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KINEMATICAL AND DYNAMIC ASPECTS OF A CYCLOIDAL PLANETARY REDUCER WITH MODIFIED STRUCTURE

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ABSTRACT
The paper deals with some representative geometrical and dynamic aspects of a new cycloidal reducer variant with modified structure, proposed by the authors: geometrical optimization of the cycloidal gear, analysis of the transmission ratio and efficiency, and analysis of the dynamic response in a relevant functioning case.
Keywords: planetary cycloidal reducer, speed ratio, efficiency, dynamic response.

1. INTRODUCTION
A new variant of a cycloidal reducer with one sun gear, proposed by the authors, is illustrated in Fig. 1; it contains a cycloidal gear pair with rollers, consisting of a fix sun gear with internal cycloidal teeth 3 and the eccentric rollers 2. The element *H* (which contains an eccentric bearing) designates the reducer's input, while the element 1 (on which the rollers 2 are eccentrically articulated) designates the output. In the premise that the reducer uses $z_2 = 20$ rollers (as teeth), then $z_3 = z_2 + 1 = 21$ teeth and, implicitly, the reducer accomplishes the transmission ratio [1]:

$$i_{H1}^{3} = \frac{\omega_{H3}}{\omega_{13}} = \frac{-\omega_{3H}}{\omega_{1H} - \omega_{3H}} = \frac{1}{1 - i_0} = -20 \text{, where } i_0 = i_{13}^{H} = i_{12}^{H} \cdot i_{23}^{H} = +1 \cdot \left(+ \frac{z_3}{z_2} \right) = \frac{21}{20} = 1.05 \,. \quad (1)$$

The aims of the geometrical and dynamic modeling, presented in the paper, are:

1. *Geometrical optimization* of an internal cycloidal gear pair with rollers, based on the premises (Fig. 2):
   a. the cycloidal gear pair has the radius of the generating circle $R_g = 90$ [mm] and the radius of the base circle $R_b = 94{,}5$ [mm];
   b. there are used as optimization parameters: the roller's radius r_r and the distance h (from the centre of the generating circle O_g to the roller's centre – the generating point P), based on which the coefficient of the cycloidal teeth addendum modification is defined $x = (h - R_g)/R_g$.
2. Modeling the *movement equation* in the premises of *friction neglecting* and respectively *friction considering,* and simulating the dynamic response of a machine obtained by connecting the *planetary cycloidal reducer* to a *motor* and a *pump,* with the following mechanical characteristics:

$$\textit{motor}:\ T_i = -a_1 \cdot \omega_i + b_1 \,;\ \textit{pump}:\ T_o = -a_2 \cdot \omega_o \,;\ a_1 = 0.1273\ N{\cdot}m{\cdot}s,\ b_1 = 25.6\ N{\cdot}m,\ a_2 = 1\ N{\cdot}m{\cdot}s. \quad (2)$$

The reducer is characterized by the following speed and acceleration transmitting functions:

$$\omega_{13} = \frac{\omega_{H3}}{i_{H1}^{3}} = (1 - i_0)\omega_{H3} = -\frac{1}{20}\omega_{H3} \,;\quad \varepsilon_{13} = -\frac{1}{20}\varepsilon_{H3} \,. \quad (3)$$

