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TORSIONAL VIBRATIONS OF THE MECHANICAL SYSTEM SUBMITTED TO THE COMPOUND EXTERNAL PERIODIC EXCITATION

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ABSTRACT
Using the bucket wheel driving system of a bucket excavator as an example, torsional vibrations of the mechanical system caused by the compound external periodic excitation during continuous operation have been investigated in this paper. Due to the periodic grip of the buckets with the excavated soil, the torque on the bucket wheel coming from excavation resistance varies considerably. This torque that represents the external periodic excitation is determined and as a periodic function described by Fourier series. The torque has been calculated for variable main working parameters which are height of the cutting layer, bucket wheel speed and the specific digging resistance of the soil. For the observed system the mechanical model in the form of a torsional chain with four degrees of freedom has been developed and the eigenvalues computed. The response of the bucket wheel driving system to the external exciting torque during steady state operation has been estimated assuming the excavated soil as homogenious. The graphic representations of both eigenvalues and the responces of the bucket wheel driving system to the external exciting torque have been given. Using the results of this research the influence of the dynamic characteristics of the system itself and the influence of the main working parameters on the torsional vibration of the bucket wheel driving system have been estimated in this paper.

Keywords: torsional vibration, external periodic excitation, bucket wheel drive, steady state response.

1. INTRODUCTION
During operation of the excuvator the digging resistance forces are acting on its bucket wheel. Due to periodic contact of the buckets with the soil the digging resistance is rather variable even if the excuvating substance is entirely homogenious. This variable digging resistance that results in forces and torques on the bucket wheel causes high dynamic streses in the bucket wheel itself, its shaft and all parts of the bucket wheel drive. The torque on the bucket wheel shaft as an external periodic excitation can cause torsional vibration of the bucket wheel and also of the elements of its drive. For steady state operation this torque can be approximately represented by a compound periodic function. The driving torque of the electric motor is also the external exciter of torsional vibration. However, during the steady state operation, this torque is not very changeable and it only insignificantly varies about working point of the electric motor. Besides these, the next significant excitations of bucket wheel torsional vibrations are the shocks that appear when a bucket collides with the embeded stone of much higher hardness and relatively big mass. By that impact the exciting torque is increasing rapidly. In order to protect the bucket wheel drive against such a strong impuls excitation, the magnetic powder- or fluid coupling is most commonly used. When the external exciting torque exceeds the set up value the coupling slips off and brakes further transmision of motion. Vibrations of the bucket wheel drive that appear after the safety coupling has slipped off are the free damped vibration of the mechanical system after action of an impuls excitation.

2. THE DIGGING RESISTANCE TORQUE ON THE EXCUVATOR BUCKET WHEEL

The total digging resistance on the excuvator bucket wheel is the sum of resistances that appear on single buckets that are in the grip with the soil. Resistance at any single bucket consists of tangential F_{Ti}, normal F_{Ni} and lateral component F_{Bi}, as shown in Fig. 1. Digging resistance torque M(t), in the plane of rotation of the bucket wheel comes from the tangential component of the resistance, F_T.

