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STRUCTURAL-KINEMATIC MODELING OF HUMAN BODY ANKLE JOINT MECHANICAL SYSTEMS – PART II

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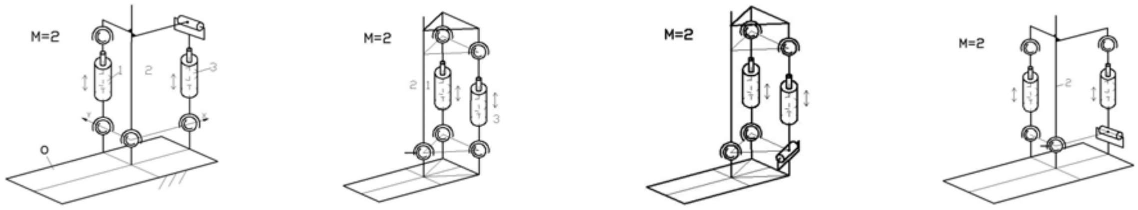
ABSTRACT
Taking into consideration the characteristic models from Part I, of the body ankle joint mechanical system, in this paper are specified their kinematic and geometrical parameters, the relations of the shank's position function, still having the foot as a fixed element of the model.
Keywords: human foot, kinematics, articulated models.

1. THE KINEMATIC MODELING OF SPHERICAL ARTICULATED STRUCTURES

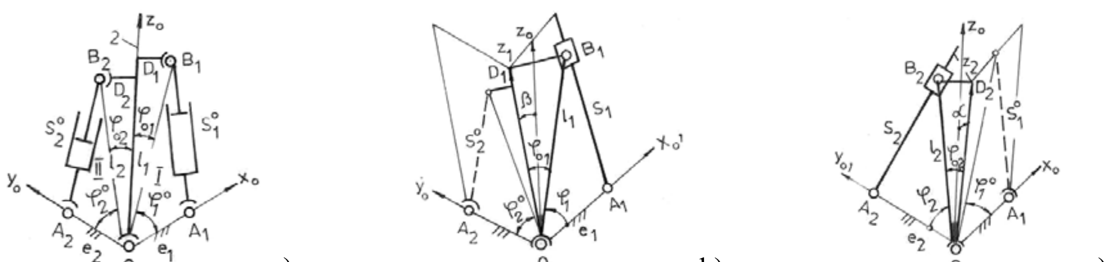
It will be taken into consideration the articulated models with two degrees of freedom for the flexion and pronation movements around the transversal axis y_0 of the foot, respectively the longitudinal one x_0 (fig. 1). The model with the mobility M=1 for only the realization of the flexion, is included in the one with M=2, only through the action of the cylinder [1].



a) b) c) d)

Figure 1.

♦ The geometry of the model from **figure 1,a** is defined by the geometrical parameters: $e_1 = \overline{OA_1}$, $e_2 = \overline{OA_2}$, $l_1 = \overline{OB_1}$, $l_2 = \overline{OB_2}$, $\varphi_{01} = \angle B_1OD_1$, $\varphi_{02} = \angle B_2OD_2$, (fig. 2).



a) b) c)

Figure 2.

The initial reference position (fig. 2, a), corresponds to the shank's position according to the vertical axis z_0, for which the actuators' runs are S_1^0 and S_2^0, respectively the angles $\varphi_1^0 = 90^\circ - \varphi_{01}$, $\varphi_2^0 = 90^\circ - \varphi_{02}$.

At a separate action of the cylinder I or II is produced the flexion around the axis y_0, respectively the pronation around the axis x_0, the two outlines – being perpendicular – do not influence one another, the plane of passive outline being folded like a rigid.

