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ABOUT KINEMATICAL DESCRIPTION IN THE POLYGONAL HOLES DRILLING

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ABSTRACT
The polygonal holes can be made in many different ways but using the rotating tool in drilling process is one of the easiest and most efficient wayst. The aim of this paper is to determinate the motion, and velocity of the tool's blade used for polygonal hole production in the manufacturing process. Given task will be solved using the equivalent mechanism and centrodes theory. Euler-Savary equations with matrix vector notation will be used for defining the velocity of tool's blade as well as geometry of polygonal contour. Presented procedure is very simple and gives precise analytical solutions.
Keywords: kinematics, polygonal holes drilling, velocity of cutting

1. INTRODUCTION
Beside well known methods for production of polygonal holes, they could be also made by drilling. During boring of these holes tool geometry and the shape of polygonal hole itself must be synchronized, which means that the hole contour represents a family of points of tool's blade tip on several consecutive tool positions. The tool during work is guided within the same shape as the hole that has to be produced. Besides rotational motion, tool during work has to perform also a vertical motion. Using Euler-Savary equation and general analytical approach polygonal holes drilling will be kinematically described.

2. ALGORITHM FOR THE KINEMATIC DESCRIPTION OF THE POLYGONAL HOLES DRILLING
One sector from single most general shape of polygonal hole is given in Fig. 1. It is enough to notice, in the observed time, two points of tool blade path which corresponds to enveloped contour leading the tool. It is shown in the Fig.1 that the path of point A and B are enveloped with contours I and II. In that case the direction of speed V_A and V_B are overlapped with contours I and II. Now the direction of speed for point C is determined in relation to the instant velocity pole P determined by turning velocity direction of V_A and V_B by ninety degrees. The same solution can be achieved by using equivalent mechanism shown in Fig.2. Considering mechanism in Fig. 2, the motion could be described as rolling the moveable centrode without sliding over fixed centrodes. The construction of fixed centrode and moveable centrode is given in Fig 3. Fixed centrode is presented with C_N curve, and moveable with C_P curve with current observed conjugation in the instant P_2 pole. The motion of moveable centrode C_P is determined from known angular velocity ω. The moveable coordinate system $P\xi\eta$ is linked to the moveable centrode C_P, while fixed system Oxy coordinates is linked to the fixed centrode C_N. Position vector of point L is given by vector $\vec{r}_L = \vec{r}_1 + \vec{r}_2$. First derivation by time of vector position $\vec{r}_L$ gives velocity of point L.

$$\vec{V}_L = \frac{d\vec{r}_L}{dt} = \frac{d\vec{r}_1}{dt} + [\vec{\varpi}, \vec{r}_2] = \vec{j}\dot{\theta}(x_L - x_p) - \vec{i}\dot{\theta}(y_L - y_p) \qquad \ldots(1)$$

