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ANALYTICAL AND EXPERIMENTAL ANALYSIS OF STRESSES IN A CONICAL ELEMENT OF WINCH MECHANISM

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ABSTRACT
This work presents comparative stress analysis in a switching element of coupling mechanism of on board winch for especially purpose. The form of switching element is compounded and consists of circular plate and conical shell. The experimental 3D photoelastic frozen stress method has been used. The experimental results have been compared with the results of performed analytical solution based on the theory of thin plates and the moment theory of shells of revolution, and the analysis was done.
Keywords: stress analysis, conical element, on board winch

1. INTRODUCTION
Axisymmetric thin-walled elements that consist of plates and shells are widely used as parts of machine and other technical constructions of different purpose. The analysis of stresses and strains of these parts is one of the most important bases of construction and plays the decisive step in their design. There are three approaches in stress analysis: analytical, numerical and experimental. Analytical and numerical approach can make analysis not reliable enough, because of problems of recognition and complete satisfaction of boundary conditions. Experimental stress analysis usually knows no problem with boundary conditions and searching for right solution depends on appropriate equipment, knowledge and skill.
This work presents comparative stress analysis in a switching element of coupling mechanism of the on board winch for especially purpose [8], Fig. 1. Performed analytical solution, which can be considered approximate due to some assumptions, is based on the theory of thin plates and the moment theory of thin shells of revolution. Experimental solution was founded by using the 3D photoelastic frozen stress method on araldite B model of switching element. On the basis of analytical and experimental solution, the curves of radial, meridional and circular stresses were comparatively drawn, and the analysis was done. It shows that the experimental frozen stress method gives very clear picture of stresses and that using this experimental method can perform reliable stress analysis of machine parts with various forms.

2. ANALYTICAL SOLUTION
Analytical model of the switching element that consists of circular plate and conical shell is supported and loaded in coordination with real working conditions, as shown in the Fig. 2. Both parts of the element are analysed separately and put together provided that boundary conditions on the point of connection are the same. Differential equation of thin circular plate [2] is:

$$\frac{d^2\vartheta}{dr^2} + \frac{1}{r}\frac{d\vartheta}{dr} + \frac{1}{r^2}\vartheta = \frac{Q}{D} \tag{1}$$

where: $D = \dfrac{Eh^3}{12(1-\mu^2)}$.

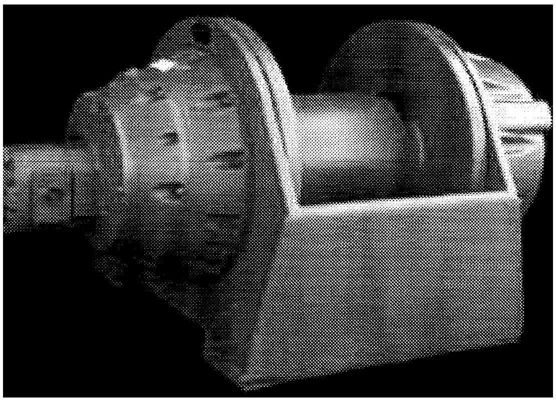

Figure 1.

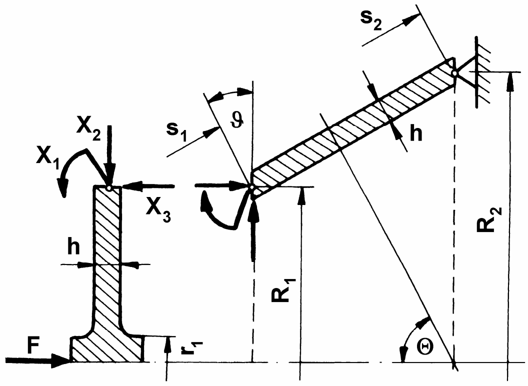

Figure 2.

Its general solution has the following form:

$$\vartheta = K_1 r + \frac{K_2}{r} + \frac{1}{Dr}\int (r\int Q dr)dr \tag{2}$$

In this case is: $Q = -\dfrac{F}{2r\pi}$. Boundary conditions on the basis of which K_1 and K_2 are defined:

for $r = r_1 \Rightarrow \vartheta = 0$, for $r = R_1 \Rightarrow X_1 = D(\dfrac{d\vartheta}{dr} + \mu\dfrac{\vartheta}{r})$. The plate is in state of plane stress, and the stresses in radial and circular directions are:

$$\sigma_r = \frac{Ez}{1-\mu^2}(\frac{d\vartheta}{dr} + \mu\frac{\vartheta}{r}); \qquad \sigma_\varphi = \frac{Ez}{1-\mu^2}(\frac{\vartheta}{r} + \mu\frac{d\vartheta}{dr}) \tag{3}$$

On the basis of theory of thin shells of revolution [1], differential equations of conical shell can be presented in following way [7]:

$$L(V) = Eh\vartheta\tan\Theta + \Phi(s), \qquad L(\vartheta) = -\frac{\tan\Theta}{D}V \tag{4}$$

where: $L(\,) = s\dfrac{d^2(\,)}{ds^2} + \dfrac{d(\,)}{ds} - \dfrac{1}{s}(\,)$; $\quad \Phi(s) = -\dfrac{F(s)}{s\sin\Theta\cos\Theta}$.

