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THE INFLUENCE OF SMALL STRENGHTENINGS AND WEAKENINGS ON LOAD CARRYING CAPICITY OF PLATES

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ABSTRACT
Lower quality plates are often manufactured by cutting or stamping sheet metal produced by cold rolling and delivered in rolls. This sheet metal cannot be ideally straightened and its thickenss is often not equal to the nominal thickness. Due to the manufacturing technology and the straightening process, places of unequal thickness can be observed in the direction of rolling, i.e. at right angle to the edge of the roll. The load carrying capacity of a square plate depends on the angle at which it is cut out from such a sheel metal plate or on the direction of strengthening or weakening with respect to the loaded edge. The paper gives an estimation of the influence of small strengthenings and weakenings on the stability of a square plate. Strengthenings and weakenings were studied with respect to the angle they form with the loaded edge. It was found that small strengthenings and weekenings forming a right angle with the loaded edge do not affect its stability. However, strengthenings and weakenings forming any other angle with the loaded edge may importantly lower the buckling stability of the plate.
Keywords: square plate, stability, small strengthenings and weakenings

1. INTRODUCTION
Plates are commonly known structural elements that can be loaded so that lateral buckling may occur. Numerous articles have been published treating lateral buckling of plates of a variety of geometric shapes for loading cases in the elastic and plastic domain [1]. Most authors solve the corresponding differential equations numerically and only rarely it is possible to find cases where an analytical solution is offered in closed form [2]. The assumptions most often made are that the plate is made of isotropic material with the modulus of elasticity E, Poission's number ν =0.3 and constant plate thickness [3,4].

The Laboratory for Non-linear Mechanics, Faculty of Mechanical Engineering, University of Ljubljana, working together in cooperation with the company SPE Lamele analysed the mechanical properties of M800-65D sheet metal plates produced by Voest Alpine, Austria and Acroni, Slovenia [5]. The sheet metal plates were produced by the standard procedure. Molten metal is first hot rolled into a strip, then cold rolled, annealed and temperrolled in the width 1000*mm* and thickness 0.65*mm*. Then the strip is cut into minor strips with a width of 216.6*mm* and wound into a roll. These rolls whose material can be taken from the middle or margin of the original strip need to be straightened and prepared for subsequent stamping. From the sheet metal fabricated in this way, a series of test samples were made for the analysis on the Zwick tensile test machine, type Z050/TH3A. The samples were cut from metal at an angle of 0^0, 45^0 and 90^0 with respect to the direction of rolling. Special treatment was devoted to samples cut from the middle and those from the margin of the original strip.

Assuming that the sheet metal has a constant thickness, aseries of experiments were made. It was found that the analyzed sheet metal is approximately orthotropic. The lowest module of elasticity, approximately 165000N/mm^2 was established in the samples that had been cut in the same direction as that of rolling, whereas the samples cut at an angle of 90^0 to the direction of rolling had a modulus of elasticity 176000N/mm^2 and those cut at an angle of 45^0 hed the highest elasticity modulus of 187000N/mm^2. The highest and the lowest modulus differ by approximately 13%. The values of elasticity modulus measured in the samples taken from the sheet metal in the same direction as that of rolling differ up to 7%. The Poission's number was found to be surprisingly high between 0.360 and 0.456, which can be explained by the fabrication process and possible errors in measuring the width and thickness of the samples during the tensile test.

The experiments have shown that low-cost sheet metal is not isotropic and can be treated as orthotropic. The sheet metal from an ideal isotropic material with weakenings or strengthenings caused by fabrication, would behave as orthotropic at constant thickness. The discrepancies between the measured Poisson's numbers and the theoretical ones are not easily explained by measurement errors only. We have reason to believe that the contractions of the samples were influenced by non-uniform thickness of samples. Assuming that the orthotropy is mostly a result of non-uniform thickness of the plate, it is interesting to ask what are the effects of fabrication inaccuracies i.e. orthotropy, on the buckling stability of the plate.

