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JOULE-THOMSON COEFFICIENT OF NITROGEN FROM SPEED OF SOUND

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ABSTRACT
A new method for deriving the Joule-Thomson coefficient from speed of sound is recommended. It is based on numerical integration of differential equations connecting the speed of sound with other thermodynamic properties. The method requires initial values of density and heat capacity at a single temperature in the pressure range of interest. It is tested by deriving the Joule-Thomson coefficient of gaseous nitrogen in the temperature range from 200 to 300 K, and in the pressure range from 2 to 10 MPa. Estimated absolute average deviation between calculated and reference values of the Joule-Thomson coefficient is 0.4%.
Key words: Joule-Thomson coefficient, nitrogen, speed of sound

1. INTRODUCTION
If the pressure of a flowing fluid is decreased by means of an adiabatic throttling process, in which kinetic and potential energy changes are negligible, the enthalpies of the fluid at the inlet and the exit of the throttling device are equal. This result can be established by applying the conservation-of-energy equation to the steady-flow throttling process. Throttling the fluid to a lower pressure can alter the temperature of the fluid. In fact, the fluid temperature can increase, decrease, or even remain unchanged. Positive values of the Joule-Thompson coefficient signify that the temperature decreases as a result of a pressure drop caused by throttling; whereas negative values signify that the temperature increases. If the coefficient is zero, throttling will not cause a change in the temperature of the fluid [1].
The speed of sound is thermodynamic property of a fluid which is readily measured with higher accuracy than majority of other thermodynamic properties. A combination of highly-accurate experimental data and a suitable method of analysis can provide a powerful tool for determining highly precise and accurate thermodynamic-property data. This connection has been made in this paper by means of a numerical integration of the equations that link the speed of sound and the Joule-Thomson coefficient. Although the gas for which the thermodynamic properties were derived from measurements of the speed of sound is neither particularly complex nor unstudied, the method developed here can be extremely useful for deriving properties of other less-studied fluids for engineering applications [2].

2. THEORY

The Joule-Thompson coefficient, used to measure the temperature change of a fluid during a throttling process, is defined by the expression [3]:

$$\mu_{\mathrm{JT}} = \left(\frac{\partial T}{\partial p} \right)_h , \tag{1}$$

where μ is the Joule-Thomson coefficient, T is the temperature, p is the pressure, and h is the enthalpy.

With aid of standard thermodynamic identities, following expression for the Joule-Thomson coefficient may be derived:

$$\mu_{\mathrm{JT}} = \frac{1}{\rho\, c_p} \left(T\alpha_p - 1 \right), \tag{2}$$

where ρ is the density, c_p is the specific heat capacity at constant pressure, and α_p is the thermal expansion coefficient.

Thermodynamic speed of sound (speed of sound at zero frequency) in a fluid, is defined by the expression [4]:

$$u^2 = \left(\frac{\partial p}{\partial \rho} \right)_s , \tag{3}$$

where u is the speed of sound, p is the pressure, ρ is the density, and s is the entropy.

The following set of partial differential equations may be derived from (3) if one takes T and p as independent variables [5]:

$$\alpha_p^2 = \frac{c_p}{T} \left[\left(\frac{\partial \rho}{\partial p} \right)_T - \frac{1}{u^2} \right], \tag{4}$$

$$c_p = T\alpha_p^2 \left[\left(\frac{\partial \rho}{\partial p} \right)_T - \frac{1}{u^2} \right]^{-1} , \tag{5}$$

$$\left(\frac{\partial \rho}{\partial T} \right)_p = -\alpha_p \rho , \tag{6}$$

and

$$\left(\frac{\partial \alpha_p}{\partial T} \right)_p = -\alpha_p^2 - \frac{\rho}{T} \left(\frac{\partial c_p}{\partial p} \right)_T . \tag{7}$$

Set of Eqs. (4) to (7) has no analytical solution, but it may be solved numerically if initial values of ρ and c_p are specified at a single temperature in the pressure range of interest. It may be solved as an initial value problem for the set of first-order ordinary differential equations if all pressure derivatives are estimated independently. The Joule-Thomson coefficient is then obtained from Eq. (2) in the range of temperature and pressure in which experimental speeds of sound are available.