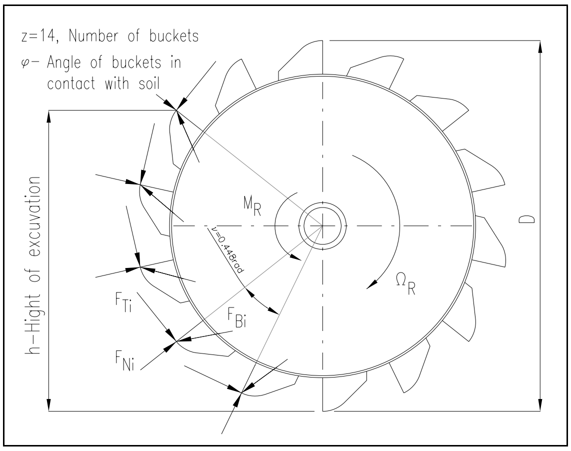

Figure 1. Forces on bucket wheel

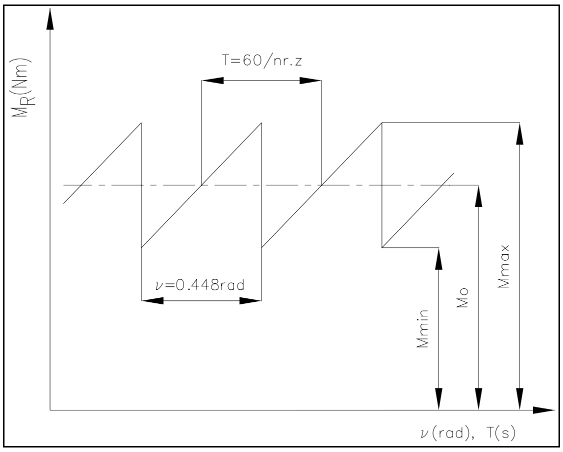

Figure 2. Approximative graphic of a torque

The determination of the digging resistance torque M(t) is complicated because many geometrical, mechanical, technological and phisical parameters are included in the calculation. The torque M(t) in this paper, is calculated by using methodology given in literature [1]. Applying that calculation procedure, by varying digging parameters such as the hight of the cutting layer h, specific digging resistance of a soil k_L, width of a cut slice, bucket wheel rotation speed and other parameters, the set of forces and torques on the bucket wheel has been obtained. Using the calculated torques M_{min} and M_{max} and taking into account a number of buckets on the bucket wheel (z=14), and the speed of rotation of the bucket wheel the graphics of the exciting torques are drawn as shown in Fig. 2, [2]. For further analysis, these torques as periodic functions, are described by Fourier series, as follows:

$$M(t) = M_0 + \sum_{n=1}^{\infty} (M_{an} \cos n\Omega t + M_{bn} \sin n\Omega t)\,, \tag{1}$$

where: $\Omega=2\pi/T_\Omega$ – angular frequency of the torque, $T_\Omega=60/n_r z$ - period of function $M(t)$,and M_0, M_{an} i M_{bn} are constants that are calculated by Euler formulae for coefficients of Fourier series, [3] :

$$M_0 = \frac{1}{T_\Omega}\int_0^{T_\Omega} M(t)dt\,,\quad M_{an} = \frac{2}{T_\Omega}\int_0^{T_\Omega} M(t)\cos n\Omega t dt\,,\quad M_{bn} = \frac{2}{T_\Omega}\int_0^{T_\Omega} M(t)\sin n\Omega t dt\,. \tag{2}$$

From Fig. 2 it is seen that function $M(t)$ is defined by the equations of the straight lines in intervals:

$$M(t) = \begin{cases} \dfrac{M_{max} - M_{min}}{T_\Omega} t + \dfrac{M_{max} + M_{min}}{2}, & 0 \le t \le T_\Omega / 2, \\ \dfrac{M_{max} - M_{min}}{T_\Omega} t - \dfrac{M_{max} + 3M_{min}}{2}, & T_\Omega / 2 \le t \le T_\Omega, \end{cases} \tag{3}$$

Inserting the calculated constants M_0, M_{an} *i* M_{bn} into (1) the next expression for the torque is obtained:

$$M(t) = M_o + \sum_1^n M_n \sin n\Omega t\,, \tag{4}$$

$$\text{where } M_0 = \frac{M_{max} + M_{min}}{2}\,, \text{ and } M_n = \sum_{i=1}^{\infty}[-\frac{2}{n\pi}(M_{max} - M_0)\cos n\pi]\,. \tag{5}$$

The funktion M(t) in this work is approximated by seven terms of Fourier series. Torques given in graphics in Fig. 3 are reduced on the axis of electric motor rotor i.e all of them are devided by gearbox ratio, for both speeds of bucket wheel (n_{R1}=6.6 RPM for i_R= *223.5*, and n_{R2}=4.9 RPM for i_R=*300.5*)