To the current runs S_1 or S_2 taken separately, respectively the angles $\varphi_1^0 \to \varphi_1$ or $\varphi_2^0 \to \varphi_2$ (fig. 2b,c),

$$\varphi_1 = \arccos\frac{l_1^2 + e_1^2 - S_1^2}{2l_1 e_1},\ \varphi_2 = \arccos\frac{l_2^2 + e_2^2 - S_2^2}{2l_2 e_2}, \tag{1}$$

the **rotation β (flexion)** or **α (pronation)** angles will be:

$$\beta = \varphi_1 + \varphi_{01} - 90^\circ,\ \alpha = \varphi_2 + \varphi_{02} - 90^\circ. \tag{2}$$

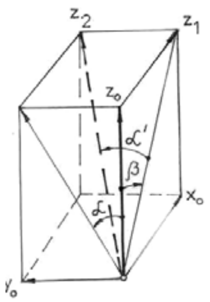
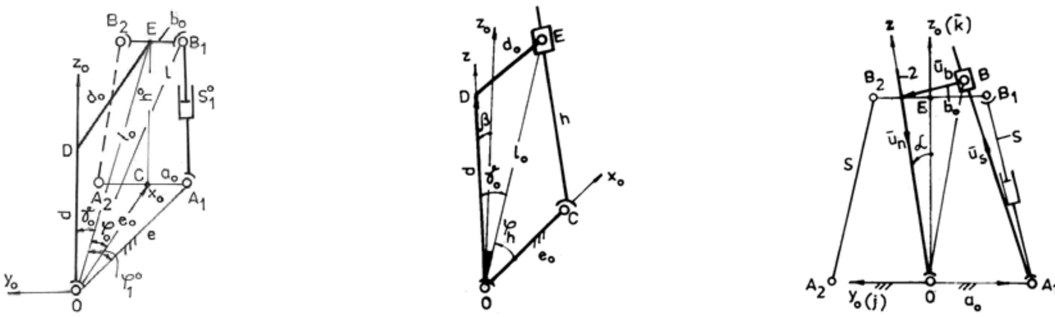


Figure 3.

At the simultaneous action of the cylinders I and II, the angles φ_1/β, φ_2/α will be calculated similar with the relations (1)-(2), only the shank 2 will have a spatial disposing, marked by the diagonal of the axis' paralelipipedon (axis z_2, fig. 3).

♦ The geometry of the model from **figure 1,b** is defined by the geometrical parameters: $e=e_1=e_2=\overline{OA}_{1,2}$, $l=l_1=l_2=\overline{OB}_{1,2}$, $a_0=\overline{CA}_{1,2}$, $b_0=\overline{EB}_{1,2}$, $d_0=\overline{DE}$, ($l_0=\sqrt{l^2-b_0^2}$, $e_0=\overline{OC}=\sqrt{e^2-a_0^2}$, $d=\sqrt{l_0^2-d_0^2}$, $\gamma_0 = \text{arctg}\, d_0/d$, $\varphi_0 = 90^\circ - \gamma_0$) – (fig. 4), the mechanism being symmetrical to the plane $(X_0Z_0) \equiv (OCE^0)$.



a) b) c)

Figure 4.

The initial, reference position (fig. 4,a), corresponds to the shank's position according to the vertical axis z_0, for which the actuator's runs $S_1^0 = S_2^0 = S^0$, respectively $h^0 = \overline{CE^0} = l_0^2 + e_0^2 - 2l_0 e_0 \cos\varphi_0$, $S_1^0 = \sqrt{h^{0^2} + (a_0 - b_0)^2}$, $\varphi_1^0 = \arccos\frac{e^2 + l^2 - (S_1^0)^2}{2el}$.

At the hydraulic actuation of the cylinders with **equal runs** $S_1 = S_2 = S \neq S^0$ takes place the flexion with the angle β (fig. 4,b) around the axis y_0, so $h^0 \to h$, $\varphi_0 \to \varphi_h$:

$$h = \sqrt{S^2 - (a_0 - b_0)^2},\ \varphi_h = \arccos\frac{l_0^2 + e_0^2 - h^2}{2l_0 e_0}, \tag{3}$$

$$\beta = \varphi_n + \gamma_0 - 90^\circ. \tag{4}$$

At the action, if cylinders with unequal curse $S_1 \neq S_2$, respectively $S_1 = S_2 + \Delta S$, the value S_2 will produce the angle β according to the relations (3)-(4), and the difference ΔS will give the pronation α

