10th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2006, Barcelona-Lloret de Mar, Spain, 11-15 September, 2006

APPLYING ADAPTED FUZZY AHP METHOD FOR QUANTIFYING RISK

Borut Buchmeister, Iztok Palcic
University of Maribor, Faculty of Mechanical Engineering
Smetanova 17, SI – 2000 Maribor
Slovenia

Krsto Pandza
Leeds University Business School
Maurice Keyworth Building, Leeds LS2 9JT
United Kingdom

ABSTRACT
In decision making under uncertainty, risk analysis aims at minimising the failure to achieve a desired result. In the paper our original method of risk estimation is presented. Each problem involving uncertainty and consecutive appearing risk is divided into identified risk categories and factors. The basic method used in the numerical part of the evaluation is the two-pass Fuzzy Analytic Hierarchy Process (applied: first for the importance and second for the uncertainty of risk factors). The process is based on the determination of relations between particular risk categories and factors. The estimates are derived from pairwise comparison of factors belonging to each category. By using fuzzy numbers the consideration of possible errors of the estimator is taken into account. In the following stages the interval results obtained by this method are used for calculating the integral uncertainty value, which, in comparison with the boundary value, defines the risk of the process in question. Based on the "Uncertainty – Importance" relations special ABC focus diagrams are created. These diagrams serve for the classification of risk factors, which provides a decision making part of the systemic approach.
Keywords: risk estimation, fuzzy AHP method, importance and uncertainty

1. INTRODUCTION
Manufacturing is and will remain one of the principal means by which wealth is created. However manufacturing has changed radically over the course of the last 20 years and rapid changes are certain to continue. The emergence of new manufacturing technologies, spurred by intense competition, will lead to dramatically new products and processes. New management and labour practices, organizational structures, and decision making methods will also emerge as complements to new products and processes. It is essential that the manufacturing industry be prepared to implement advanced manufacturing methods in time.
These changes have led organizations to search for new approaches in organization models and in production management. Uncertainty and fast changing environment are making long-term planning next to impossible. This uncertain environment is leaving only time and risk as means for survival. Decision-making has become one of the most challenging tasks in these unpredictable global conditions, demanding competency in understanding these complicated processes [1,2].
Managers employed in industrial companies, the public sector and service industry cope with high levels of uncertainty in their decision-making processes, due to rapid, large-scale changes that define the environment their companies operate in. Decision-making in high-risk conditions is becoming a common area for research within strategic management organizational theory, research and development management and industrial engineering.
Tackling uncertainty involves developing heuristic tools that can offer satisfactory solutions. The problem of decision-making in uncertain conditions is only partially presented in relevant literature

[3,4,5]. Intensive research in the area of multi-level decision-making, supported by expert systems is currently under way.

2. FUZZY AHP METHOD

Used procedure with the application of fuzzy triangular numbers is described in the following steps [6,7,8].

1st step: pairwise AHP comparison (using triangular fuzzy numbers from $\tilde{1}$ to $\tilde{9}$ - see Fig. 1) of the elements at the same hierarchy level. Triangular fuzzy number is described as $\tilde{M} = (a, b, c)$ and by defining the interval of confidence level α, we get:

$$\forall \alpha \in [0,1] \tilde{M}_\alpha = \left[a^\alpha, c^\alpha\right] = \left[(b-a)\cdot\alpha + a, -(c-b)\cdot\alpha + c\right] \quad \dots (1)$$

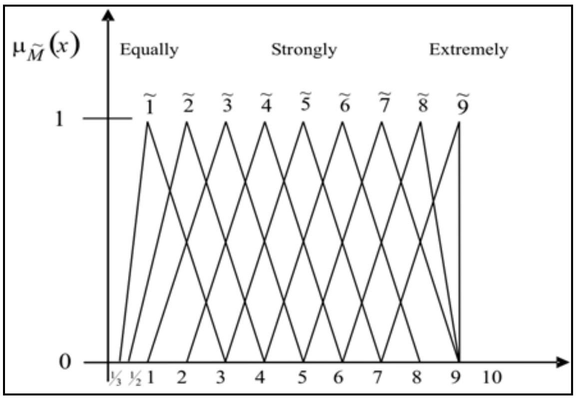

Figure 1. Triangular membership functions of numbers $\tilde{1}$ to $\tilde{9}$.

2nd step: constructing the fuzzy comparison matrix $\tilde{A}$ (a_{ij}) with triangular fuzzy numbers:

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \tilde{a}_{13} \dots \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \tilde{a}_{23} \dots \tilde{a}_{2n} \\ : & & \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \tilde{a}_{n3} \dots 1 \end{bmatrix}, \text{ where } \tilde{a}_{ij} = \begin{cases} 1, \text{if } i = j \\ \tilde{1}, \tilde{2}, \dots, \tilde{9} \text{ or } \tilde{1}^{-1}, \tilde{2}^{-1}, \dots, \tilde{9}^{-1}, \text{if } i \neq j \end{cases}$$

It is presumed that the evaluator's mistake in quantitative evaluation might be ± one class to the left or to the right.

