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MODELLING AND SIMULATION OF DYNAMIC BEHAVIOUR OF REFRIGERATED PRODUCTS IN A COLD STORAGE ROOM

Matko Bupic
University of Dubrovnik, Department of Mechanical Engineering
Cira Carica 4, HR-20000 Dubrovnik
Croatia (Hrvatska)

Branimir Pavkovic
University of Rijeka, Faculty of Engineering
Vukovarska 58, HR-51000 Rijeka
Croatia (Hrvatska)

Tomislav Jemric
University of Zagreb, Faculty of Agriculture
Svetosimunska 25, HR-10000 Zagreb
Croatia (Hrvatska)

ABSTRACT

The presented paper describes the development of mathematical model suitable for calculating temperature and humidity response and for simulation of dynamic behaviour of refrigerated products in a cold storage room.

Moist air, circulating around refrigerated products within cold chamber is continuously in direct contact with surface of the goods. Simultaneous heat and mass exchange occurs between refrigerated goods and moist air. During such process mass exchange is considered as evaporation at the product surface exposed to air circulation. Only top surface of the product is therefore drying. This paper is limited to consideration of drying of surface moisture, but not the hygroscopic moisture.

Developed mathematical model, describing the dynamic changes of temperature and humidity of refrigerated goods caused by multiple influential factors, is based on mass and energy balances. Such model provides simulation of dynamic behaviour of refrigerated product between temperature ranges of above and below 0°C (including 0°C).

Keywords: modelling, simulation, refrigerated product

1. INTRODUCTION

Industry of refrigerated cargo is constantly increasing in capacity and traffic. Optimisation of refrigeration systems based on different methods of simulation includes modelling of all components, especially refrigerated products. Presented model is part of the dynamic model of the whole refrigeration system that includes: chamber walls, refrigerated moist air in the chamber, condensation unit, evaporator, chamber outfit, evaporator fans and defrosting device.

According to the marks at figure 1., the refrigerated product (P) has the temperature $\vartheta_P\,(t)$ and the total mass $m_P\,(t)$, that is consisting of the dry substance mass $m_{d,P}$ and of the moisture contained in the product $m_{w,P}\,(t)$. Refrigerated product is placed in a cold storage room and it is circulated by moist air

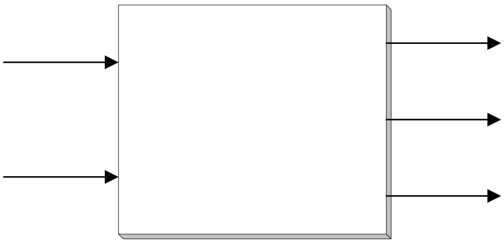

Figure 1. Structural model of refrigerated product

(A), which temperature is $\vartheta_A(t)$ and partial vapour pressure is $p_{s,A}(t)$. Simultaneous heat ($\dot{Q}_{PA}, \dot{H}_{s,PA}$) and mass exchange ($\dot{m}_{s,PA}$) occurs between refrigerated product and moist air. During such process mass exchange is considered as evaporation at the product surface exposed to air circulation. Only top surface of the product is therefore drying. This paper is limited to consideration of drying of surface moisture, but not the hygroscopic moisture. The aim is to develop mathematical model of refrigerated product, that is describing dynamics of temperature $\vartheta_{P}.(t)$ and dynamics of moisture of refrigerated product $m_{w,P}.(t)$, as well as dynamics of heat load $\dot{Q}_{PA}(t)$ and moisture load $\dot{m}_{s,PA}(t)$ of air in the cold chamber – all depending on the changes $\vartheta_A(t)$ and $p_{s,A}(t)$. Apart to the supposition on only top surfacing coarse moisture, that is dried from the refrigerated product, during dynamic mathematical model setting, following assumptions and simplifications are introduced:

- refrigerated product temperature ϑ_P is equal across whole product surface,
- specific heat capacities of dry refrigerated product $c_{d,P}$ have constant value,
- heat transfer coefficient onto the product surface α_P is not a function of temperature,
- refrigerated product is homogeneous, with constant physical characteristics,
- heat exchange due to radiation is not taken into account,
- mass transfer after frosting due to sublimation is disregarded.

2. EQUATIONS OF ENERGY AND MASS CONSERVATION

Internal energy balance of the refrigerated product can be expressed with equation:

$$\frac{dU_P(t)}{dt} = -\dot{Q}_{PA}(t) - \dot{H}_{s,PA}(t), \tag{1}$$

where: U_P, J – is internal energy of the refrigerated product; $\dot{Q}_{PA}$, Js^{-1} – is heat flow from product surface to the refrigerated moist air; $\dot{H}_{s,PA}$, Js^{-1} – is enthalpy flow drained off by evaporated moisture (vapour) from product surface to the refrigerated moist air.

