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AHP METHOD FOR OPTIMAL DECISION MAKING PROBLEM IN MULTI-CRITERIA PROGRAMMING

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ABSTRACT
Analytic Hierarchy Process (AHP) developed by Saaty represents a tool in decision making analysis, created to help decision-makers solve complex problems using a greater number of, decision criteria, within multiple periods of time.
This paper demonstrates the application of the AHP method by choosing a winter tourist centre in Montenegro, using qualitative and quantitative criteria in their ranking.
Keywords: Multi-criteria analysis, multi-attributive decision making, AHP method, attributes, decision- makers, tourist centres

1. INTRODUCTION
Multicriteria decision making (MDM) is one of the most important branches in the theory of decision making. It refers to situations by which there are a number of, most often conflicting criteria on the grounds of which the optimal decision is to be made. Unlike single criteria optimisation models where only one criterion in decision making is used, thus significantly reducing the actual problem to be solved, the presence of a greater number of criteria in decision making models represents an important step towards actuality. Namely, with most decision making problems the generated results are to be analysed from many angles and evaluated on numerous criteria. The main characteristic of all multicriteria decision making problems being the existence of more criteria in decision making, and accordingly, creating more alternatives, the decision-maker, in such situations does not try to maximize stated goals, but works to achieve them to the greatest possible extent. This requirement is not easy to meet due to the fact that the stated goals are often mutually conflicted, so the optimisation of one goal usually means that the other goals will be achieved to a smaller extent.
Multicriteria decision making can be multiattributive decision making (MADM) to which the criteria is specified by attributes, and multiobjective decision making (MODM) with explicitly defined analytic form for each separate criterion. On the following pages, we will use the multiprogramming decision making model.

2. ATTRIBUTES
Attributes are the chosen characteristics in which the alternatives differ and represent the characteristics of the alternatives relevant to the actual choice of the observed decision making problem. Each attribute is supposed to provide a means for assessment of the levels of one criterion (goal). Unlike alternatives, which have been previously defined, we always choose and formulate attributes independently. This means that the choice of attributes is subjective because the set of

attributes reflects the individual opinion of the decision-maker, that is, reveals specific goals which the decision-maker wants to achieve by making the decision. This is why the sets of attributes will be different for each decision-maker, and will differ in numbers, content, or the assigned meaning. According to measurability levels attributes can be:

- Quantitative attributes – these are the characteristics of alternatives which can be measured with precision on so called cardinal scales (cost, number of rooms, distance)
- Qualitative attributes – these are characteristics of alternatives whose modalities can not be expressed in numbers, i.e. they cannot be measured with precision, but can be ranked by their intensity (service quality, cultural life and entertainment)

3. FORMULATING OF THE MATHEMATICAL MODEL OF MADM

The multiattributive decision-making model (MADM) is suitable for poorly structured problems and is mathematically represented in this way:

$$Max\{f_1(x), f_2(x), ..., f_n(x), n \geq 2\}$$

Subject to the constraints $x \in A = [a_1, a_2, ..., a_m]$,

Where: n – the number of criteria (attribute); m – the number of alternatives (actions); f_j – criteria (attributes), j = 1, 2, ..., n; a_i – alternative(actions) to be observed, i = 1, 2, ..., m; A – the set of all alternatives (actions).

Each attribute depends on $j-$ criterion and $i-$ alternative (it is of two-dimensional character), i.e. the values f_{ij} of each observed criterion fj generated by each possible alternative a_i:

$$f_{ij} = f_j(a_i), \quad i = 1,2,...,m; \; j = 1,2,.....,n.$$

Multiattributivity should be characteristic of each alternative and the attributes are chosen on the basis of the criteria selected by the decision-maker.

A typical way of representing the problem is the MADM matrix form. In the matrix of decision making, the criteria values for particular alternatives are shown, as follows:

$$\begin{array}{cc} & \begin{array}{cccc} \max\limits_{f_1} & \max\limits_{f_2} & ... & \max\limits_{f_n} \end{array} \\ A = \begin{array}{c} a_1 \\ a_2 \\ ... \\ a_m \end{array} & \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ ... & ... & ... & ... \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{bmatrix} \end{array}$$

The first condition for solving MADM problems is the qualification of quantitative methods performed [1]. In practice several ways of attribute transformation are used, among which are (a) transforming attributes into scale intervals (b) normalisation of attributes, and (c) assignment of corresponding sets of weights.

4. ANALYTIC HIERARCHY PROCESS METHOD (AHP)

Analytic Hierarchy Process, developed by Saaty (1980), is one of the best known and most used decision making methods when the decision is based on a number of attributes which are used as criteria on which a choice is to be made. The results obtained using the AHP method are helpful with solving complex decision making problems with more decision-makers, more criteria and within multiple time periods. The solving of complex decision making problems with this method is based on their decomposing into a hierarchic structure with elements being goals, sub-goals, and alternatives. Four stages in the application of the model are noted:

1. **Problem structuring.** The first step, in any multicriteria analysis problem, is the decomposing of complex decision making problems into a hierarchic series. The goal is placed at the top of a pyramid and then sub-goals are defined at lower levels while the alternatives are at the base of the pyramid.
2. **Data gathering and measuring.** The mathematical model for computing the priorities (weights) of elements which are at the same level of hierarchic structure is formed. The

mathematical model represents the basis for generating the ranking scale. Based on pairwise comparison at each level of hierarchic structure, the elements of the structure are compared. The preferences of the decision maker are represented by a scale. This scale is defined as a ratio scale, and it is supposed that the intensity of preferences between two alternatives can be expressed using the scale [1].