3rd step: solving fuzzy eigenvalues $(\tilde{\lambda})$ of matrix, where: $\tilde{A} \cdot \tilde{x} = \tilde{\lambda} \cdot \tilde{x}$ … (2)
and $\tilde{x}$ is a non-zero $n\times1$ fuzzy vector.
To be able to perform fuzzy multiplication and addition with interval arithmetic and level of confidence α the equation (2) is transferred into:

$$\left[a^\alpha_{i1l}\cdot x^\alpha_{1l}, a^\alpha_{i1u}\cdot x^\alpha_{1u}\right] \oplus \cdots \oplus \left[a^\alpha_{inl}\cdot x^\alpha_{nl}, a^\alpha_{inu}\cdot x^\alpha_{nu}\right] = \left[\lambda\cdot x^\alpha_{il}, \lambda\cdot x^\alpha_{iu}\right] \quad \dots (3)$$

where:

$$\tilde{A} = \left[\tilde{a}_{ij}\right], \ \tilde{x} = \left\{\tilde{x}_1 \dots \tilde{x}_n\right\}^T \ \text{ and } \ \tilde{a}^\alpha_{ij} = \left[a^\alpha_{ijl}, a^\alpha_{iju}\right], \ \tilde{x}^\alpha_i = \left[x^\alpha_{il}, x^\alpha_{iu}\right], \ \tilde{\lambda}^\alpha = \left[\lambda^\alpha_l, \lambda^\alpha_u\right] \quad \dots (4)$$

for $0 < \alpha \leq 1$ and all i, j, where $i = 1 \dots n, j = 1 \dots n$.

Degree of satisfaction for the matrix $\tilde{A}$ is estimated by the index of optimism μ. The larger index value μ indicates the higher degree of optimism calculated as a linear convex combination (with upper and lower limits), defined as: $\hat{a}^\alpha_{ij} = \mu \cdot a^\alpha_{ijn} + (1-\mu)\cdot a^\alpha_{ijl}, \ \forall \mu \in [0,1]$ … (5)

At optimistic estimates that are above average value ($\mu > 0,5$) $\hat{a}_{i,j}$ is higher than the middle triangular value (b) and vice versa.
While α is fixed, the following matrix can be obtained after setting the index of optimism μ:

$$\widetilde{A}=\begin{bmatrix}1 & \hat{a}_{12}^{\alpha}\ldots\hat{a}_{1n}^{\alpha}\\ : & :\\ \hat{a}_{n1}^{\alpha} & \hat{a}_{n2}^{\alpha}\ldots 1\end{bmatrix} \qquad \ldots (6)$$

The eigenvector is calculated by fixing the value μ and by identifying the maximal eigenvalue.

4th step: determining total weights. By synthesizing the priorities over all hierarchy levels the overall importance weights of uncertainty factors are obtained by varying α value.

Upper and lower limits of fuzzy numbers considering α are calculated by application of the appropriate equation, for example:

$$\widetilde{3}_{\alpha}=\left[1^{\alpha},5^{\alpha}\right]=\left[1+2\cdot\alpha,5-2\cdot\alpha\right] \qquad \ldots (7)$$

$$\widetilde{3}_{\alpha}^{-1}=\left[\frac{1}{5-2\cdot\alpha},\frac{1}{1+2\cdot\alpha}\right] \qquad \ldots (8)$$

2.1 Calculation of the importance of factors

Ratios between categories or factors are expressed with a question: “How many times is the category/factor *i* more important than category/factor *j*?” By pair wise comparison [9] of the factors and categories (according to the AHP estimation scale) and the use of triangularly distributed fuzzy numbers, we get fuzzy matrixes on all levels of hierarchy.

2.2 Calculation of the uncertainty of factors

Ratios between categories or factors are expressed with a question: “How many times is the category/factor *i* more uncertain than category/factor *j*?” By pair wise comparison of the factors and categories (according to the AHP estimation scale, adapted for the level of uncertainty) and the use of triangularly distributed fuzzy numbers, we get fuzzy matrixes on all levels of hierarchy.

Normally we calculate the importance and uncertainty of categories and factors at different levels of confidence (α = 0, 0.5, 1) and optimism (μ = 0.05, 0.5, 0.95). Variations in the results indicate some possible mistakes of the estimation process (human impact). Introduction of fuzzy numbers allows the compensation of the possible errors of the estimator.