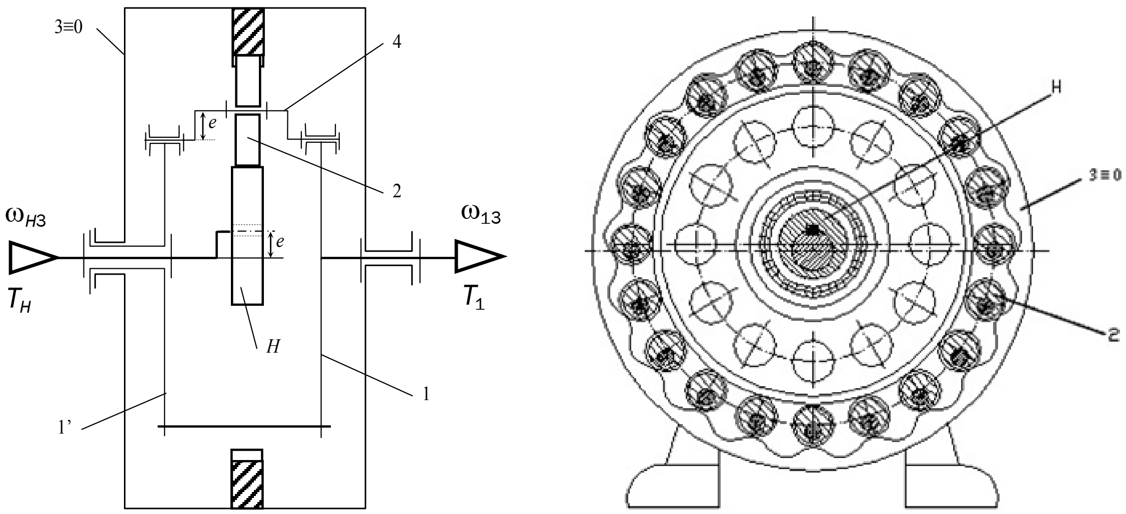

Figure 1. The cycloidal reducer equipped with a sun gear 3 and eccentric rollers 2.

2. ON THE GEOMETRICAL OPTIMIZATION OF THE CYCLOIDAL GEAR PAIR WITH ROLLERS

According to Fig. 1 and 2 and the previous premises, it outcomes that the cycloidal gear pair, that will be optimized, has:

1) a generating circle (of radius R_g = 90 [mm]), in which plane there are z_2 = 20 equidistant generating points *P*, materialized by the centers of the z_2 = 20 rollers;
2) a base circle (of radius R_b = 94,5 [mm]); the z_2 = 20 generating points describe in its plane a hypocycloid with $z_3 = z_2+1 = 21$ loops (Fig. 3,*a*);
3) a cycloidal gear with z_3 = 21 teeth, which resulted as the envelope curve of a family of circles of r_r radius and the centers on the hypocycloid (Fig. 3,*b*) with z_3 = 21 loops (Fig. 3,*c*).

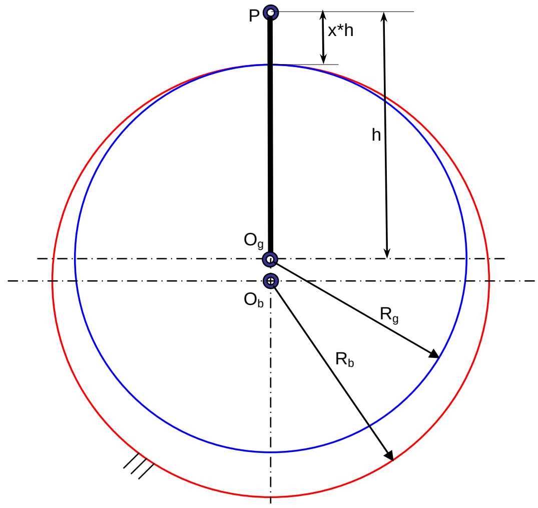

Figure 2. The geometrical parameters of a hypocycloid.

The geometrical optimization of this gear was performed by applying the following algorithm:

a) Firstly, a class of hypocycloids is generated using a set of values (properly chosen) for the coefficient *x* (and, implicitly, for the distance *h* – see Fig. 2);
b) Then, a subclass of gears is generated from each hypocycloid that was previously obtained (as envelope curve), using a set of values (properly chosen) for the roller's radius r_r;
c) The optimal variant (described through the values of the quantities *x* and r_r that complies with the geometrical and the constructive requirements) is extracted from the obtained set of generated gears.

From the analysis of the obtained gears, it outcomes that the gear with the parameters x = +0,333 and r_r = 15 [mm] represents the optimal solution (Fig. 3), both from the geometrical and constructive point of view.