Equation left side (1) represents the change of refrigerated product internal energy that can be expressed by equation:

$$\frac{dU_P(t)}{dt} = m_{d,P}\frac{du_{d,P}(t)}{dt} + m_{w,P}(t)\frac{du_{w,P}(t)}{dt} + u_{w,P}(t)\frac{dm_{w,P}(t)}{dt}, \tag{2}$$

where: $m_{d,P}$, kg – is dry refrigerated product mass ($m_{d,P}$ = const.); $m_{w,P}$, kg – is moisture mass contained in the refrigerated product; $u_{d,P}$, Jkg^{-1} – is dry product specific internal energy ($u_{d,P} = c_{d,P}\,\vartheta_P$); $u_{w,P}$, Jkg^{-1} – is moisture specific internal energy on the product ($u_{w,P} = h_{lw,P} = c_{lw}\,\vartheta_P$); c_{lw}, Jkg^{-1}K^{-1}– is liquid water specific heat capacity.

First and second member at right side of equation (1) may be expressed as follows:

$$\dot{Q}_{PA}(t) = \alpha_P\left[\vartheta_P(t) - \vartheta_A(t)\right]A_P, \tag{3}$$

$$\dot{H}_{s,PA}(t) = \dot{m}_{s,PA}(t)\,h_{s,P}, \tag{4}$$

where: A_P, m^2 – is product surface exposed to the air current; $\dot{m}_{s,PA}$, kgs^{-1} –is mass flow of evaporated moisture from product surface in the refrigerated air; $h_{s,P}$, Jkg^{-1} – is water vapour specific enthalpy.

Taking into account equations (2), (3) and (4), the equation of internal energy balance of refrigerated product is obtained in its extended form:

$$m_{d,P}\,c_{d,P}\frac{d\vartheta_P(t)}{dt} + m_{lw,P}(t)\,c_{lw}\frac{d\vartheta_P(t)}{dt} + h_{lw,P}\frac{dm_{lw,P}(t)}{dt} = -\alpha_P\left[\vartheta_P(t) - \vartheta_A(t)\right]A_P - \dot{m}_{s,PA}(t)\,h_{s,P}. \tag{5}$$

Whereas the moisture mass change contained in the refrigerated product is developing due to moisture evaporation from product surface into the refrigerated moist air, the moisture mass balance is:

$$\frac{dm_{lw,P}(t)}{dt} = -\dot{m}_{s,PA}(t). \tag{6}$$

Thus, reordering equation (5), the following equation is obtained:

$$\frac{d\vartheta_P(t)}{dt} = -\frac{\alpha_P\left[\vartheta_P(t) - \vartheta_A(t)\right]A_P + \dot{m}_{s,PA}(t)\left[h_{s,P} - h_{lw,P}\right]}{m_{d,P}\, c_{d,P} + m_{lw,P}(t)\, c_{lw}}, \tag{7}$$

describing dynamics of the refrigerated product temperature $\vartheta_{P}(t)$.

The specific enthalpy of superheated vapour $h_{s,P}$, Jkg^{-1} in equation (7) is counted according to the following equation:

$$h_{s,P} = \begin{cases} 2\,500\,357 + 1830\,\vartheta_P & \dot{m}_{s,PA} > 0 \\ & \text{for} \\ 2\,500\,357 + 1830\,\vartheta_A & \dot{m}_{s,PA} < 0 \end{cases} \tag{8}$$

Total moisture content in the refrigerated product is changed according to equation (6), and its transient quantity $m_{lw,P}(t)$ after time t has elapsed, from initial moment t_0, is determinated according to the following equation:

$$m_{lw,P}(t) = m_{lw,P}(t_0) - \int_{t_0}^{t} \dot{m}_{s,PA}\, dt\,. \tag{9}$$

Based on heat and mass transfer analogy, mass flow of the evaporated moisture from refrigerated product surface into refrigerated moist air $\dot{m}_{s,PA}$, kgs^{-1} may be expressed as:

$$\dot{m}_{s,PA} = \frac{\beta_P\, A_P}{R_s\, T_{m,PA}}\left(p_{s,P} - p_{s,A}\right), \tag{10}$$

where: β_P, ms^{-1} – is mass transfer coefficient by moisture evaporation from product surface:

$$\beta_P = \frac{\alpha_P}{c_{p\,A}\, \rho_A} Le^{n-1}\,; \tag{11}$$

R_s – is water vapour gas constant (R_s = 461,52 Jkg^{-1}K^{-1}); $p_{s,P}$, Pa – is water vapour partial pressure that is originating in moisture evaporation from the product; $p_{s,A}$, Pa – is water vapour partial pressure in the air; $T_{m,PA}$, K – is arithmetic mean of product temperature and air temperature values; Le – is Lewis's dimensionless number.