3. UNCERTAINTY – IMPORTANCE RELATIONS AND ABC FOCUS DIAGRAMS

Categories and factors are mutually compared according to importance and uncertainty in the diagram. In the diagram we define three areas (low, medium, high) with boundaries around ± 50 % from average share. The position of every category / factor in one of the nine fields of the diagram is defined with an ellipsis, whose boundaries are minimal and maximal shares of category / factor of importance or uncertainty (see Fig. 2). The diagrams enable selection of factors that need special attention, or vice versa, the factors which can be partially neglected, which is shown in ABC focus diagram (see Fig. 3).

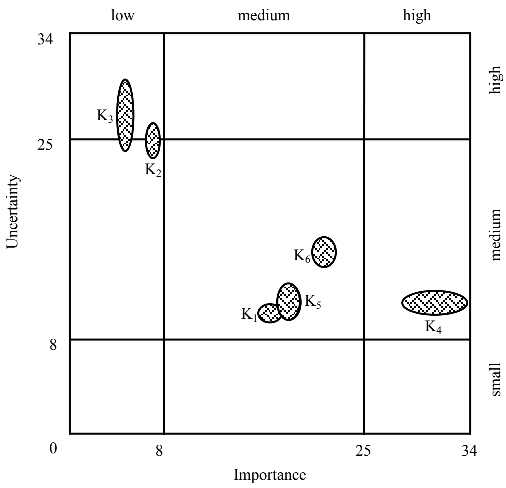

Figure 2. Position of 6 categories (example).

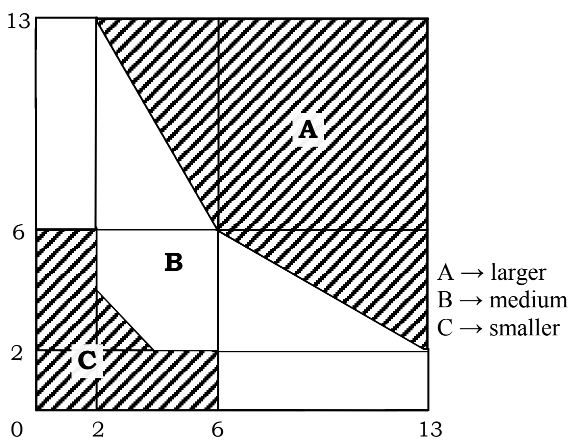

Figure 3. ABC areas of attention.

4. INTEGRAL UNCERTAINTY VALUE

We would like to evaluate the problem of uncertainty, represented with the categories and factors, numerically. With the fuzzy AHP we have determined the intervals of the level of importance and uncertainty for every factor, which have given us the opportunity for selection of more or less critical factors. The mentioned intervals of values can be used for:

- Design of the factor importance vector $\vec{P}$, whose elements are weights of factor importance, obtained as arithmetical mean value between the lowest and highest value of the importance of the factor (multiplied by 100),
- Design of the factor uncertainty vector $\vec{N}$, whose elements are weights of factor uncertainty, obtained also as arithmetical mean value between the lowest and highest value of the uncertainty of the factor (multiplied by 100).

Integral uncertainty value (*IUV*) is a scalar product of vectors $\vec{P}$ and $\vec{N}$: $IUV = \vec{P} \cdot \vec{N}$ … (9)

Boundary integral uncertainty value can be obtained by using the same mean weights at all vector components, therefore, with n factors:

$$IUV_b = \sum_{i=1}^{n} \left[\left(\frac{100}{n} \right) \cdot \left(\frac{100}{n} \right) \right] = n \cdot \frac{100}{n} \cdot \frac{100}{n} = \frac{10000}{n} \qquad \ldots (10)$$

In practice, real bounds of *IUV* depend upon the number of factors (from 5 to about 100), which gives: IUV_b = 100 … 2000. Integral uncertainty value, which exceeds boundary value, means that we are dealing with an activity of higher risk, or vice versa.

5. CONCLUSION

The companies are exposed to various risks every day. Risk management in the quickly changing environment is essential, for it contributes to achieving the strategic advantage of the company. The article encompasses the original synthesis of risk management, modelling uncertainty, method of analytic hierarchy process and fuzzy logic, and it represents a contribution to the construction of tools for decision-making support in organisational systems.

Human assessment on qualitative attributes is always subjective and thus imprecise. We should take into account the uncertainty associated with the mapping of one's perception or judgment to a number. The original contribution in this article is comprised by:

- Completion of heuristic approach for effective interpretation of numerical results and their support to decision-making process,
- Use of fuzzy AHP method for determining uncertainty level is an entirely original idea, for the abovementioned method is used only for defining the importance (weights),
- Original combined diagrams 'Importance – Uncertainty' and ABC diagram of attention enable the selection and classification of factors,
- Integral uncertainty value (*IUV*) and its boundary value represent an original contribution for estimating uncertainty and risk of discussed activities.

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