10th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2006, Barcelona-Lloret de Mar, Spain, 11-15 September, 2006

SYMMETRIZING CIRCUITRY UNDER NON-SINUSOIDAL CONDITIONS

dr.eng. CZIKER Andrei, prof. dr.eng. CHINDRIS Mircea
Technical University of Cluj-Napoca
Faculty of Electrical Engineering
Power Systems Departments
15, C. Daicoviciu Str. 400020, Cluj-Napoca
Romania
Andrei.Cziker@eps.utcluj.ro

ABSTRACT
LV distribution networks operate in unbalanced conditions and supply many non-linear loads. Taking into account the harmful effects that accompany these real conditions, the balancing of electrical networks is an important challenge. Circuitries consisting of supplementary reactive elements represent a common solution for industrial customers. The paper deals with the design of these elements for loads having a reduced value of power factor in harmonic polluted networks.
Keywords: unbalanced loads, harmonic polluted power systems, balancing circuitries

1. INTRODUCTION
Industrial distribution networks are supposed to operate in sinusoidal balanced conditions. However the practice has demonstrated that these networks supply a great number of high-power single-phase receivers and, as a result, they work severely unbalanced. The unbalance of line current system produces some unlikable effects [1]:

- the unbalance of the supply voltages, and as a result, the malfunction of three-phase rotating motors;
- excessive heating of alternators, saturation of transformers and ripple in rectifiers;
- supplementary power losses in phase and neutral lines;
- diminishing of power factor.

A common solution to mitigate the above mentioned effects is the compensation of unbalanced loads that is, the conversion of a single-phase receiver into a balanced three-phase load by the means of supplementary reactive elements suitable connected to the receiver. So far, the value of these elements is calculated for a sinusoidal shape of the supply voltage and is closely related to the parameters of the load equivalent circuit.

Nowadays, the electrical power systems are severely harmonic polluted and depending on the network behaviour, more or less dominant voltage distortions exist in industrial power systems. Under these circumstances, the existing design relationships are no more accurate and the obtained values of reactive elements do not guarantee the network balance. The main aim of the paper is to calculate the supplementary reactive elements included in the balancing circuitry (when the power factor is less than 0.86) if the supply voltages are harmonic distorted. The corresponding mathematical relationships are obtained and a new parameter to asses the efficiency of the symmetrizing circuitry in harmonic polluted power systems is proposed.

2. BALANCING CIRCUITRY IN NONSINUSOIDAL CONDITIONS

If the load power factor is less than 0.86, the balancing circuitry consists in two reactors Δ connected with the load (expressed by its equivalent impedance $\underline{Z}_0$) - Figure 1 [1]; let us suppose that the customer is supplied by a balanced non-sinusoidal voltage system that can be expressed as

$$u_1 = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot \sin(k \cdot \omega \cdot t); \tag{1.a}$$

$$u_2 = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot \sin\left(k \cdot \omega \cdot t - k \cdot \frac{2 \cdot \pi}{3}\right); \tag{1.b}$$

$$u_3 = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot \sin\left(k \cdot \omega \cdot t + k \cdot \frac{2 \cdot \pi}{3}\right), \tag{1.c}$$

where k is the harmonic order (in power systems $n \le 40$);
U_k - the rms. value of the k harmonic voltage.

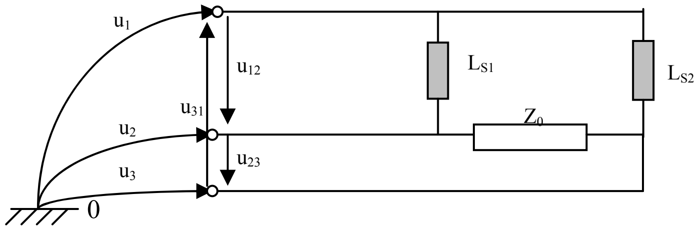

Figure 1. Balancing circuitry, for single phase load with less power factor, using two reactors

Considering the system (1), the line voltages are:

$$u_{12} = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \sin\left(k \cdot \omega \cdot t - k \cdot \frac{\pi}{3} + \frac{\pi}{2}\right); \tag{2.a}$$

$$u_{23} = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{2 \cdot \pi}{3}\right) \cdot \sin\left(k \cdot \omega \cdot t - \frac{\pi}{2}\right); \tag{2.b}$$

$$u_{31} = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \sin\left(k \cdot \omega \cdot t + k \cdot \frac{\pi}{3} + \frac{\pi}{2}\right). \tag{2.c}$$

If the industrial single-phase load (an induction furnace) is characterized by the equivalent impedance $\underline{Z}_0 = R_0 + j \cdot X_0$, the goal of the study is to determine the supplementary elements that are required to balance the load, namely L_{s1} and L_{s2}.

