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COMPARATIVE STUDY OF PRESSURE ANGLE IN CAM FOLLOWER SYSTEMS USING BÉZIER AND TRADITIONAL CURVES

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ABSTRACT
With the purpose of avoid that the force between follower and cam surface bends follower shaft, pressure angle must keep as low as possible. A high pressure angle will increase the friction effect then it is possible that follower displaces or even blocks. This paper compares degree 5, 7 y 9 Bézier curves with traditional curves in order to rationalize cam profile. Graphics and tables are developed to calculate pressure angle with Bézier curves. In similar conditions, cams designed by degrees 5, 7 y 9 Bézier curves have higher pressure angle and prime circle radius than those designed by traditional curves.
Keywords: cams, pressure angle, Bezier curves

1. INTRODUCTION
In cam-follower mechanisms, cam design demands to have previously displacement function in order to know velocities and acceleration. There are two important factors necessaries to define cam geometry: Pressure Angle and Radius of Curvature. This paper discusses pressure angle of cams designed by degree 7 Bézier curves. Comparison with cams designed by traditional curves (harmonic and cycloidal functions) is presented.

2. PRESSURE ANGLE
Pressure angle is defined as angle between the direction of motion (velocity) of the follower and the direction of the axis of transmission [1,2,3,4]. In this paper, pressure angle in radial follower is analyzed.
In order to avoid that the force between follower and cam surface bends follower shaft, pressure angle must keep as low as possible. A high pressure angle will increase the friction effect then it is possible that follower displaces or even blocks. Experience shows that pressure angle higher than 30º are not satisfactory.

3. GENERAL EQUATION FOR PRESSURE ANGLE
In cam design process, first at all designer have to define motion functions. Then, pressure angle can be calculated by next expression:

$$\varphi = \arctan \frac{v - \varepsilon}{s + \sqrt{R_0^2 - \varepsilon^2}} \qquad (1)$$

where, R_O is the prime circle radius,   is the first derivate of motion function, s is the law of displacement and ε is the cam eccentricity (Fig. 1).

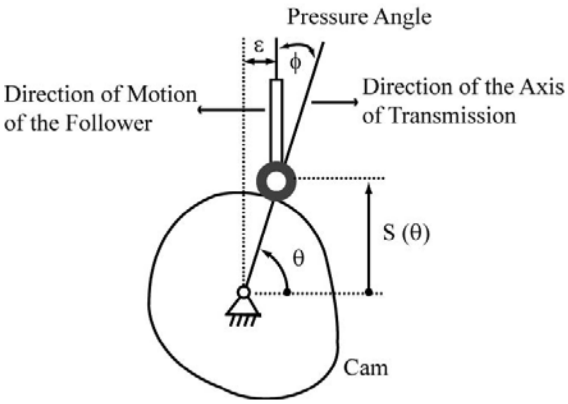

Figure 1. Cam pressure angle

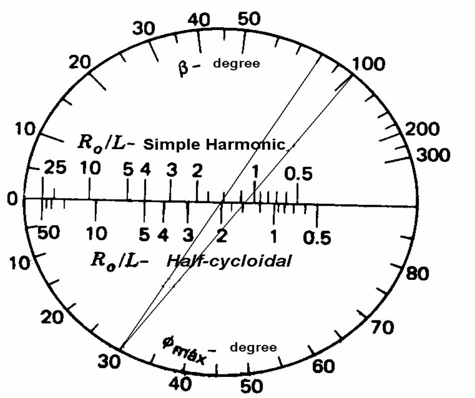

Figure 2. Nomogram for determine maximum cam pressure angle

4. CALCULATION OF HARMONIC AND CYCLOIDAL FUNCTIONS

From equation (1) can be observed that pressure angle depend on eccentricity. In rise motion is positive then a small eccentricity reduce pressure angle. But in return motion, when is negative, pressure angle will be increased. It is possible to keep pressure angle under 30º with variation of eccentricity. Due to force magnitudes are almost always higher in rise motion, is a common practice use a small eccentricity in the follower to reduce cam pressure angle [3,5,6,8].

If there is no eccentricity, ε =0, equation (1) becomes in:

$$\varphi = \arctan \frac{v}{s+R_0} \tag{2}$$

To determine maximum pressure angle, it is necessary find out the maximum of function (1) or (2), depends if there is eccentricity or not. In order to avoid this calculation and to simplify, Varnum developed a nomogram (fig. 2.) to determine maximum pressure angle which ε =0 [1,7].

For to use this nomogram Ro/L relation must be determined where Ro is the prime circle radius and L is the maximum displacement of follower, which are known data in cam design. With this relation and with camshaft angle , pressure angle for a specific segment can be read from nomogram. Although nomogram allow working with pressure angle up to 90º, angles higher than 30º are not recommendable.

For example, if a rise segment will be designed with camshaft angle of 90º and Ro/L = 1,5, cycloidal curves must no be used due to minimum angle camshaft allowed, in order to keep pressure angle under 30º, is 100º. By the contrast, with harmonic functions this problem do not exist because minimum camshaft angle allowed is 80º, lower than target angle of 90º.

