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BUSSINESS COOPERATION AND INTEGRATION AS PROBLEM OF LINEAR PROGRAMMING

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ABSTRACT
Organizations go in various businesses cooperation among themselves. They come to an agreement about materials distributions, market appearance together, production programs coordinating, etc. Cooperation in production takes important role in relations between organizations. At least, stronger and more stabile organizations are formed by integration. Before beginning business cooperation or integration, it is need to examine if intended idea is economic for all participants. An estimate of expected results can be made by linear programming methods. The aim of this article is to show possibilities of defining an optimal production program for small and medium enterprises, which have to satisfy some joint goals, on the basis of nets organization. At this way it is possible to overcome some problems, as poverty of internal resources, limited managements capacities, insufficient knowledge and skills, with aim to develop them according to the open market criteria.
Keywords: Business cooperation, Linear programming, Decomposition.

1. INTRODUCTION
During the 20th century the opinion that success of the company is connected to its size was prevailing in economic theory and practice. However, the practice indicates tat the number of big companies has been increasingly reducing since 80s, while at the same time the number of the small ones has been increasing. The technological and economical development alone enables the appearance of a new type of organization, combining good parts of both, big and small companies. In this organizational model, the size is not that important, but the quality of business relations between the companies. The key production unit is not an individual company any more, but a decentralized network of the companies that have mutual interests. Sometimes these networks consist of vertical networks connecting small suppliers with big final fitters. In other cases, connections are horizontal and unite a certain number of more or less equal small companies. In both cases, the networks enable constant innovations, overcoming of the problems related to poor resources, limited capacities, and a better managing problem resolving.
There are various forms of business cooperation among the companies, such as material and raw material distribution, common appearance in the home and foreign markets, bringing in line, i.e. coordination of their own production programs, etc. When production is in question, cooperation is getting more and more important in relations among the companies. Finally, stronger and steadier companies are formed by means of integration. Before entering any kind of business cooperation, i.e. before starting the integration, it is necessary to investigate whether such integration is economically

justified for all the participants. The expected results estimation can be done by means of linear programming methods, after the intended cooperation, i.e. integration had been performed.
The objective of this work is to show the possibilities of optimum production program determination of small and medium sized companies, which at the same time have to meet some common aims, on the basis of network organization. Thus, it is possible to overcome the problems related to poor internal resources, limited managing capacities, insufficient knowledge and skills, all with the purpose of their development towards the open, market economy criteria.

2. DECOMPOSITION IDEAS

Three major decomposition ideas [1] are:
a) *Dantzig-Wolfe decomposition*: was originally applied to large LPs (Linear Programming). It is based on the idea of complicating constraints [2]. The "easy" constraints (non-complicating) must define a feasible region that is easy to optimize over. The original problem is reformulated in terms of extreme points and extreme rays of this feasible region. Usually, Dantzig-Wolfe decomposition is used for large LPs where a subset of the constraints has a nice structure (the non-complicating constraints).
b) *Lagrangian relaxation:* traces its origins to Lagrange multipliers from multivariable calculus. It is based on the idea of complicating constraints for an LP or IP (Integer Programming) [2,3,4]. A solution to a Lagrangian relaxation gives a bound on the optimal objective value for the original problem. The problem of finding the best such bound is the Lagrangian dual problem. For an IP, the value of the Lagrangian dual is between the value of the optimal IP solution and the value of the LP relaxation. In some cases it gives an optimal solution to the IP, but this is not guaranteed.
c) *Benders decomposition:* was originally used for integer programs. It is based on the idea of complicating variables - originally meaning the integer variables, though this is not the only interpretation [2,3,4]. Removing the non-complicating variables and replacing them with a large number of constraints, called feasibility constraints and optimality constraints, reformulate the problem. A restricted master problem is optimized using only some of these constraints. The non-complicating ("easy") variables are placed in a sub-problem. The dual values from this solution are used to create a new optimality constraint or feasibility constraint, which is added to the restricted master. Benders decomposition is usually used for large IPs.