3. DYNAMIC MODELLING OF THE CYCLOIDAL PLANETARY REDUCER

The *main objective* of the dynamic modeling of the reducer is to obtain the torque transmitting function: $T_H = T_H(\varphi_H, T_1)$. This function is established using *Lagrange* method $\frac{d}{dt}\left(\frac{\partial E_c}{\partial \omega}\right)-\left(\frac{\partial E_c}{\partial \varphi}\right)=Q$ in the case of friction neglecting (Fig. 4,*a*), and *Newton – Euler* method (Fig. 4,*b*,*c*) in the case of friction considering. The dynamic models are derived neglecting the inertial effects of the component kinematical elements, excepting the input and output elements (*H* and 1, Fig. 4).

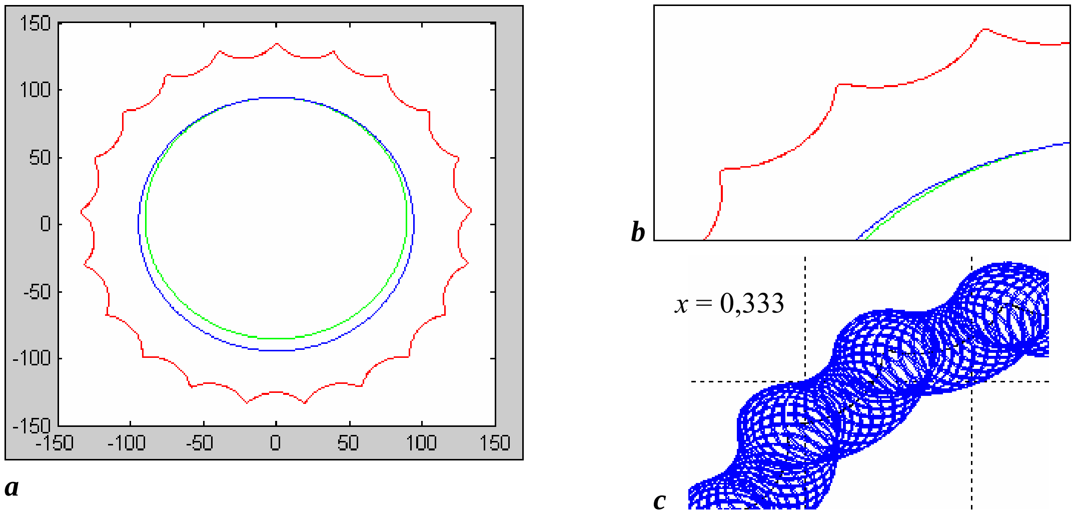

Figure 3. Hypocycloid generated with the addendum coefficient x = 0,333 (h = 120 mm) and r_r = 15 mm.

a) Friction neglecting case

Conformity with Fig. 4,*a*, the kinetic energy E_C and the generalized force Q are:

$$E_c=\frac{1}{2}\left(J_H\cdot\omega_H^2+J_1\cdot\omega_1^2\right)=\frac{1}{2}\cdot\omega_H^2\cdot\left[J_H+\left(1-i_0^2\right)\cdot J_1\right];\; Q=\;T_i+T_o\cdot\left(1-i_0\right), \qquad (4)$$

where J_i, i = 1,H is the axial inertial moment of the input element H, and respectively of the output element 1, with following values considered in numerical simulations: J_1 = 0.02 and J_H = 0,06 [$kg\cdot m^2$].

