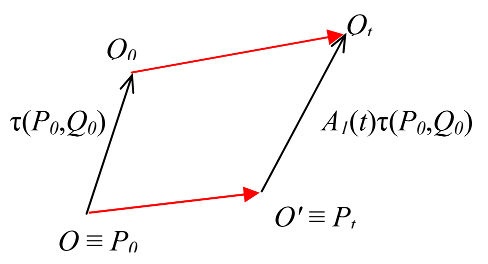

Figure 2. Motion and deformation

The vector

$$\Delta\tau(O, Q_0) = A_1\ \tau(O, Q_0) - \tau(O, Q_0) \Rightarrow$$
$$\Delta\tau(O, Q_0) = (A_1 - E)\ \tau(O, Q_0) = A\ \tau(O, Q_0) \quad (11)$$

is marked as the deformation vector. It is obtained with application lineal operator $A = (A_1 - E)$: $\mathbf{X}_0 \to \mathbf{X}_0$ to the vector $\tau(P_0, Q_0)$. He is equal to differentiations the displacements of the material points P and Q, i.e. is worth

$$A\ \tau(P_0, Q_0) = \Delta\,\mathbf{u} \qquad (P,Q \in E). \quad (12)$$

This vector (12) characterizes state of uniform deformation at country material point P.

On the base (10), (11) and (12) the displacement **u**' material point Q is obtained, for interval of time $\Delta t = t_1 - t_0$, i.e.

$$\mathbf{u}' = \tau(O,Q_1) - \tau(O,Q_0) = \mathbf{u} + \Delta\mathbf{u}, \quad (13)$$

with indirectly of the change position vector of a material point Q in the space E (E') to respect the fix point O or in the function of the displacement **u** of the material point P.

8. CONCLUSION

The deformation vector is defined as the change of position between two points in progress motion and deformation. The sole conditional for selection of the observed points is that belong to zone which is deformed uniform [5]. With the experimental results, in the process simple tensile test, it is shown that is the change of the distance between selected points the function exclusively of the initial distance these points. On the base these results it is shown [4] that deformation vector is the linear vector function of the position vector between the points in the initial state (non deformed state).

In this base, in the Euclidean space for uniform deformed region, it is described montions and deformation, against the selection the fixed point, with the displacement vector and linear operator. With the deformation vector characterize the deformation state the surroundings of a material point. The deformation vector is defined without call on the coordinate system. At all events that is of this access possible and in corresponding coordinate system.

9. REFERENCES

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