(fig. 4,c). For tit will be taken into consideration the outline of the mechanism in the plane (A_1A_2E), the trapezium $A_1B_1B_2A_2$ corresponding to the previous case $S_1 = S_2 = S$.
The equation of the loop A_1BECA_1,

$$(S+\Delta s)\overline{u}_s + b_0\overline{u}_b + h\overline{u}_h - a_0\overline{j} = 0$$

contains the versors

$$\overline{u}_b = \overline{j}\cos\alpha - \overline{k}\sin\alpha\,,\ \overline{u}_h = -\overline{j}\sin\alpha - \overline{k}\cos\alpha\,,$$

which through arrangement takes the shape:

$$A\sin\alpha + B\cos\alpha + C = 0 \Rightarrow \operatorname{tg}\frac{\alpha}{2} = \frac{A \pm \sqrt{A^2+B^2-C^2}}{B-C}\,, \tag{5}$$

where:

$$A = 2a_0h\,,\ B = -2a_0b_0\,,\ C = a_0^2 + b_0^2 + h^2 - (S+\Delta S)^2 \tag{6}$$

2. THE KINEMATIC MODELING OF CARDANIC ARTICULATED STRUCTURES

The model with two linear actuators and cardanic joint between the foot and the shank (fig. 3) is geometrically defined by the distances to the joints $e_1 = \overline{OA_1}$, $e_2 = \overline{OA_2}$, $l_1 = \overline{OB_1}$, $l_2 = \overline{OB_2}$, respectively the angles φ_{01}, φ_{02}. The representation from figure 5 corresponds to the shank's 2 reference position, meaning a vertical display according to the axis z_0, for which the actuators runs are $S_1^0 = A_1B_1^0$, $S_2 = A_2B_2^0$, position in which the angles $\alpha = 0$, $\beta = 0$.

To the current runs S_1 and S_2, the angles $\varphi_1^0 \to \varphi_1, \varphi_2^0 \to \varphi_2$, concordant with the relation (1).

At the action only with S_1, the shank 2 rotates itself in the joint C_1 – around the axis y_0 – dproducing the flexion without repercussions for the loop II (fig. 6.)

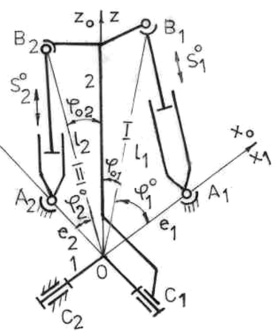
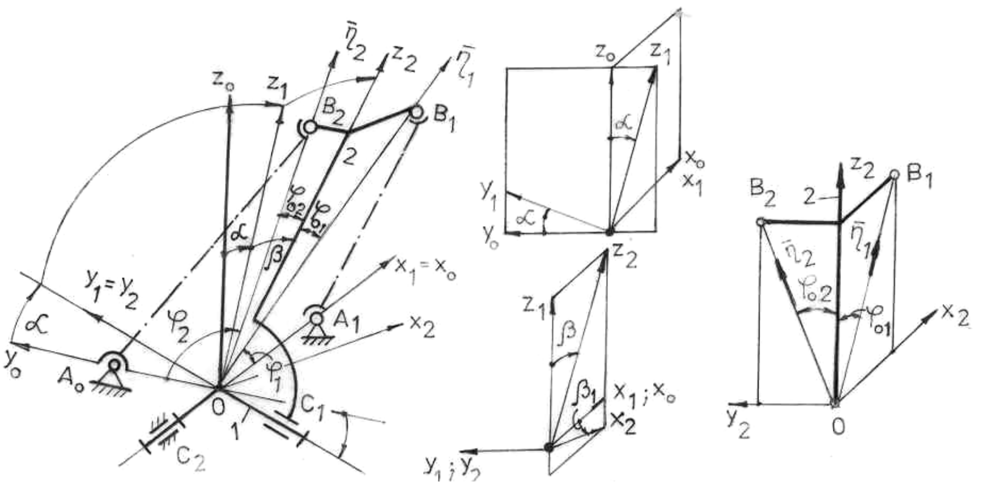


Figure 5.



a) b) c)

Figure 6.