2. PROBLEM FORMULATION AND SOLUTION

The paper treats elastic stability of homogeneous square plates,(Fig. 1) whose orthotropic behavior is achieved by equally spaced strengthening bands of increased plate thickness, or weakenings if the plate thickness of these bands is decreased, $t<h$. The average weight of the sheet metal plate per unit area is stated by the manufacturer and is constant. Thus it is possible to assume that despite the weakenings and strengthenings the volume of the plate remains all the time the same. The plates are simply supported along all four edges. Two different cases are considered. In the first case the two edges, parallel to the y axis, are loaded by an external uniformly distributed compressive in-plane force per unit length N_x, (Fig. 1a), and the width of the strengthening/weakening bands is $a/11$. The direction of strengthening/weakening bands is parallel to the x axis. If the strenghthenings and weakenings ran parallel to the y axis, the lateral buckling stability the plate would be reduced. In the second case all four edges are loaded with uniformly distributed tangential in-plane forces per unit length $N_{xy} = N_{yx}$, (Fig. 1b), and the width of the strengthening/weakening bands is $a\sqrt{2}/11$. The direction of strengthening/weakening bands in this case is parallel to the direction $y = x$, and only at this direction of the strengthening/weakening bands we can expect an increased lateral buckling stability (Fig. 1b). The aim of this paper is to study the influence of small strengthening/weakening bands on the buckling load in both cases.

For a reference square plate with constant thickness h_0 and edge length a, the buckling stress can be written

$$\sigma_{cr} = \frac{N_{cr}}{h_0} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{h_0}{a}\right)^2,$$

where k is the buckling coefficient, E the elastic modulus and ν the Poisson coefficient.

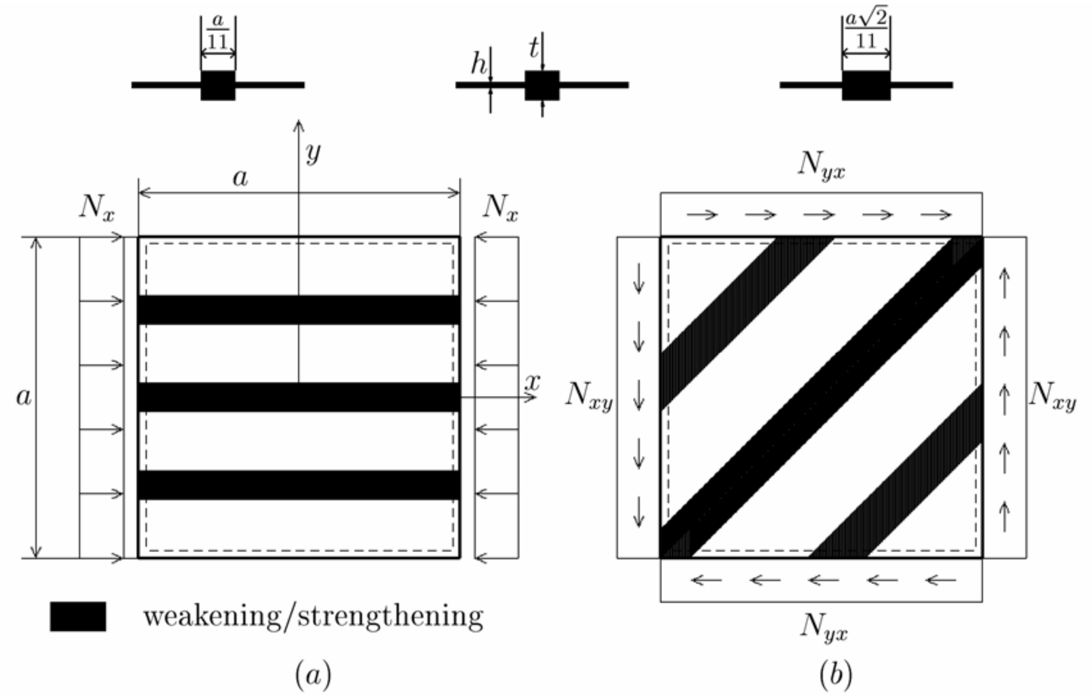

Figure 1. Studied cases: a) compressive forces N_x; b) tangential forces N_{xy}

For stability computations the finite element discretisation to 2.*n.n* triangular elements were used where n = 22. In total 968 finite elements were used. Because of the symmetric loading of the plate, symmetric boundary conditions and the chosen length/width ratio of the bands it is possible to study the load carrying capacity only on one-fourth of the plate. The boundary conditions on the symmetry axes should be chosen in the way they would grant symmetric buckling of the plate.