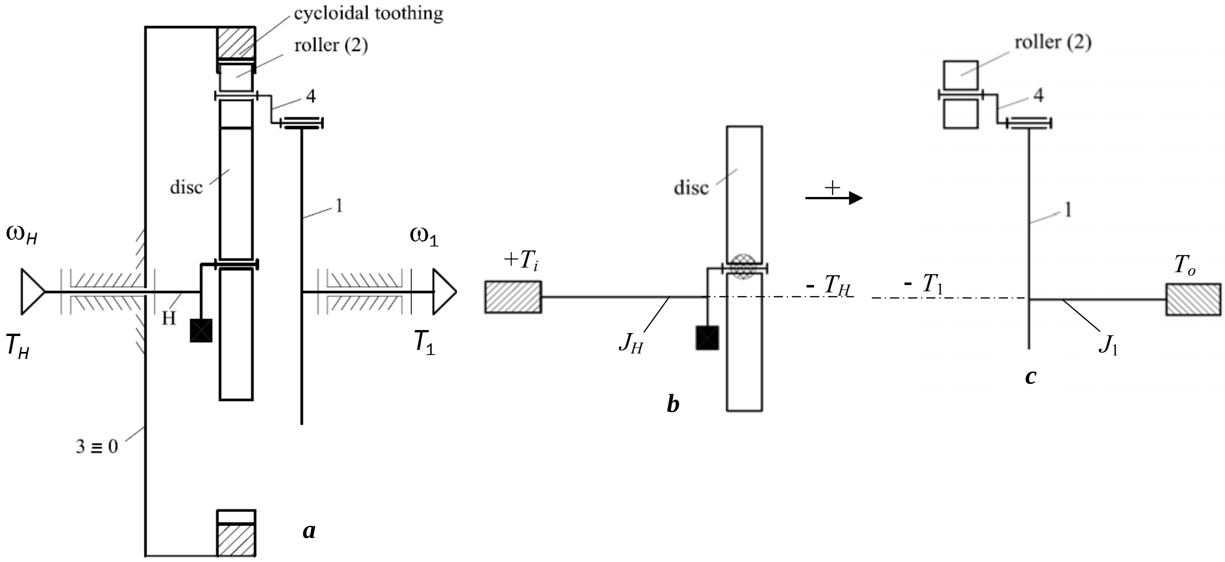

Figure 4. Kinematical scheme a) and dynamic scheme (Newton-Euler method): b) element H, c) element 1.

The movement equation of the considered machine *CC motor – planetary reducer – pump*, in the premise of friction neglecting, is finally obtained:

$$\left[J_H+J_1(1-i_0)^2\right]\cdot\varepsilon_H+\left[0.1237+(1-i_0)^2\right]\cdot\omega_H-25.6=0\;\Rightarrow 0.06005\cdot\varepsilon_H+0.1262\cdot\omega_H-25.6=0\,.(5)$$

b) Friction considering case

In this case, conformity with Fig. 4,*b* and *c*, the following equations can be written:

$$J_H\cdot\varepsilon_H=T_i-T_H\,,\;J_1\cdot\varepsilon=-T_1+T_0\,; \qquad (6)$$

these relations must be completed with the torque equations of planetary unit (see Fig. 4,*a*) [1, 2]:

$$T_1+T_3+T_H=0,\; T_1\cdot i_0\cdot\eta_0^{\,w}+T_3=0. \qquad (6')$$

Finally, the machine *movement equation* in the premise of *friction considering* is obtained:

$$\left[J_H + (1-i_0)\cdot(1-i_0\eta_0^w)\cdot J_1\right]\cdot \varepsilon_H + \left[0.1237 + (1-i_0)\cdot(1-i_0\eta_0^w)\right]\cdot \omega_H - 25.6 = 0 \Rightarrow$$

$$0.06006\cdot\varepsilon_H + 0.1267\cdot\omega_H - 25.6 = 0\,, \tag{7}$$

where we considered $\eta_0 = \eta_{13}^H = \eta_{12}^H \cdot \eta_{23}^H \cong 0.99$ and $w = sgn(\omega_{1H}\cdot T_1) = -1$.

In the premise of $\varepsilon_H = 0$, the *operating point* of the machine in stationary stage is obtained from eq. 5 and 7, in the premises of:

a) *friction neglecting:* $\omega_H \cong 202.85\ s^{-1}$, $T_H \cong 0.507\ Nm$, $\omega_1 \cong -10.14\ s^{-1}$, $T_1 \cong 10.14\ Nm$.

b) *Friction considering*: $\omega_H \cong 202.02\ s^{-1}$, $T_H \cong 0.606\ Nm$, $\omega_1 \cong -10.1\ s^{-1}$, $T_1 \cong 10.1\ Nm$.