At the action loop II, next to the pronation α around the axis x_0 is being produced also the position modification of the transversal axis $y \to y_1$.
The crossing matrix from system „2" in „0" will be:

$$T_{2\to 0}=T_{1\to 0}\cdot T_{2\to 1}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & -\sin\alpha\\ 0 & \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix}. \quad (7)$$

As the versors $\overline{\eta}_1$ and $\overline{\eta}_2$ of the directions $\overline{OB_1}$, respectively $\overline{OB_2}$ in the shank's system are:

$$\overline{\eta}_{1(2)}=\begin{bmatrix}\sin\varphi_{01}\\ 0\\ \cos\varphi_{01}\end{bmatrix}, \overline{\eta}_{2(2)}=\begin{bmatrix}0\\ \sin\varphi_{02}\\ \cos\varphi_{02}\end{bmatrix}\Rightarrow \eta_{1(0)}=T_{2\to 0}\eta_{1(2)},\ \eta_{2(0)}=T_{2\to 0}\eta_{2(2)}, \quad (8)$$

meaning

$$\eta_{1(0)}=\begin{bmatrix}\sin(\varphi_{01}+\beta)\\ -\sin\alpha\cos(\varphi_{01}+\beta)\\ \cos\alpha\cos(\varphi_{01}+\beta)\end{bmatrix},\ \eta_{2(0)}=\begin{bmatrix}\cos\varphi_{02}\sin\beta\\ \sin\varphi_{02}\cos\alpha-\cos\varphi_{02}\sin\alpha\cos\beta\\ \sin\varphi_{02}\sin\alpha+\cos\varphi_{02}\cos\alpha\cos\beta\end{bmatrix}. \quad (9)$$

The demanded unknowns are the angles α and β.
The versor's $\overline{\eta}_1$ proiection on the axis x_0 will be $\cos\varphi_1$, so the first line from its matrix:

$$\cos\varphi_1=\sin(\varphi_{01}+\beta)\to\beta=\arcsin(\cos\varphi_1)-\varphi_{01}. \quad (10)$$

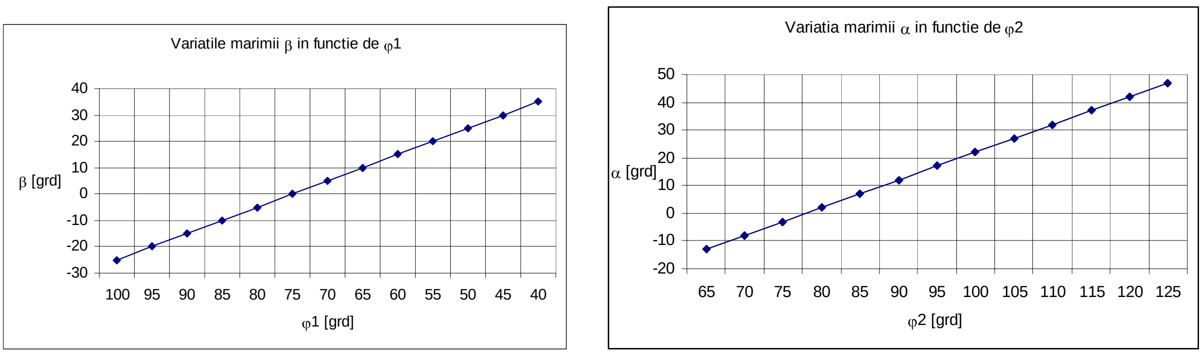
The proiection of the versor η_2 on the axis y_0 will be $\cos\varphi_2$, so the second line from its matrix:

$$\cos\varphi_2=\sin\varphi_{02}\cos\alpha-\cos\varphi_{02}\sin\alpha\cos\beta, \quad (11)$$

resulting:

$$\mathrm{tg}\frac{\alpha}{2}=\frac{-\cos\beta\cos\varphi_{02}\pm\sqrt{(\cos\beta\cos\varphi_{02})^2-(\cos\varphi_2+\sin\varphi_{02})(\cos\varphi_2-\sin\varphi_{02})}}{\cos\varphi_2+\sin\varphi_{02}}. \quad (12)$$

For a concrete scheme, to which the parameters l_1 = 200 mm, l_2 = 200 mm, e_1 = 50 mm, e_2 = 40 mm, φ_{01} = 15°, φ_{02} = 12° and the varaibles ΔS_1 = O – 50 mm $\to$ φ_1 = O – 60°, respectively ΔS_2 = O – 40 mm $\to$ φ_2 = O - 65° is presented the graphic from figure 7.



a) b)

Figure 7.

3. REFERENCES

[1] Alexandru, P., Ştefan, I. Cinematic modeling of the articulated structures which are equivalent to the human foot. International Conference IMT'07, Oradea University, 2007.