In this work the general solutions of homogeneous equations are expressed by second-order Thomson functions with real variable [3]. The particular solutions were performed by the way of loading the shell. The solutions are:

$$V = \frac{Eh^2}{\sqrt{12(1-\mu^2)}}(C_1\varphi_1 + C_2\varphi_2 + C_3\varphi_3 + C_4\varphi_4); \; \vartheta = -C_1\varphi_2 + C_2\varphi_1 - C_3\varphi_4 + C_4\varphi_3 - \frac{X_3 R_1}{Ehs\sin^2\Theta} \tag{5}$$

where: $\varphi_1 = ber_2(x)$, $\varphi_2 = bei_2(x)$, $\varphi_3 = ker_2(x)$, $\varphi_4 = kei_2(x)$, $x = 2\sqrt[4]{12(1-\mu^2)(\dfrac{s\tan\Theta}{h})^2}$.

In the given case, boundary conditions, which determine constants C_1, C_2, C_3 i C_4, are:

for $x = x_1 (C_1 = C_2 = 0) \Rightarrow X_1 = -D(\dfrac{d\vartheta}{ds} + \mu\dfrac{\vartheta}{s})$, $X_2 = \dfrac{V}{s(\sin^2\Theta/\cos\Theta + 1)} - \dfrac{F(s)}{s\sin\Theta}$

for $x = x_2 (C_3 = C_4 = 0) \Rightarrow \dfrac{d\vartheta}{ds} + \mu\dfrac{\vartheta}{s} = 0$, $\dfrac{dV}{ds} - \mu(\dfrac{V}{s} - \Phi(s)) = 0$.

The shell is in the state of plane stress, and stresses in meridional and circular directions are:

$$\sigma_\vartheta = -\frac{1}{h}\left(\frac{V}{s} - \Phi(s)\right) - \frac{Ez}{1-\mu^2}\left(\frac{d\vartheta}{ds} + \mu\frac{\vartheta}{s}\right); \qquad \sigma_\varphi = -\frac{1}{h}\frac{dV}{ds} - \frac{Ez}{1-\mu^2}\left(\frac{\vartheta}{s} + \mu\frac{d\vartheta}{ds}\right) \tag{6}$$

On the basis of expression (3) and (6), the maximum radial, meridional and circular stresses on the inner surface of switching element were determined. The following values of basic quantities were taken: r_1 = 5mm, R_1 = 60mm, s_1 = 70mm, s_2 = 120mm, h = 5mm, Θ = 60^0, μ = 0,48, F = 14N, E= 18MPa, $X_3 = \frac{F}{2R_1\pi}$, $F(s) = -\frac{F}{2\pi}$, $z = \frac{h}{2}$.

3. EXPERIMENTAL SOLUTION

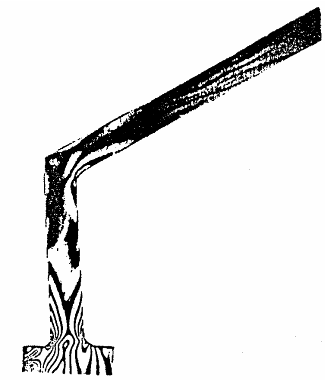

Figure 3.

Model of the conical element was made of photoelastic material Araldite B. In an especially designed and very precisely made device, the model was supported and loaded in coordination with real operating conditions [8,10]. After being heated in oven up to the freezing temperature of 150^0C, which has been maintained for 2 hours, the model is cooled under load at rate of 5^0C per hour to room temperature. Afterwards the model was cutted into slices and subslices in radial, meridional and circular directions, and they are photographed in light and dark field of circularly polarised light. The photography of radial-meridional slice in light field is given on Fig. 3. In radial - meridional slices, and in circular slices the basic equation of photoelasticity [5,6], is at follows:

$$\sigma_1 - \sigma_3 = \frac{f_\sigma}{b} N_r; \qquad \sigma_2 - \sigma_3 = \frac{f_\sigma}{b} N_\varphi \tag{7}$$

The maximum values of radial and circular stresses of the plate-like part, and meridional and circular stresses of the shell-like part were the result of equation (7).

The following values were taken: $\sigma_1 = \sigma_r$ (plate); $\sigma_1 = \sigma_\vartheta$ (shell); $\sigma_2 = \sigma_\varphi$; $\sigma_3 = 0$ (stress perpendicular to the free edge), b = 3,5mm, f_σ = 0,28kN/m (t_Z = 150^0C – calibration specimen).