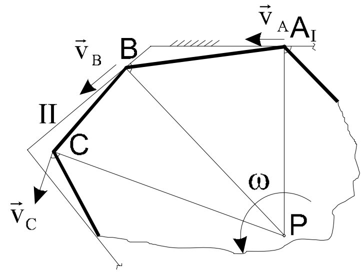

Figure 1. Contour of polygonal hole

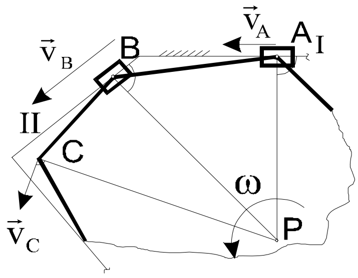

Figure 2 . Equivalent mechanism

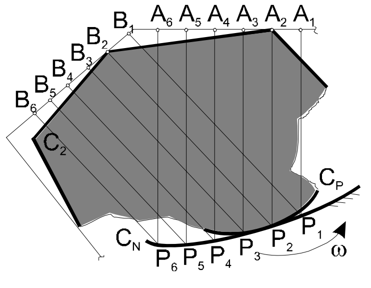

Figure 3. Design of centrodes

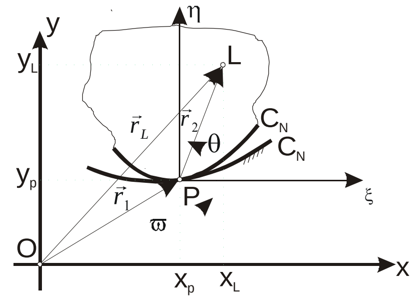

Figure 4. Kinematic of centrodes curvature

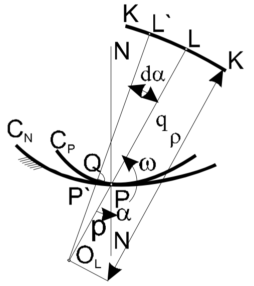

Figure 5. Position and radius of roulette k-k

3. ANALYTICAL METHOD OF KINEMATIC DESCRIPTION OF THE POLYGONAL HOLE DRILLING

The fixed centrodes points (instantaneous pole of velocity), as intersection point between lines $y_{1n}(x,\theta,n)$ and $y_{2n}(x,\theta,n)$ are found. Parametric equations of centrodes are:

$$X^{l}{}_{PN}(\theta,n)=\frac{b_{1n}(\theta,n)-b_{2n}(\theta,n)}{a_{2n}(n)-a_{1n}(n)}\ ,\quad Y^{l}{}_{PN}(\theta,n)=a_{1n}(n)\cdot X^{l}{}_{PN}(\theta,n)+b_{1n}(\theta,n) \qquad \ldots (2)$$

Where the equations of the straight lines $y_{1n}(x,\theta,n)$, $y_{2n}(x,\theta,n)$ are given by:

$$y_{1n}(x,\theta,n)=a_{1n}(n)\cdot x+b_{1n}(\theta,n),\quad y_{2n}(x,\theta,n)=a_{2n}(n)\cdot x+b_{2n}(\theta,n) \qquad \ldots(3)$$

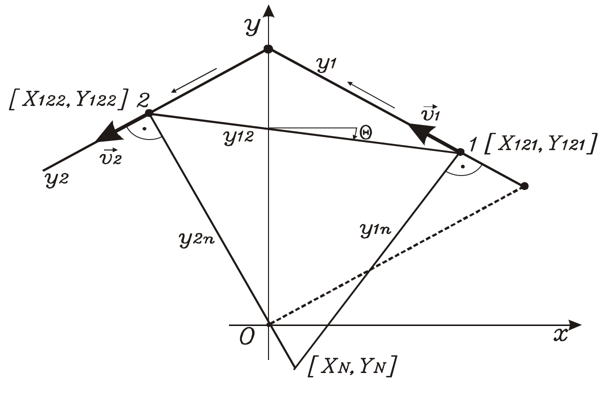

Figure 6. Velocity and instantaneous pole of velocity

Where: $a_{1n}(n)=-\dfrac{1}{k(n)}$, $a_{2n}(n)=\dfrac{1}{k(n)}$ are the coefficients of direction.

Parameters $b_{1n}(\theta,n)$, $b_{2n}(\theta,n)$ are determined from the condition that the directions $y_1(x,\theta,n)$, $y_{1n}(x,\theta,n)$ have intersection point (X_{121} , Y_{121}) , and directions $y_2(x,\theta,n)$, $y_{2n}(x,\theta,n)$ have intersection in *(*X_{122} , Y_{122}*).*

$$b_{1n}(\theta,n)=Y_{121}(\theta,n)-a_{1n}(\theta,n)\cdot X_{121}(\theta,n),\quad b_{2n}(\theta,n)=Y_{122}(\theta,n)-a_{2n}(\theta,n)\cdot X_{122}(\theta,n) \qquad \ldots(4)$$

These equations are valid for $(-\theta_g(n) \le \theta \le \theta_g(n))$ surrounding of the axis of the bisectors of the angle hole, and from this reason we labeled them as local coordinates of the fixed centrodes (index "l" is used).