3. **Relative weights assessment –** Generated local goal, sub goal and alternative priorities are synthesized into a total of alternative priorities. Let us suppose that each hierarchy level A_i, $i = 1,2,...,n$ has n-attributes, whose weights, that is, priorities w_i are to be determined on the basis of their relations assessment, for each pair A_i, A_j . If the decision-maker compares each pair A_i, A_j of all attributes , whereas level A_i dominates (over) level A_j, that is w_i / w_j, a new matrix A (whose elements will be of w_i / w_j size) and which, in case of consistent assessment, for which $a_{ij} = a_{ik} a_{kj}$, fulfils the equation $Aw = nw$, can be formed. In practice, however, it might happen that matrix A has inconsistent evaluation, in which case, the weight vector w can be generated by solving the equation $Aw = \lambda_{\max} w$, under the condition that $\sum w_i = 1$, where $\lambda_{\max}$ represents the highest eigenvalue of matrix A (due to the matrix property $\lambda_{\max} \geq$ n). Using consistency index $CI = (\lambda_{\max} - n)/(n-1)$, as the measure of the consistency in deviation n from $\lambda_{\max}$ the consistency ratio can be generated, whereas RI is a random index.
4. **Sensitivity analysis** is the last stage in *Saaty's* model application.

5. THE CHOICE OF WINTER SKIING CENTRES USING THE AHP METHOD

There are five winter skiing centres in Montenegro offering tourists a variety of activities. These are «Durmitor»-Žabljak, «Bjelasica»-Kolašin, «Lokve»-Berane, «Turjak»-Rožaje, and «Vučje»-Nikšić. The criteria which will be used in choosing a winter skiing centre are:

- A_1- skiing conditions (qualitative assessment)
- A_2 - accommodation conditions (qualitative assessment)
- A_3 - the offer for cultural and social life (qualitative assessment)
- A_4 - accommodation cost (quantitative assessment)
- A_5 - distance to skiing-trails (quantitative assessment)

Table 1. Decision making matrix

	A_1	A_2	A_3	A_4	A_5
«Durmitor» (D)	very high	high	very high	22 eur	5 km
«Bjelasica» (B)	high	very high	high	150 eur	12 km
«Lokve» (L)	average	below average	average	15 eur	0.200 km
«Turjak» (T)	low	below average	below average	14 eur	0.100 km
«Vučje» (V)	average	average	below average	9 eur	17 km

Quantified matrix of decision making generated by *Saaty*'s scale is:

$$
\begin{array}{c} \\ O = \end{array}
\begin{array}{c} A_1 \; A_2 \; A_3 \; A_4 \; A_5 \\
\begin{bmatrix} 9 & 7 & 9 & 22 & 5 \\ 7 & 9 & 7 & 150 & 12 \\ 5 & 3 & 5 & 15 & 0.2 \\ 1 & 3 & 3 & 14 & 0.1 \\ 5 & 5 & 3 & 9 & 17 \end{bmatrix}\end{array}
$$

We will suppose that the highest participation and importance in the set model has the criterion A_1. Since we are not in a position of showing the whole process of assessment and ranking of tourist centres in Montenegro we shall only present the comparison of centres based on the first criterion, which is considered the most important. The comparison of the winter tourist centres by criterion A_1 is shown in Table 2, and a modified pair-wise weight comparison table in Table 3.

Table 2. Comparison of alternatives

	D	B	L	T	V
D		2	4	8	4
B			2	6	2
L				4	1
T					(4)
V					

Table 3: Weights in pairs of alternatives

	D	B	L	T	V
D	1	2	4	8	4
B	0.5	1	2	6	2
L	0.25	0.5	1	4	1
T	0.125	0.166	0.25	1	0.25
V	0.25	0.5	1	4	1
∑	**2.125**	**4.166**	**8.25**	**23**	**8.25**

Table 4: The eigenvectors of corresponding eigenvalues

	D K_1	B K_2	L K_3	T K_4	V K_5	∑ K_6	K_7
D	0.47	0.48	0.48	0.35	0.48	2.268	0.4536
B	0.24	0.24	0.24	0.26	0.24	1.2209	0.24418
L	0.12	0.12	0.12	0.17	0.12	0.6539	0.13078
T	0.06	0.04	0.03	0.04	0.03	0.2027	0.04054
V	0.12	0.12	0.12	0.17	0.12	0.6539	0.13078

The research results, which are the eigenvectors of corresponding eigenvalues is shown in Table 4.
The order of alternatives by the criterion A_1 is:

SKI centre «Durmitor»-Žabljak,	D = 0.4536	**************
SKI centre «Bjelasica»-Kolašin,	B = 0.2442	*******
SKI centre «Lokve»-Berane,	L = 0.1308	****
SKI centre «Vučje»-Nikšić,	V = 0.1308	****
SKI centre «Turjak»-Rožaje,	T = 0.0405	**

Thus, if only the skiing conditions (A_1) are taken as the criterion for the choice of winter tourist centre, a tourist will choose SKI centre «Durmitor»-Žabljak, because it received the highest rating. (0,4536).

6. REFERENCES

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