4. CONCLUSION

On the basis of analytical and experimental solution, the curves of stresses on the inner surface of the switching element were comparatively drawn, as shown in Fig. 4. The stresses are presented in non-dimensional form dividing by $\sigma = \frac{F}{5h^2}$.

The similarities of given curves are evident except for the point of connection between the plate and shell. At this point the analytical values of stress are higher than the experimental ones, for 34% at radial and meridional stress, and for 59% at circular stress. The extreme value of experimental radial stress is 24% lower than analytical value, while the extreme analytical and experimental values of circular stress are the same.

Different results at the point of connection of plate and shell can be explained by assumption of approximate analytical solution. It is that contact of these parts is realised in the point lies on their central surfaces. It is not true of course, because the contact is realised on the surface with help of radius of curvature, so the connecting conditions are not satisfied in expected point. Besides the stress concentration appear on the place of connection, and it is not taken into account in this analytical solution. The experience and engineering practice shows that at h (thickness of plate and shell) to $2h$ distance from the point of connection, other approximate solutions based on theory of thin plates and shells give almost exact values [1,2,3,7], which is confirmed also for the performed solution by the experimental results in this case.

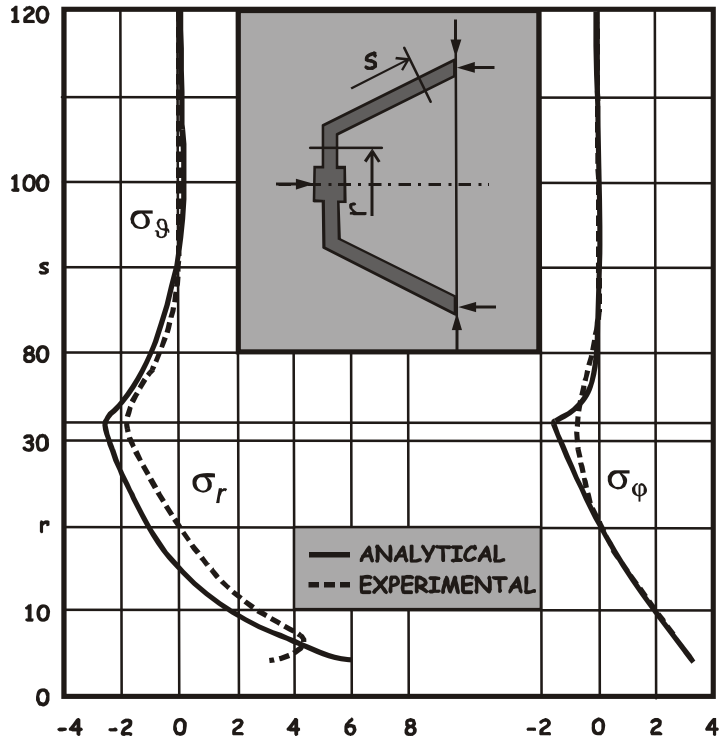

Figure 4.

Finally, the work shows that the experimental frozen stress method gives very clear picture of stresses and that using this experimental method can perform reliable stress analysis of this and similar designed construction elements.

Symbols

b - slice width; C - constant; D - flexural rigidity; E - modulus of elasticity; F - force; f_σ - stress optic coefficient; h - thickness; K - constant; L - differential operator; N - fringe order; R - radius of shell; r - radius of plate; s - meridian length of shell; V - Meissner variable; X - reaction; x - axial coordinate, real variable; z - distance from central surface; Θ - angle between normal on the central surface and axis of symmetry of shell; ϑ - angle of twist of normal; μ - Poisson's ratio; σ - normal stress; Φ - function depending on stress; φ - Thomson's function.

5. REFERENCES

[1] Chatterjee, B. K.: Theory and design of concrete shells, Chapman and Hall, New York, 1988.,
[2] Gould, P. L.: Analysis of shells and plates, Springer-Verlag, New York 1988.,
[3] Bojaršinov, S. V.: Osnovi strojiteljnoj mehaniki mašin, Mašinostroenie, Moskva, 1973.,
[4] Ozretić, V.: Brodski pomoćni strojevi i uređaji, Split Ship Management Ltd., Split, 1998.,
[5] Alfirević, I., Jecić, S.: Fotoelasticimetrija, FSB Sveučilišta u Zagrebu, Zagreb, 1982.,
[6] Wolf, H., Spannungsoptik, Springer-Verlag, Berlin, 1976.,
[7] Kulenović Z.: Comparative stress analysis in a thin-walled axisymmetric machine part, Proc. of 2nd
[8] International Congress of Croatian Society of Mechanics, Supetar 1997.,
[9] Oceanographic/Hydrographic Winch LZ 750, Design Report 9 - 01, VPS, Split, 2001.,
[10] http://www.tel.hr/adria-winch/indekslogo2.htm,
[11] http://lantecgear.com/lantec/winches.html