3. RESULTS AND CONCLUSION

Recommended numerical method is used for deriving the Joule-Thomson coefficient of gaseous nitrogen from its speed of sound [6, 7], in the temperature range from 200 to 300 K, and in the pressure range from 2 to 10 MPa. The temperature range is divided into 5 isotherms (e.g. 200 K, 225 K, 250 K, 275 K, and 300 K), and the pressure range is divided into 5 isobars (e.g. 2 MPa, 4 MPa, 6 MPa, 8 MPa, and 10 MPa). The set of Eqs. (4) to (7) is solved numerically by combined Adams-

Moulton [8] and Runge-Kutta [9] method. All pressure derivatives are estimated by Lagrange interpolating polynomial [10] of fourth-degree. Figure 1 gives an impression of the results obtained, while more detailed insight may be obtained from Table 1. Initial values of ρ and c_p [11] are specified along isotherm at 200 K, and therefore this isotherm is omitted. Estimated absolute average deviation of calculated values of the Joule-Thomson coefficient, with reference to corresponding reference values [11], is 0.4%.

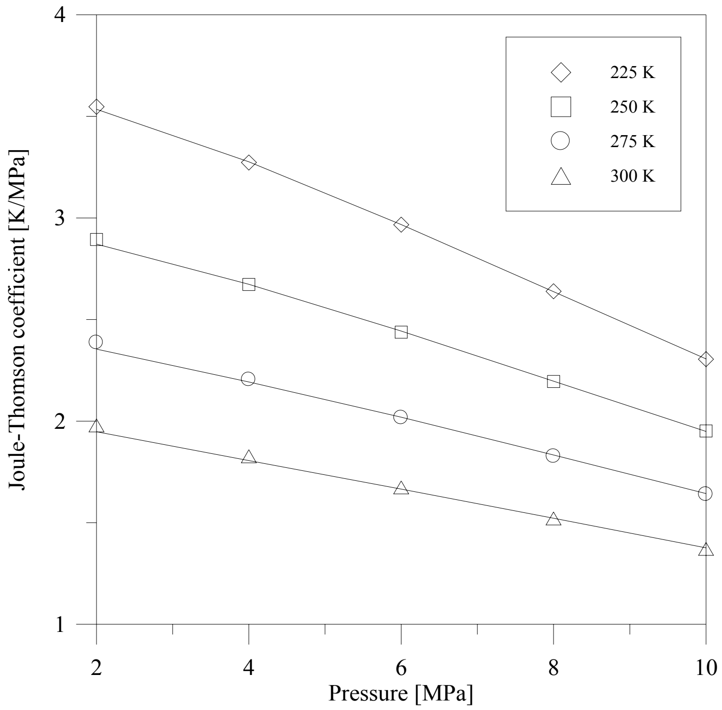

Figure 1. Joule-Thomson coefficient vs. p; full line this work; ◇ □ ○ △ Ref. [11].

Table 1. Results of numerical integration vs. reference values of the Joule-Thomson coefficient

Temperature	Pressure	$\mu_{JT,\,calc}$	$\mu_{JT,\,ref}$	$\mu_{JT,\,calc} - \mu_{JT,\,ref}$	$\mu_{JT,\,calc} - \mu_{JT,\,ref}$
K	MPa	K/MPa	K/MPa	K/MPa	%
225.0	2.0	3.535	3.547	-0.013	-0.364
225.0	4.0	3.277	3.273	0.003	0.104
225.0	6.0	2.967	2.967	0.001	0.019
225.0	8.0	2.637	2.639	-0.002	-0.065
225.0	10.0	2.306	2.305	0.002	0.069
250.0	2.0	2.871	2.894	-0.023	-0.804
250.0	4.0	2.673	2.673	0.000	0.015
250.0	6.0	2.443	2.438	0.005	0.212
250.0	8.0	2.196	2.195	0.001	0.060
250.0	10.0	1.949	1.951	-0.002	-0.127
275.0	2.0	2.354	2.386	-0.032	-1.336
275.0	4.0	2.193	2.204	-0.011	-0.492
275.0	6.0	2.019	2.016	0.002	0.122
275.0	8.0	1.833	1.827	0.006	0.331
275.0	10.0	1.643	1.639	0.004	0.245
300.0	2.0	1.948	1.978	-0.031	-1.552
300.0	4.0	1.805	1.827	-0.021	-1.165
300.0	6.0	1.665	1.673	-0.008	-0.481
300.0	8.0	1.522	1.521	0.001	0.085
300.0	10.0	1.376	1.371	0.005	0.400

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