Movable centrodes with coordinates $X^l_{PP}(\theta,n)$, $Y^l_{PP}(\theta,n)$ are rolling over the fixed centrodes *without* sliding. With coordinates of the tool center and coordinates of the fixed centrodes known, it is easy to find the coordinates of the movable centrodes. Movable centrode is obtained if fixed centrode is expressed in movable coordinate system ($x_1O_1y_1$). System ($x_1O_1y_1$) is connected to the tool. Axis (y_1) presents the axis of the tool side being led along the hole sides, and the beginning of the coordinate (O_1) is in the center of the tool. It can be seen from the Fig. 7 that the vector equation: $\vec{r}_{PN} = \vec{r}_C + \vec{r}_{PP}$ is valid.

Coordinates of the movable centrodes present local values because they exclusively stand for the surroundings of bisectors axis of the observed holes angle. Using matrix transformation between coordinate system ($x_1O_1y_1$) and (xOy) following expression is obtained:

$$\begin{bmatrix} X_{PP}(\theta,n) \\ Y_{PP}(\theta,n) \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & -X_c(\theta,n)\cdot\cos(\theta) - Y_c(\theta,n)\cdot\sin(\theta \\ -\sin(\theta) & \cos(\theta) & X_c(\theta,n)\cdot\sin(\theta) - Y_c(\theta,n)\cdot\cos(\theta) \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_{PN}(\theta,n) \\ Y_{PN}(\theta,n) \\ 1 \end{bmatrix} \quad \ldots(5)$$

If the trajectory of a single tip point of the tool blade depicted by vector position $\vec{r}(s)$ at the holes corner point is known, then the radius of curvature can be obtained in the form (5):

$$\rho(s) = \frac{\left|\dot{\vec{r}}(s)\right|^3}{\left|\dot{\vec{r}}(s) \times \ddot{\vec{r}}(s)\right|} \qquad \ldots(5)$$

s is a parameter defining the trajectory of the tool's point. In this case the tip point of the tool has the following coordinates: $X_m(\theta,n,m)$, $Y_m(\theta,n,m)$ and its position in fixed coordinate system Oxy is:

$$\vec{r}_m(\theta,n,m) = X_m(\theta,n,m)\cdot\vec{i} + Y_m(\theta,n,m)\cdot\vec{j} \qquad \ldots(6)$$

The curvature radius as a function of parameter (θ) is given by the expression (7):

$$\rho(\theta,n,m) = \frac{\left|\dot{\vec{r}}_m(\theta,n,m)\right|^3}{\left|\dot{\vec{r}}_m(\theta,n,m) \times \ddot{\vec{r}}_m(\theta,n,m)\right|} = \frac{\left|\sqrt{\dot{X}_m(\theta,n,m)^2 + \dot{Y}_m(\theta,n,m)^2}\right|^3}{\left|\dot{X}_m(\theta,n,m)\cdot\ddot{Y}_m(\theta,n,m) - \ddot{X}_m(\theta,n,m)\cdot\dot{Y}_m(\theta,n,m)\right|} \quad \ldots(7)$$

The obtained radius represents non-dimensional value. The real radius is obtained using following relation: $\rho(\theta,n,m)_{real} = a\cdot\rho(\theta,n,m)$.

Velocity of the tip points can be easily obtained through general kinematics relation:

$$\vec{v}_m(\theta,n,m) = \dot{\vec{r}}_m(\theta,n,m) = \dot{X}_m(\theta,n,m)\cdot\vec{i} + \dot{Y}_m(\theta,n,m)\cdot\vec{j} \qquad \ldots(8)$$

Absolute velocity of the tool blade is obtained as:

$$v_m(\theta,n,m) = \sqrt{\dot{X}_m(\theta,n,m)^2 + \dot{Y}_m(\theta,n,m)^2} \quad \text{(m/rad)} \qquad \ldots(9)$$

Since all length measures are non-dimensional compared to the length of holes side (a), the real tool blade velocity is obtained as: $v(\theta,n,m)_{real} = a\cdot v(\theta,n,m)$. While drilling, the tool is rotating with a constant angular velocity $(\dot{\theta})$, so relation $\theta(t) = \dot{\theta}\cdot t$ can be used and velocity is obtained as:

$$v_m(t,n,m) = \dot{\theta}\cdot\sqrt{\dot{X}_m(\theta,n,m)^2 + \dot{Y}_m(\theta,n,m)^2} = \dot{\theta}\cdot v_m(\theta,n,m) \quad \text{(m/s)} \qquad \ldots(10)$$