For the analyzed circuitry, the following relationships can be written:

$$u_{12} = L_{S1} \cdot \frac{di_{L1}}{dt}; \tag{3.a}$$

$$u_{23} = R_0 \cdot i_S + L_0 \cdot \frac{di_S}{dt}; \tag{3.b}$$

$$u_{31} = L_{S2} \cdot \frac{di_{L2}}{dt}. \tag{3.c}$$

From (2) and (3), the instantaneous phase currents i_{L1}, i_S, i_{L2} are expressed as [2]:

$$i_{L1} = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{1}{k \cdot \omega \cdot L_{S1}} \cdot \sin\left(k \cdot \omega \cdot t - k \cdot \frac{\pi}{3}\right); \tag{4.a}$$

$$i_{L2} = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{1}{k \cdot \omega \cdot L_{S2}} \cdot \sin\left(k \cdot \omega \cdot t - k \cdot \frac{\pi}{3}\right); \tag{4.b}$$

$$i_S = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{2\pi}{3}\right) \cdot \frac{\sin\left(k \cdot \omega \cdot t - \frac{\pi}{2} + a\tan\left(\frac{k \cdot \omega \cdot L_0}{R_0}\right)\right)}{\sqrt{R_0^2 + (k \cdot \omega \cdot L_0)^2}}. \tag{4.c}$$

In order to simplify the computation, we can use the complex representation of instantaneous currents:

$$\underline{I}_{L1} = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{1}{k \cdot \omega \cdot L_{S1}} \cdot e^{j\left(k\cdot\omega\cdot t - k\cdot\frac{\pi}{3}\right)}; \tag{5.a}$$

$$\underline{I}_{L2} = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{1}{k \cdot \omega \cdot L_{S2}} \cdot e^{j\left(k\cdot\omega\cdot t - k\cdot\frac{\pi}{3}\right)}; \tag{5.b}$$

$$\underline{I}_{S} = \sum_{k=1}^{n} U_k \cdot \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{2\pi}{3}\right) \cdot \frac{1}{\sqrt{R_0^2 + (k\cdot\omega\cdot L_0)^2}} \cdot e^{j\left(k\cdot\omega\cdot t - \frac{\pi}{2} + a\tan\left(\frac{k\cdot\omega\cdot L_0}{R_0}\right)\right)}; \tag{5.c}$$

consequently, we can determine the line currents from:

$$\underline{I}_1 = \underline{I}_{L1} - \underline{I}_{L2}; \tag{6.a}$$

$$\underline{I}_2 = \underline{I}_S - \underline{I}_{L1}; \tag{6.b}$$

$$\underline{I}_3 = \underline{I}_{L2} - \underline{I}_S. \tag{6.c}$$

The authors propose a new coefficient for characterization of unbalance in harmonic polluted systems; this coefficient is named *harmonic unbalance factor* and can be expressed as [3]:

$$k_{aI} = \frac{I^-}{I^+} + \frac{I^0}{I^+} = k_{aI}^- + k_{aI}^0, \tag{7}$$

where

- I^0 represents the zero sequence rms. value of the distorted line current system (this component equals zero when the load is Δ connected and in three wires distribution networks);
- I^- - the negative sequence rms. value of the distorted line current system;
- I^+ - the positive sequence rms. value of the same system;
- k_{aI}^- - negative sequence factor of distorted line current system;
- k_{aI}^0 - zero sequence factor of the distorted line current system.

The harmonic unbalance factor has to equalize the negative sequence factor of distorted line current system when the balancing circuitry is used. The value of supplementary elements results from the condition of canceling the harmonic unbalance factor, that is:

$$L_{S1} = \frac{G_1 \cdot H_2 - G_2 \cdot H_1}{G_2 \cdot H_3 - G_3 \cdot H_2}; \tag{8}$$

$$L_{S2} = \frac{G_2 \cdot H_1 - G_1 \cdot H_2}{G_1 \cdot H_3 - G_3 \cdot H_1}. \tag{9}$$

where

$$G_1 = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{\frac{3}{2} \cdot \cos\left(k \cdot \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} \cdot \sin\left(k \cdot \frac{\pi}{3}\right)}{k \cdot \omega};$$