3. PROBLEM DEFINING (DANTZIG-WOLFE DECOMPOSITION METHOD)

Let's take in consideration one company consisting of two production sectors. The aim of the company business dealing is defined with total profit maximization. A part of restrictions, i.e. limitations, referring to the production of only one sector, and totally independent of the other sector, shall be named as *'sector limitations'*, in difference to so called common or '*central limitation'*.
The sector limitation referring to the first sector is

$$A_1x_1 \le b_1, \quad x_1 \ge 0 \qquad ...(1)$$

and the sector limitation referring to the second one is

$$A_2x_2 \le b_2, \quad x_2 \ge 0 \qquad ...(2)$$

where x_1 and x_2 have no common components.
However, apart from sector limitations, optimum resolution is to satisfy central limitations as well, which unite the sectors, and can be represented by the following inequality forms

$$B_1x_1 + B_2x_2 \le b, \quad x_1 \ge 0,\ x_2 \ge 0. \qquad ...(3)$$

and which express the general limitation of the resources that can be used.
At the end, suppose that the company's objective function is expressed in the following form

$$(\max) z = c_1x_1 + c_2x_2, \qquad ...(4)$$

where: $-c_1x_1$ expresses the first sector efficiency, and $-c_2x_2$ expresses the second sector efficiency.
Accordingly, the so-called *'complete problem'* of the linear programming, i.e. the main task, shall be defined in the following way:
It is necessary to calculate the value of the vectors x x_1 and x_2, which maximizing the objective function

$$(\max) z = c_1 x_1 + c_2 x_2 \qquad \text{...(5)}$$

with the following limitations:

$$\begin{aligned} &A_1 x_1 \qquad \le b_1 \\ &\qquad A_2 x_2 \le b_2 \\ &B_1 x_1 + B_2 x_2 \le b, \quad x_1 \ge 0,\ x_2 \ge 0 \end{aligned} \qquad \text{...(6)}$$

4. ALGORITHM METHOD

To make the problem of linear programming, expressed by the objective function (5) and the limitation system (6) solvable by the method of decomposition, the existence of at least two possible basic solutions for each sector are required.

The first possible basic solution for the first sector is acquired by resolving the following sub-task

$$\begin{aligned} &(\max) z_1 = c_1 x_1 \\ &A_1 x_1 \le b_1, \quad x_1 \ge 0 \end{aligned} \qquad \text{...(7)}$$

and the initial basic solution for the second sector, by resolving the sub-task given below

$$A_2 x_2 \le b_2, \quad x_2 \ge 0\,. \qquad \text{...(8)}$$

The solution of the sub-tasks (7) and (8), most often do not satisfy the central requirement (3), either for the reason that the limited resource *b* is surpassed or the products are no more produced in the required quantities, so the obtained sub-task (7) and (8) results do not represent the optimal solution of the main task (5) – (6) as well.

Let us now consider the case of the general resource deficit. Namely, due to the general resource deficit, the sectors are liable to perform some ‘compensation‘ equal to the multiplication of the price by the deficit resource unit (p) and the corresponding resource quantity. That is why the sub-tasks objective functions, for the first, i.e. second sector shall be

$$(\max) z_1 = \left[c_1 x_1 - (pB_1) x_1 \right] \qquad \text{...(9)}$$

namely

$$(\max) z_2 = \left[c_2 x_2 - (pB_2) x_2 \right] \qquad \text{...(10)}$$

So the second basic solution, for the first sector, is obtained by resolving the problems (9) and (7), and (10) and (7) for the second sector. By the same analogy, the rest of the required possible basic solutions are determined. Having obtained the possible sub-task basic solutions, it is possible to determine the main task optimal solution, needed to maximize the objective function (5) and fulfill the limitation system (6).