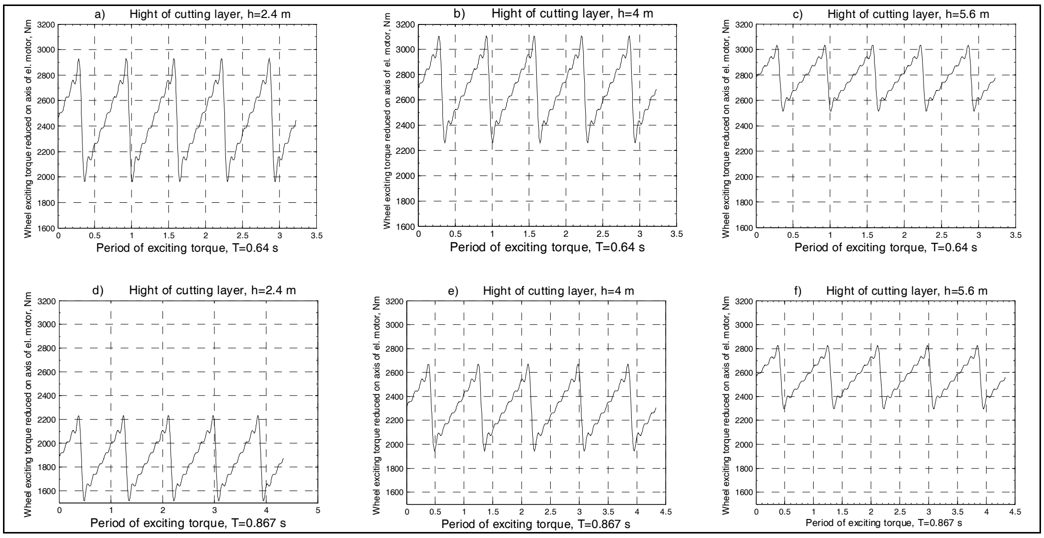

Fig 3. A wheel exciting torque for k_L = 80 kN/m, and different hights of cutting layers a), b),c) for i_R=223.5 (n_R=6.6 RPM), d), e), f) for i_R=300.5 (n_R=4.9 RPM)

3. MECHANICAL MODEL OF THE BUCKET WHEEL DRIVING SYSTEM

Mechanical model of the bucket wheel driving system is given in Fig. 4, in the shape of a free torsional chain that consists of the shaft with four discs on it. Moments of inertia of the discs J_i, i=1(1)4 and torsional stiffnesses of the shafts ci, i=1(1)3, are moments of inertia and stiffnesses of the elements of a real bucket wheel drive reduced on the axis of rotation of electric motor rotor. This reduction is based on the condition that kinetic and potential energy of a real system is equal to the kinetic and potential energy of the reduced mechanical model. In the model in Fig. 4 (taken from [4]), the first disc represents the moment of inertia of the electric motor rotor and the second one, the moment of inertia of the coupling. The 3rd and 4th disc represent inertia of gearbox and bucket wheel, respectively, both reduced on the axis of rotation of the bucket wheel drive electric motor. For simplicity reasons the damping in the system is not taken into account in this analysis.

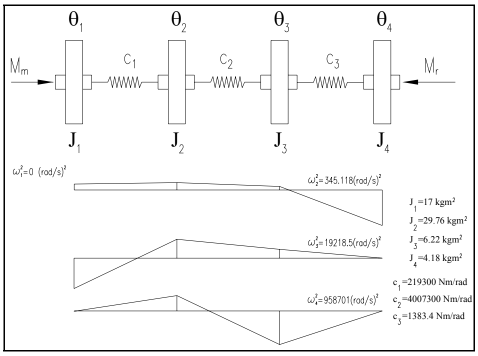

Fig. 4. Mechanical model of a wheel drive

Table 1. Eigenvalues of a system in Fig. 4, for i_R=223.5

Natural angular frequencies ω^2[rad/s]^2			
1	2	3	4
0.02595	345.118	19218.50	958701
Eigenvectors			
0.56134	0.12119	-0.81865	-0.00389
0.74271	0.15606	0.53059	0.37747
0.30045	0.06287	0.21973	-0.92601
0.20733	-0.97826	-0.00265	0.00022

4. DIFERENCIAL EQUATIONS OF TORSIONAL VIBRATIONS OF THE SYSTEM

Equations of motion of a system represented by the mechanical model in Fig. 4, are the following:

$$J_1\ddot{\theta}_1 + c(\theta_1 - \theta_2) = M_m$$

$$J_2\ddot{\theta}_2 - c_1(\theta_1 - \theta_2) + c_2(\theta_2 - \theta_3) = 0$$

$$J_3\ddot{\theta}_3 - c_2(\theta_2 - \theta_3) + c_3(\theta_3 - \theta_4) = 0$$

$$J_4\ddot{\theta}_4 - c_3(\theta_3 - \theta_4) = (M_o + M_n \sin n\Omega t)/i_R \,, \tag{6}$$

It should be noticed here that the electric motor torque, M_m (see Fig. 4) is annuled by the mean torque, M_O, so that only variable part of the exciting torque is taken into account in the set of equation (6).