3. MODIFICATION OF THE MODEL FOR THE PHASE CHANGES PHENOMENA

At moment when a final temperature of water freezing is reached, $\boldsymbol{\vartheta_P (t) = 0}$ **°C**, residual quantity of liquid moisture in the product $m_{lw,P}(t)$, kg is continuing to evaporate in one part, and in the other starts to freeze, while produced ice sublimates. At that time, product temperature is not changed. The dynamics of liquid moisture residual mass in the product are described in the following equation:

$$\frac{dm_{lw,P}(t)}{dt} = -\frac{\alpha_P\left[\vartheta_P - \vartheta_A(t)\right]A_P + 2\,834\,357\, \dot{m}_{s,PA}(t)}{334\,000}, \tag{12}$$

and dynamics of ice mass created on the refrigerated product, in the following equation:

$$\frac{dm_{i,P}(t)}{dt} = \frac{\alpha_P\left[\vartheta_P - \vartheta_A(t)\right]A_P + 2\,500\,357\, \dot{m}_{s,PA}(t)}{334\,000}. \tag{13}$$

After last liquid drop transforms into ice, liquid moisture does not exist at all in the product, so as the refrigerated product, together with created ice, continues to refrigerate with continual temperature diminution.

In the temperature field $\boldsymbol{\vartheta_P (t) < 0}$ **°C** dynamics of the refrigerated product temperature is described in the following equation:

$$\frac{d\vartheta_P(t)}{dt} = -\frac{\alpha_P\left[\vartheta_P(t) - \vartheta_A(t)\right]A_P}{m_{d,P}\, c_{d,P} + m_{i,P}\, c_i}\,. \tag{14}$$

4. SIMULATION RESULTS AND MODEL VERIFICATION

The method of *System Dynamics*, known as Forrester's Dynamics, was used for development of dynamic model, and *Powersim* program was used for simulation. The simulation is applied as per following scenario:

- air temperature in cold chamber ϑ_A falls linearly with inclination coefficient –0,01 °Cs^{-1} from initial value of 20 °C till up value of –28 °C, and thereafter effected value is retained constant until the simulation process ending,
- relative humidity of air φ_A during entire process has constant value of 0,9,
- initial temperature value of the refrigerated product ϑ_P is 30 °C,
- dry substance product mass $m_{d,P}$ is 200 kg,
- initial value of moisture mass contained in the product $m_{w,P}$ is 40 kg,
- refrigerated product surface A_P is 13,5m^2,
- specific heat capacity of dry product $c_{d,P}$ = 1675 Jkg^{-1}K^{-1},
- heat transfer coefficient on object surface α_P has value 20 Wm^{-2}K^{-1}.

Simulation results are graphically presented by the diagrams in figure 2. The verification of refrigerated product model validity is accomplished by the comparison of simulation results with empirical results in similar situations.

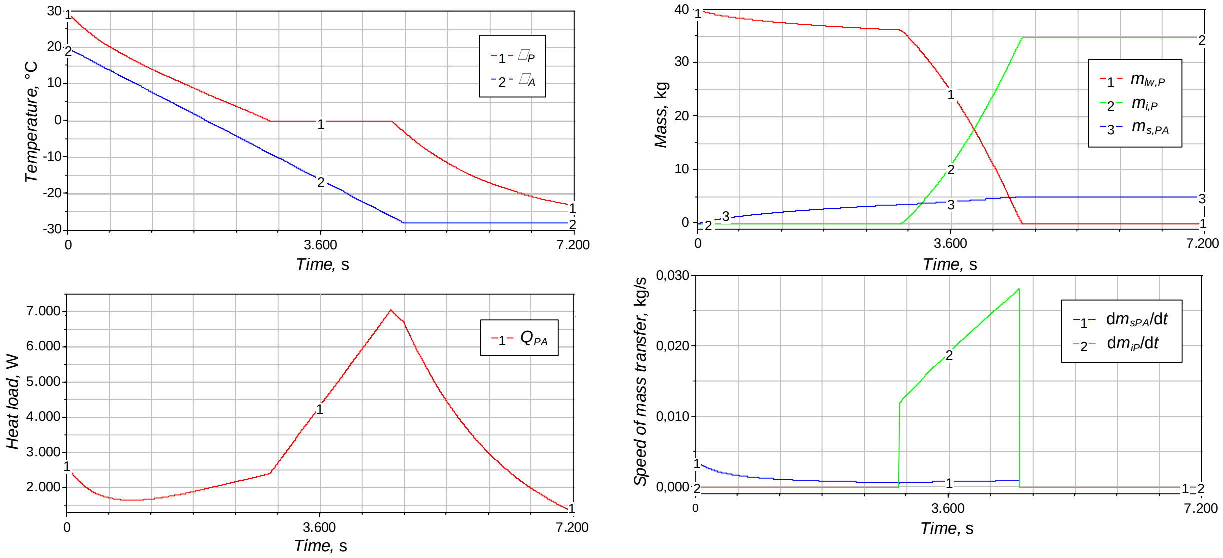

Figure 2. Simulation results of dynamic behaviour of the refrigerated product in a cold storage room

5. CONCLUSIONS

Dynamical behaviour of the refrigerated product model, in accordance with relevant knowledge on real model and obtained simulation results, are entirely compatible in qualitative sense with empirical results. Therefore, it may be concluded: this way of verification, when the deficiency in experimental results occurred, has verified the validity of extended refrigerated product model.

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