$$G_2 = -\sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{\frac{3}{2} \cdot \cos\left(k \cdot \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} \cdot \sin\left(k \cdot \frac{\pi}{3}\right)}{k \cdot \omega};$$

$$G_3 = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{2 \cdot \cos\left(k \cdot \frac{\pi}{3}\right)}{\sqrt{R_0^2 + (k\cdot\omega\cdot L_0)^2}} \cdot \left(-\sqrt{3}\cos\left(a\tan\left(k\frac{\omega \cdot L_0}{R_0}\right)\right)\right);$$

$$H_1 = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{\frac{\sqrt{3}}{2} \cdot \cos\left(k \cdot \frac{\pi}{3}\right) - \frac{3}{2} \cdot \sin\left(k \cdot \frac{\pi}{3}\right)}{k \cdot \omega};$$

$$H_2 = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{\frac{\sqrt{3}}{2} \cdot \cos\left(k \cdot \frac{\pi}{3}\right) - \frac{3}{2} \cdot \sin\left(k \cdot \frac{\pi}{3}\right)}{k \cdot \omega};$$

$$H_3 = \sum_{k=1}^{n} U_k \sqrt{2} \cdot 2 \cdot \sin\left(k \cdot \frac{\pi}{3}\right) \cdot \frac{2 \cdot \cos\left(k \cdot \frac{\pi}{3}\right) \cdot \sqrt{3} \sin\left(a \tan\left(k \frac{\omega \cdot L_0}{R_0}\right)\right)}{\sqrt{R_0^2 + (k \cdot \omega \cdot L_0)^2}}. \quad (10)$$

Under these circumstances, the load is supplied by a balanced distorted line current system. Although this problem is solved, this solution worsens the value of the system power factor, i.e. the presence of capacitor banks is required.

3. SYMMETRIZING CIRCUITRY IN SINUSOIDAL CONDITIONS

In order to validate the above obtained relationships, let us consider the basically case where the supply voltage system is balanced and sinusoidal. For these conditions, (8) and (9) turn into:

$$L_{s1} = \frac{\sqrt{3}}{\omega} \cdot \frac{R_0^2 + X_0^2}{R_0 + \sqrt{3} \cdot X_0}; \quad (11)$$

$$L_{s2} = \frac{1}{\omega} \frac{\sqrt{3}(R_0^2 + X_0^2)}{\sqrt{3} X_0 - R_0} \quad (12)$$

that is the well-known relationships indicated in the literature [1, 3]. Furthermore, the harmonic unbalance factor becomes the unbalance factor defined by the existing norms [4].

The values of the line currents and the global power factor can be determined as in above presented circuitry. As normally $\lambda_S \leq 0.5$, a capacitor bank, in a delta connection, should be attached to the network; its line capacitance is

$$C = \frac{3X_0}{\left(R_0^2 + X_0^2\right) \cdot \omega}. \quad (13)$$

4. CONCLUSIONS

Modern power systems supply a great number of high-power unbalanced and/or non-linear receivers. The working conditions imposed by these loads are characterized by a series of phenomena that negative effect both the power system and other loads supplied by the same network; as a result, the existing norms propose some limits for the operation in unbalanced and harmonic distorted electrical systems and the implementation of special devices aimed to mitigate these unpleasant effects.

The balancing circuitries represent nowadays a convenient solution for the balancing of industrial distribution networks supplying high-power single-phase receivers. The paper proposes a new parameter to indicate the level of unbalance in harmonic polluted power systems. The appropriate relationships for the computation of the supplementary reactive elements belonging to the balancing circuitries for loads having a reduced power factor are also indicated.

6. REFERENCES

[1] Cziker A.: Mitigation of unbalance in harmonic polluted industrial electrical networks. PhD Thesis, Cluj-Napoca, 2002.

[2] Chindris M., Stefanescu S., Cziker A.: Symmetrizing Systems under Non-sinusoidal Conditions. Proceedings of the 12th International POWER QUALITY'99 Conference, 9-11 November, 1999, Chicago, USA, pp. 512 – 522.

[3] Cziker A. and Chindris M.: Design of Steinmetz circuitry elements in non-sinusoidal conditions. Energetica Revue, No. 7, 2004, pp. 331 - 334.

[4] SREN 50160: Characteristics of supplied voltage in public distribution networks. October 1998.

7. ACKNOWLEDGMENTS

This work was supported by the National Council for the Scientific Research in Universities (CNCSIS) of Romania, under Grants No. 1003/2005 and CEEX 109/2005.