The machine responses in relation with time for speed, acceleration and torque, in both premises of friction considering and respectively friction neglecting, are represented in Fig. 5.

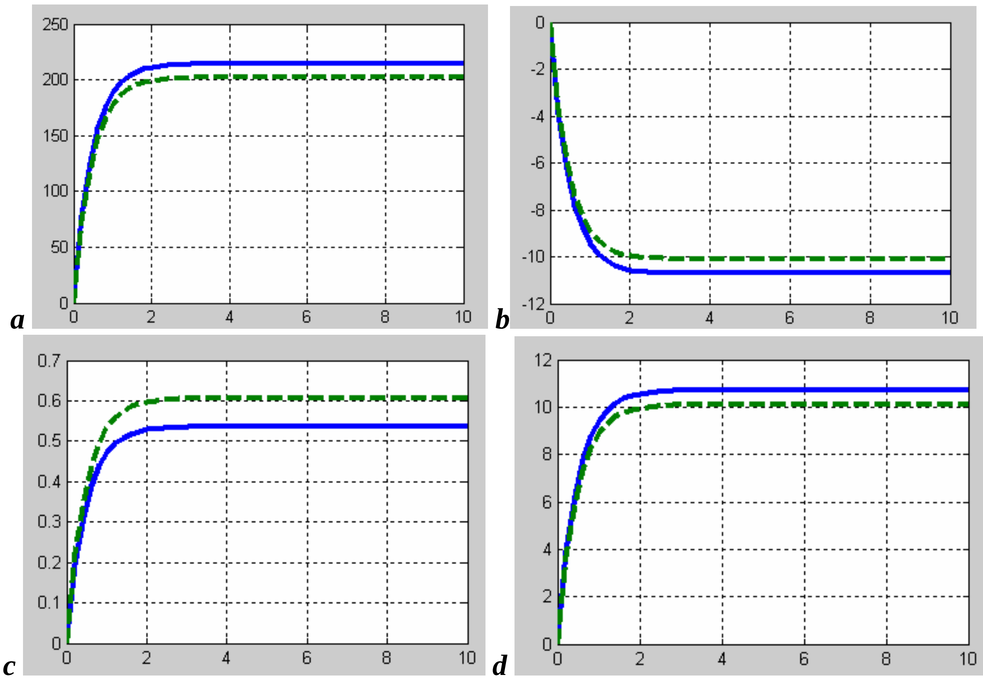

Fig. 5. *The variation in time [s] of the speeds a) ω_H [s^{-1}] and b) ω_1[s^{-1}], and torques c) T_H [Nm] and d) T_1 [Nm], friction considering (dashed line) and friction neglecting (continuous line).*

4. CONCLUSIONS

a) The optimal values of the geometrical parameters x and r_r, considering the radiuses R_g and R_b and the number of rollers as known parameters, were established using a proper Matlab application.

b) The analysis of the generated gears highlights the fact that there are preferred the relative high positive values of the addendum modification coefficient x, together with the high values of the roller's radius r_r.

c) Based on the dynamic models obtained in both cases of friction neglecting and friction considering, numerical simulations were performed and the results are systematized in Fig. 5. Accordingly to the Fig. 5, the machine has a starting time of about 2 [*s*]; after this time the machine achieves the stationary stage. At any time *t,* the speed ratio and torque ratio, *without friction*, are identically with the transmitting ratio (in modulus): $|\omega_H/\omega_1| = |T_1(\eta_0 = 1)/T_H(\eta_0 = 1)| = 20$, while the torque ratio *with friction* (in modulus) is lower than the transmitting ratio: $|T_1(\eta_0 < 1)/T_H(\eta_0 < 1)| < 20$.

5. REFERENCES

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