11th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2007, Hammamet, Tunisia, 05-09 September, 2007.

NEW ANALITICAL SOLUTION TO PREDICT THE PRESSURE IN THE UPSETTING PROCESS

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ABSTRACT
The objective of this paper is obtaining a solution to predict the average pressure in the upsetting process. The most important contribution of the obtained solution consists of considering the shear stress in the von Mises yield criterion. The comparison of the simulation with the experimental data shows that the new solution correctly predicts the behaviour during the upsetting process.
Keywords: Upsetting, analytical solution, plastic forming.

1. INTRODUCTION

Different analytical, empirical, numerical, and experimental methods were developed to predict the pressure in the upsetting process, and consequently to be able to determine the best combination of the process parameters. The analytical methods most used for analysis and simulation in upsetting are: homogeneous deformation, slab method, and the upper bound technique [1].

The distribution of pressure in the upsetting process is obtained by a new analytical solution of this process using a material with axisymmetric geometry. This work consists of developing a model of upsetting pressure based on a method of equilibrium of forces on an infinitesimal free body. The forces balance is made on a piece whose thickness is of infinitesimal material. This technique generates ordinary differentials equations, where dependent variables are function of only one space coordinate, among other problem variables. The main innovation of this work relative to existing works and to the "classic slab method" is the introduction of the effect of the internal distortions of the material, produced by friction forces, on the differential equation that rules the problem. The internal distortions of the material have an influence on the orientation of the main directions in an infinitesimal element of matter. The internal distortions are introduced in the model by the shear stress τ that appears in the von Mises equation, which is not considered negligible.

2. MATHEMATICAL MODEL

A model predicting the average stress in the upsetting process for a cylindrical geometry is developed. This model is obtained by making an equilibrium of forces on a differential element. The differential equation associated with the problem of forces is analytically solved and integrated, and thus a function of the average pressure P_a is obtained.

These are the hypothesis made to obtain a model of the upsetting process:

I) The materials of the bodies under study are considered as rigid-plastic.
II) The plastic deformation is plane strain.
III) The average stresses are distributed uniformly within elements.
IV) It is assumed that there is friction in the punch-material interface, and the constant friction coefficient is considered.

2.1 Equilibrium Equation

Making an equilibrium of forces in the radial direction allows to obtain the following differential equation:

$$\frac{\partial \sigma_r}{\partial r} - \frac{2\mu}{h} P = 0 \tag{1}$$

To solve (1), it is necessary to have a function relating the pressure P and the stress σ_r. It is assumed that pressure relates to the radial stress through a linear equation like the one shown in (2), where A and B are constants to determine:

$$P = A - B\sigma_r \tag{2}$$

Replacing (2) in (1), a common differential equation is obtained. The solution of this differential equation with the conditions (3) is shown in the equation (4).

$$P\big|_{r=0} = P_{\max} \qquad\qquad P\big|_{r=R} = A = 2K \tag{3}$$

This equation (4) relates the upsetting pressure to the independent variables which rule the process: radial position r, radius of the disc R, dynamic friction coefficient μ, and thickness of the disc h.

$$P(r) = 2Ke^{\frac{B2\mu(R-r)}{h}} \tag{4}$$

Defining the average pressure as:

$$P_a = \frac{2}{R^2}\int_0^R P(r)\cdot r \cdot dr\,, \tag{5}$$

an equation for the average pressure P_a is obtained

$$P_a = 2\frac{2K}{B}\frac{h}{\mu D}\left[\frac{h}{B\mu D}\left(e^{\frac{B\mu D}{h}} - 1\right) - 1\right] \tag{6}$$

where D: diameter of the disc.

2.2 Yield Criterion

Normal stresses and shear stresses in a differential element relate through the von Mises yield criterion.

Assuming the hypothesis, and supposing that the increase in the plastic deformation depends on the stress deviatoric tensor, the von Mises criterion (7) is used as a yield or plastic discontinuity criterion.

$$\left|\sigma_x - \sigma_y\right| = 2k\sqrt{1 - 4\left(\frac{\tau}{2k}\right)^2} \tag{7}$$

Where k is the yield limit for pure shear.

Two approximations to determine the yield criterion are described below.

2.2.1 First Yield Criterion

The upsetting pressure P is related to the stress σ_r through the equation (8), assuming that the shear stress τ is negligible.

$$P = 2k - \sigma_r \tag{8}$$

Then B=1, replacing this constant in the general solution (6), the first approximation for the dimensionless average pressure is obtained.

$$\frac{P_a}{2k} = \frac{2h}{\mu D}\left[\frac{h}{\mu D}\left(e^{\frac{\mu D}{h}} - 1\right) - 1\right] \tag{9}$$