Particular solutions for the system of differencial equations of motion (6) are assumed in the form:

$$\theta_k = \sum_{i=1}^{10} A_{kn} \sin n\Omega t\ , \quad k=1(1)4. \tag{7}$$

By computing derivations and substituting them into (6) we obtain the next set of algebraic equations:

$$\begin{aligned}
&(c_1 - J_1 n^2\Omega^2)A_{1n} - c_1 A_{2n} + 0A_{3n} + 0A_{4n} = 0 \\
&- c_1 A_{1n} + [(c_1 + c_2) - J_2 n^2\Omega^2]A_{2n} - c_2 A_{3n} + 0A_{4n} = 0 \\
&0A_{1n} - c_2 A_{2n} + \left[(c_2 + c_3) - J_3 n^2\Omega^2\right]A_{3n} - c_3 A_{4n} = 0 \\
&0A_{1n} + 0A_{2n} - c_3 A_{3n} + \left(c_3 - J_4 n^2\Omega^2\right)A_{4n} = M_n
\end{aligned} \tag{8}$$

from which the amplitudes of forced torsional vibrations of the discs are calculated. Substituting them into equations (7) we obtain the laws of forced torsional vibrations of the system, or the steady state response to the compound periodic excitations given in Fig. 3, in the form:

$$\theta_1 = \sum_{i=1}^{10} A_{1n} \sin n\Omega t\ , \quad \theta_2 = \sum_{i=1}^{10} A_{2n} \sin n\Omega t\ , \quad \theta_3 = \sum_{i=1}^{10} A_{3n} \sin n\Omega t\ , \quad \theta_4 = \sum_{i=1}^{10} A_{4n} \sin n\Omega t\ , \qquad (9).$$

The steady state responses of the system are given in Fig. 5. Each picture, respectively, contains the responses of all four discs for the given set of working parameters. In the calculation the soil specific digging resistance was constant while the hight of cutting layers and the speed of rotation of the bucket wheel were varied.

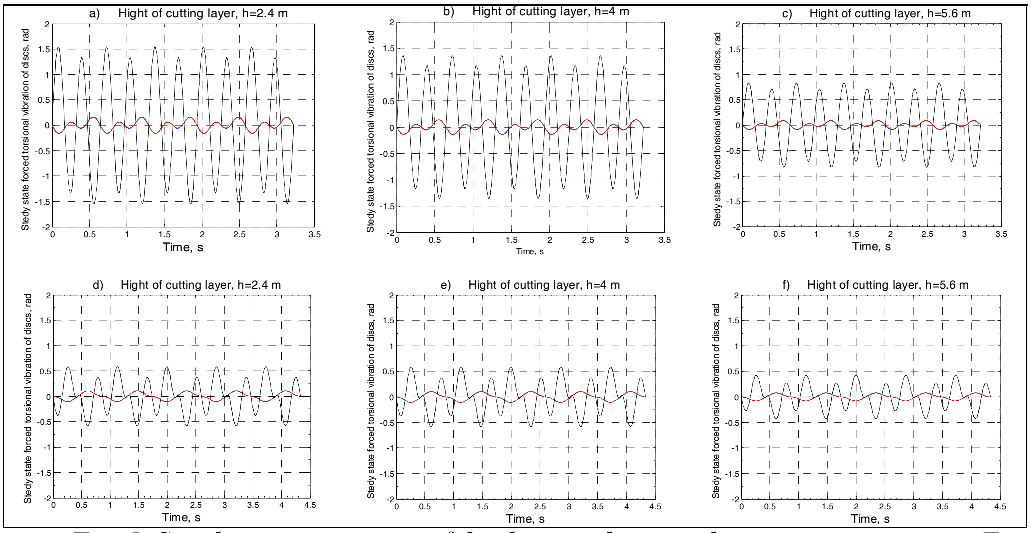

Fig. 5. Steady state responses of the discs to the periodic excitations given in Fig. 3.
a), b),c) for i_R=223.5 (n_R=6.6 RPM), d), e), f) for i_R=300.5 (n_R=4.9 RPM)

5. CONCLUSION

The exciting torques on bucket wheel have been calculated for different working parameters, see Fig. 3. As it can be seen, for the given soil specific digging resistance, the mean torque increases with the increase of the hight of cutting layer while its amplitude decreases. So do the amplitudes of steady state forced torsional vibration. These amplitudes are much smaller if the excavation is performed with lower bucket wheel speed, Fig. 5. d), e),f). Since amplitudes of first three discs differ very litle, mechanical model can be reduced on model with two degree of freedom. Results of the given analysis can be used in the design stage of a bucket wheel drive and for judgement of the existing equipment.

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