Starting from the assumption that the allowed sub-tasks limitation sets are limited and convex, the vectors $x_1 \ge 0$ i $x_2 \ge 0$, that meet / fulfill the limitations (1), namely (2) may be expressed as one convex linear combination of the possible solution sets final points, that is

$$x_1 = \sum_{i=1}^{m} \lambda_i\, x_i,\ \sum_{i=1}^{m} \lambda_i = 1,\ \lambda_i \ge 0\ ; \qquad x_2 = \sum_{j=1}^{n} \mu_j\, x_j,\ \sum_{j=1}^{n} \mu_j = 1,\ \mu_j \ge 0\,.$$

The objective function (5) of the main task, is transformed into the function

$$(\max) z = \left[\sum_{i=1}^{m} \lambda_i (c_1 x_i) + \sum_{j=1}^{n} \mu_j (c_2 x_j) \right]$$

with limitations

$$\begin{aligned} &\sum_{i=1}^{m} \lambda_i (B_1 x_i) + \sum_{j=1}^{n} \mu_j (B_2 x_j) \le b \\ &\sum_{i=1}^{m} \lambda_i \qquad\qquad\qquad = 1 \\ &\qquad\quad \sum_{j=1}^{n} \mu_j \qquad\quad = 1, \quad \lambda_i \ge 0, \quad \mu_j \ge 0. \end{aligned} \qquad \text{...(11)}$$

The solution of the task is the values for λ_i i μ_j,, so that the possible solution of the main task is

$$x_1 = \lambda_1 x_i^{(1)} + \lambda_2 x_i^{(2)} + \ldots + \lambda_m x_i^{(m)}, \quad \sum_{i=1}^{m} \lambda_i = 1,\ \lambda_i \geq 0$$

$$x_2 = \mu_1 x_i^{(1)} + \mu_2 x_i^{(2)} + \ldots + \mu_n x_i^{(n)}, \quad \sum_{j=1}^{n} \mu_j = 1,\ \mu_j \geq 0 \qquad \ldots(12)$$

The further procedure is related to dual problem. The optimal solution of the dual problem should be calculated from the primary problem, where, the dual variable *u* expresses the price by the limited general resource unit, and dual variables v_1 i v_2, realized profits for individual sectors, reduced for the amount of the 'compensation' now calculated as the multiplication of the price by the limited resource unit (u) and its quantity. Newly formed functions of sub-tasks objectives, for the sectors are as follows

$$(\max) z_1 = \left[c_1 x_1 - (uB_1) x_1\right], \qquad \ldots(13)$$

namely

$$(\max) z_2 = \left[c_2 x_2 - (uB_2) x_2\right] \qquad \ldots(14)$$

with unchanged sub-tasks limitation system.

Now there are two separate problems of linear programming to be solved. The first one is defined by the objective function (13) and the limitation (1), while the second one is defined by the objective function (14) and the limitation (2). The values obtained for the objective function (13), namely (14) should be now compared with the values of dual variables. If it is

$$(\max) z_1 = \left[c_1 x_1 - (uB_1) x_1\right] > v_1, \quad \text{or} \quad \text{-}(\max) z_2 = \left[c_2 x_2 - (uB_2) x_2\right] > v_2$$

the procedure is to be repeated, and if it is

$$(\max) z_1 = \left[c_1 x_1 - (uB_1) x_1\right] = v_1, \quad \text{or} \quad (\max) z_2 = \left[c_2 x_2 - (uB_2) x_2\right] = v_2$$

The solution (11) is the optimal one.

5. CONCLUSION

Decomposition method can be successfully utilized for the optimization problem resolving, in all those cases, when it is required to bring in accord the individual programs with some common objectives and requirements or, reversal, some common requirements correct with those available, individual resources.

The entire problem of linear programming, namely the main task, can be solved by means of simplex method. But if the problem dimensions are really big, the number of arithmetical operations that have to be carried out can be so big, that might cause giving up of the problem solving. Regarding these difficulties we wanted to show that by separating the sector problems from the central ones, the procedure of determining an optimal solution of the main problem can be decentralized into a bigger number of smaller problems, making them significantly simpler.

When considered on LPs, the three approaches have some strong relationships. Dantzig-Wolfe decomposition and Benders decomposition may be viewed as dual operations, so that taking the dual and then doing Benders decomposition is equivalent to doing Dantzig-Wolfe decomposition and then taking the dual. It is also easy to show that the Lagrangian dual problem is equivalent to this same formulation.

6. REFERENCES

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