5. CALCULATION OF BÉZIER CURVES

Theoretical base for pressure angle with Bézier curves is similar to theory with traditional curves. Now, in equations (1) and (2) functions will be Bézier curves [3,5].

Functions for this rise motion are

C^2

$$b(u) = L\left[10u^3 - 15u^4 + 6u^5\right]$$
$$b'(u) = \frac{L}{\beta}\left[30u^2 - 60u^3 + 30u^4\right]$$
$$b''(u) = \frac{L}{\beta^2}\left[60u - 180u^2 + 120u^3\right]$$

C^3

$$b(u) = L\left[35u^4 - 84u^5 + 70u^6 - 20u^7\right]$$
$$b'(u) = \frac{L}{\beta}\left[140u^3 - 420u^4 + 420u^5 - 140u^6\right]$$
$$b''(u) = \frac{L}{\beta^2}\left[420u^2 - 1680u^3 + 2100u^4 - 840u^5\right]$$

C^4

$$b(u) = L\left[126u^5 - 420u^6 + 540u^7 - 315u^8 + 70u^9\right]$$
$$b'(u) = \frac{L}{\beta}\left[630u^4 - 2520u^5 + 3780u^6 - 2520u^7 + 630u^8\right] \qquad (3)$$
$$b''(u) = \frac{L}{\beta^2}\left[2520u^3 - 12600u^4 + 22680u^5 - 17640u^6 + 5040u^7\right]$$

where . $u = \frac{\theta}{\beta}$ and $\beta = \theta_f - \theta_i$

Replacing (3) in (2) and differentiating respect to cramshaft angle θ, Ro/L for degree 7 Bézier curves

$$\frac{R_0}{L} = \frac{\left[\frac{1}{\beta}\left(140u^3 - 420u^4 + 420u^5 - 140u^6\right)\right]^2}{\frac{1}{\beta^2}\left(420u^2 - 1680u^3 + 2100u^4 - 840u^5\right)} - \left(35u^4 - 84u^5 + 70u^6 - 20u^7\right) \qquad (4)$$

Table 1 (equivalent to nomogram used for traditional curves) present results for equation (4). Table has three columns. First column contains Ro/L relation for rise motion with degree 7 Bézier curves. In third column, minimum camshaft angle, (required in order to pressure angle do not exceed 30º) to can be read.

Fig. 4 shows result of minimum camshaft angle for pressure angle of 25º and 30º.

Table 1. Camshaft angle in rise motion.

R_o/L	*u máx*	β_{min} for $\phi \leq 30º$
0.75	0.423	186.09º
1.0	0.437	151.65º
1.5	0.453	111.36º
2.0	0.463	88.25º
2.5	0.469	73.17º
3.0	0.473	62.53º
3.5	0.477	54.61º
4.0	0.479	48.48º
4.5	0.481	43.59º
5.0	0.483	39.60º
6.0	0.485	33.47º
7.0	0.487	28.99º

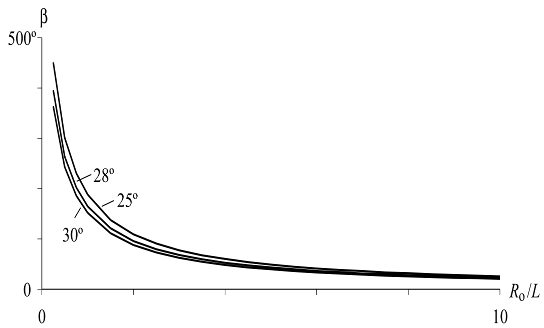

Figure 3. Fig 4. Graphic for calculate minimum camshaft angle in cams designed by Bézier curves degree 7 with pressure angle of 30º, 28º y 25º degrees.

For example, with a Ro/L = 1.5 and target pressute angle of 30º, minimum camshaft angle must be 111º with degree 7 Bézier curves.

In table 2, results for minimum camshaft angle (rise motion) using three different curves are shown. Just some Ro/L relations have been included. For cycloidal and harmonic curves, nomogram in figure 2 was used. Table 1 was employed for Bézier curves.

Table 2. Comparative results of minimum camshaft angle with pressure angle of 30º using different curves.

R_o/L	Degree 5 Bézier Curves	Degree 7 Bézier Curves	Degree 9 Bézier Curves	Harmonic Curves	Cycloidal Curves
1.5	95º	111º	117º	80º	100º
2	75º	88º	95º	60º	80º
3	53º	62º	69º	45º	56º
4	41º	48º	54º	35º	43º

6. CONCLUSIONS

Maximum pressure angle is a function of type of curve used. Degree 9 Bézier curves present pressure angle higher than results obtained by traditional curves in rise motion. As a consequence, minimum camshaft angle, β , is higher in degree 9 Bézier curves.

To avoid elevated pressure angle in rise motion, degree 9 Bézier curves need to use higher prime circle radius than traditional curves in order to get similar cam's performance. Due to primer circle radius variation, radius of curvature change and cam's profile too.

In order to rationalize cam design, other degrees Bézier curves should be evaluated to compare with traditional curves. Degree 9 Bézier curves, evaluated in this paper, have not present advantage comparing with traditional curves.

7. REFERENCES

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