3. BUCKLING OF PLATE UNDER COMPRESSIVE IN-PLANE FORCE

The buckling coefficients for plates with one, two and three strengthening/weakening bands and different ratios *t/h* were calculated considering the condition of constant volume. To fulfill the condition of constant volume of the plate $V_0 = h_0 a^2 = const.$, the thickness of strengthening/weakening bands t and also the thickness of the remaining plate h can be calculated. In the case of three strengthening/weakening bands and t/h =1.2 we get h =0.948h_0 and t =1.138h_0.

Table 1. Buckling coefficients k at constant volume of the plate, compressive in-plane force

t/h	$k^{(1)}$	$k^{(2)}$	$k^{(3)}$
0.6	3.58	3.86	3.68
0.8	3.87	3.98	3.89
1.0	3.99	3.99	3.99
1.2	4.02	3.98	4.01
1.4	4.03	3.98	4.01

$k^{(1)}$ -plate with one central band
$k^{(2)}$ -plate with two side bands
$k^{(3)}$ -plate with three bands
compare Fig. 1

From Table 1, we can see that if the plate's thickness in a certain direction varies by a few per cent its lateral buckling stability is not significantly lowered if the plate is correctly loaded. It is interesting to study the shape of the buckled plate as a function of different number and thickness of strengthening/weakening bands. In the interval approximately $0.2 \le t/h \le 5.0$ the plate buckles globally in a shape which is slightly similar to the buckling shape of the constant thickness reference plate.

4. BUCKLING OF PLATE UNDER TANGENTIAL IN-PLANE FORCES

The buckling coefficients for plates, Fig. 1b, with one, two or three strengthening/weakening bands for different ratios *t/h*were calculated considering the condition of constant volume once again. To fulfill the condition of constant volume of the plate $V_0 = h_0 a^2 = const.,$ the thickness of

Table 2. Buckling coefficients k at constant volume of the plate, tangential in-plane force

t/h	*k*(1)	*k*(2)	*k*(3)
0.6	4.71	8.49	4.84
0.8	7.26	9.18	7.39
1.0	9.27	9.27	9.27
1.2	10.29	8.99	10.08
1.4	10.27	8.55	8.96

$\overset{(1)}{k}$ -plate with one central band
$\overset{(2)}{k}$ -plate with two side bands
$\overset{(3)}{k}$ -plate with three bands
compare Fig. 1

strengthening/weakening bands *t* and also the thickness of the remaining plate *h* can be calculated. In the case of three strengthening/weakening bands and *t/h*=1.2 we get $h = 0.936 h_0$ and $t = 1.124 h_0$. From Table 2 we can see that if the thickness of the plate in the chosen direction varies by a few per cent, its buckling stability in the case of tangential in-plane forces can be significantly reduced, therefore the plates loaded by tangential in-plane forces should be more carefully made. It is interesting to study the shape of the buckled plate as function of different number and thickness of strengthening/weakening bands. In the interval approximately $0.4 \le t/h \le 1.2$ the plate buckles globally in a shape which is slightly similar to the buckling shape of the constant thickness reference plate.

5. CONCLUSION

The load carrying capacity of a square plate depends on the angle at which it is cut out from a sheet metal strip or on the direction of strengthening or weakening with respect to the loaded edge. It was found that small strengthenings and weakenings forming a right angle to the edge subjected to compressive in-plane force do not affect its stability. However, strengthenings and weakenings forming an angle with the same loaded edge that is different from the right angle may importantly lower the buckling stability of the plate.

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