2.2.2 Second Yield Criterion

In this second approximation, the shear stress τ is considered different from zero in the equation (7), assuming that the shear stress τ is equal to $-\mu P$, thus obtaining:

$$\sigma_r + P = 2k\sqrt{1 - 4\left(\frac{\mu P}{2k}\right)^2} \tag{10}$$

Finding the value of the pressure P from (10):

$$P = \frac{2K}{\sqrt{1+4\mu^2}} - \frac{1}{1+4\mu^2}\sigma_r \tag{11}$$

Where the term multiplying σ_r in (11) is equal to B. Replacing this constant in the general solution (6), a new approximation for the average pressure is obtained.

$$\frac{P_a}{2K} = 2\left(1+4\mu^2\right)\frac{h}{\mu D}\left[\frac{h\left(1+4\mu^2\right)}{\mu D}\left(e^{\frac{\mu D}{h\left(1+4\mu^2\right)}} - 1\right) - 1\right] \tag{12}$$

3. RESULTS

This section shows the results of the simulations. Figure 1 shows the graphs of the equations (9) and (12), which are compared to the experimental data obtained by Hu *et al*. [2]. The broken line corresponds to the simulation made with the first yield criterion, the continuous line corresponds to the second yield criterion, and dots correspond to the experimental data.

This figure shows the influence of the shear stress on the von Mises criterion. Considering the shear stress, the yield criterion is reached earlier. In this case the shear stress existing in the continuous medium contributes to the plastic deformation. The influence of the shear stress on the von Mises criterion increases as the relation D/h increases too.

Figure 2 shows the error defined by the dimensionless Euclidean norm between the simulation made with the second yield criterion and the experimental data. The error observed is lower than 16 % for both values of μ. The error gets higher as the friction coefficient μ increases. Therefore, the monodimensional model works properly when μ is small, which is the expected situation.

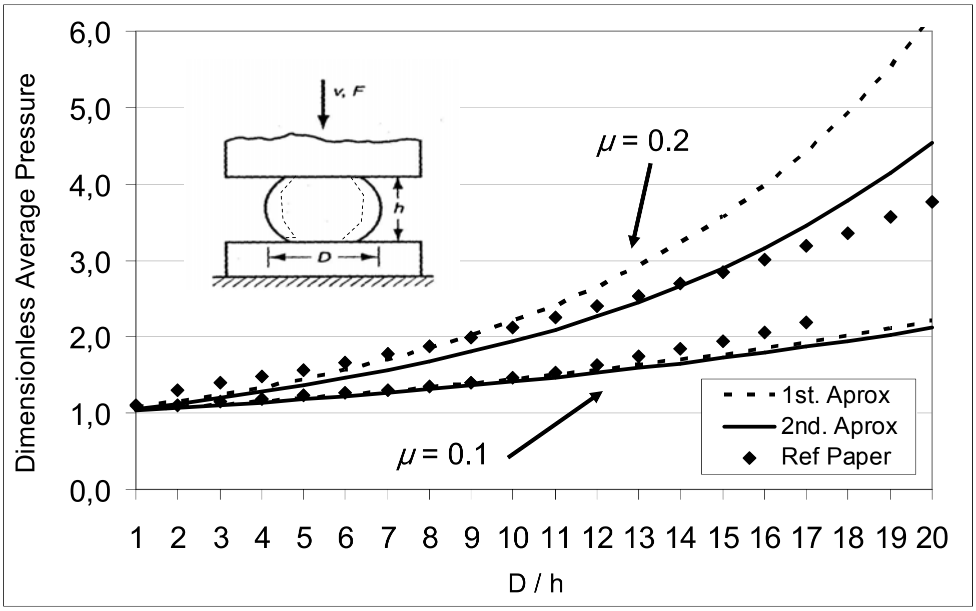

Figure 1. Comparison between the simulated and the experimental average pressure for different values of D/h, and for two friction coefficients.

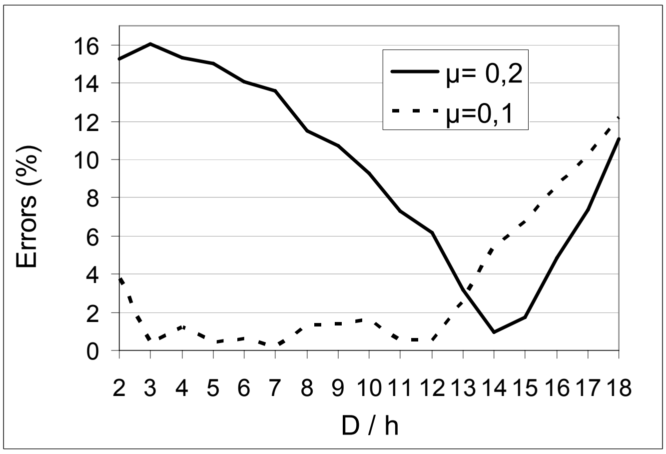

Figure 2. Error in the forecast made with the second yield criterion with regard to the experimental results obtained by Hu et al.

4. CONCLUSIONS

A new predictive model of the upsetting stress was successfully developed. This is an analytical model with an easy numerical implementation.

The analytical model includes the shear stresses in the von Mises criterion, thus improving the forecasts made by other analytical methods of simulation.

5. REFERENCES

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