With described analytical procedure it is possible to analyze geometry and motion of tool during drilling n-angle polygonal holes. Length of the side of polygonal tool is given as:

$$a(n) = A\cdot(\cos(\delta_1(n)) + \sin(\delta_1(n))\cdot\tan(\delta_2(n))) \qquad \ldots(11)$$

if n>4, otherwise a(4)=A. In Fig. 7 some possible polygonal holes shapes are presented.

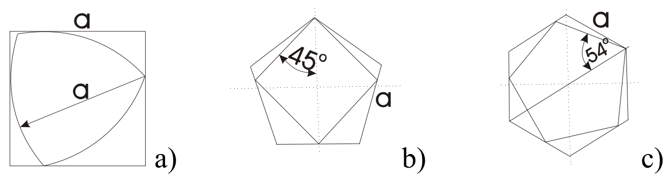

Figure 7. Different shapes of the polygonal holes

4. EXAMPLE

Problem of boring of the seven angles hole will be solved as an example of the mentioned procedure. The results of the analysis were obtained using program MATHCAD.

Results:

Side of a polygonal (six angles) tool: $a(7) = A \cdot (\cos(\delta_1(7)) + \sin(\delta_1(7)) \cdot \tan(\delta_2(7)) = 1.067 \cdot A$

Eccentricity of the pathway of the tool during drilling of the square holes: e=1.005.

Real shape obtained during drilling of the seven angles hole is given in Fig. 8.

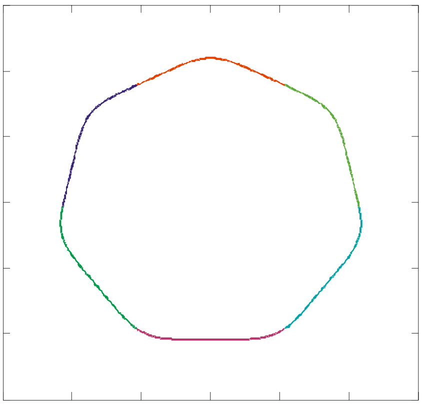

Figure 8. Real shape of the seven angles hole

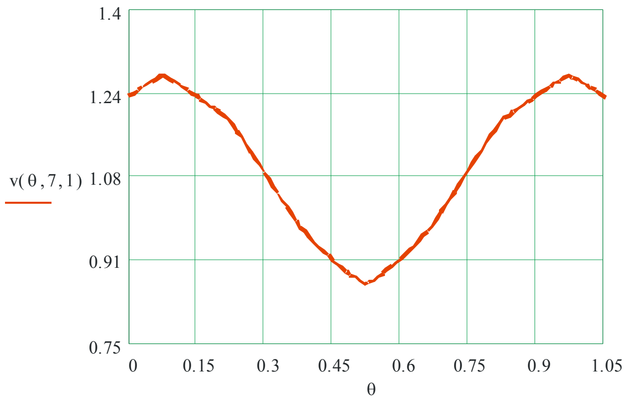

Figure 9. Diagram of the velocity of the blade cutting tool

Minimal radius of the contour hole curve is obtained at the point crowns of the hole. For seven angles hole the minimum radius is: $\rho_{min}(\theta = 7 \cdot \theta_g, 7, 1) = 0.35 \cdot A$

In Fig. 9, velocity of the six angles tool in the boring of the seven angles polygonal hole is given. Maximal and minimal (non-dimensional) speeds of the cutter are: $v_{max} = 1.272 \cdot A$; $v_{min} = 0.868 \cdot A$.

5. CONCLUSION

With kinematically described analytical procedure it is possible to analyze geometry and movement of tool during boring polygonal holes with n angles. Analytical approach and method are used for problem solving which makes the application easy and at the same time generalizes the procedure.

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