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ANALYTICAL SOLUTION AND EXPERIMENTAL DATA TO PREDICT THE PRESSURE IN THE DEFORMATION PROCESS OF A CYLINDRICAL BODY IN CASE OF PLANE TENSION

Travieso Rodríguez, J.A. (1) and González Rojas, H.A. (2)
(1) Universitat Politècnica de Catalunya (UPC). Escola Universitària d'Enginyeria Tècnica Industrial de Barcelona (EUETIB), C/ Comte d'Urgell 187, 08036-Barcelona, Spain
(2) Universitat Politècnica de Catalunya (UPC). Escola Tècnica Superior d'Enginyeria Industrial de Barcelona (ETSEIB), Avda. Diagonal 647, 08028-Barcelona, Spain

ABSTRACT
The objective of this paper is obtaining a solution to predict the average pressure in the process of deformation of a cylindrical body, and consequently be able to determine the best combination of the process parameters. The most important contribution of the obtained solution consists of considering the shear stress in the von Misses yield criterion. The results are shown in a graph where is possible to observe the values of medium pressures during the process versus the relation diameter / height of the work piece. They are compared with another solution obtained consider the phenomenon as a plane deformation problem, and also with experimental data. We also show a graph with the results of the one-dimensional model.
Keywords: Plastic deformation, Tension, Upsetting

1. INTRODUCTION
Different analytical, empirical, numerical and experimental methods have been developed to predict the pressure in the process of deformation of a cylindrical body, and consequently be able to determine the best combination of the process parameters. The analytical methods more used for analysis and simulation are: homogeneous deformation, slab method and the upper bound technique.
The distribution of pressure in this process is obtained by an analytical solution using a material with axisimetrical geometry. This work consists in the development of a model of this pressure based on a method of equilibrium of forces on an infinitesimal free body. The forces balance is made on a piece or thickness of infinitesimal material. This technique produces ordinary differentials equations, where the dependent variables are function of only one space coordinate, among other problem variables. The main newness of this work respect to the already existing works, are first of all that it is consider a plane tension problem and not a plane deformation problem and in second as far as the one-dimensional solution is correct.
The results are shown in a graph where is possible to observe the values of medium pressures during the process versus the relation diameter / height of the work piece. They are compared with another solution obtained consider the phenomenon as a plane deformation problem, and also with experimental data. We also show a graph with the limits of the one-dimensional model.

2. MATHEMATICAL MODEL
A model predicting the average stress in the process of deformation of a cylindrical body for a cylindrical geometry is developed. This model is obtained by making equilibrium of forces on a differential element. The differential equation associated with the problem of forces is analytically solved and integrated, and thus a function of the average pressure P_a is obtained.
These are the hypothesis made to obtain a model of the upsetting process:

i) The materials of the bodies under study, such as rigid-plastic, are considered.
ii) The plastic deformation is plane stress.
iii) The average stresses are distributed uniformly within elements.
iv) It is assumed that there is friction in the punch-material interface, and the constant friction coefficient is considered.

2.1. Equilibrium Equation

Equation (1) obtained:

$$\frac{\partial \sigma_r}{\partial r} - \frac{2\mu}{h} P = 0 \qquad (1)$$

Where*:* μ*:* Friction coefficient
h: Height of the work piece

To solve (1) it is necessary to have a function relating the pressure *P* and the stress σ_r. It is assumed that the pressure relates to the radial stress through a linear equation like the one shown in (2), where *A* and *B* are constants to determine:

$$P = A - B\sigma_r \qquad (2)$$

Replacing (2) in (1), a common differential equation is obtained.

This equation (3), relates the pressure in the proces of deformation to the independent variables which rule the process: radial position r, radius of the work piece R, dynamic friction coefficient μ, and thickness of the disc h.

$$P(r) = 2Ke^{\frac{2\mu B(R-r)}{h}} \qquad (3)$$

Defining the average pressure as:

$$Pa = \frac{2}{R^2}\int_0^R P(r)\cdot r \cdot dr \qquad (4)$$

An equation for the average pressure Pa is obtained

$$Pa = 2\frac{2K}{B}\frac{h}{\mu D}\left[\frac{h}{B\mu D}\left(e^{\frac{B\mu D}{h}} - 1\right) - 1\right] \qquad (5)$$

Where D is the diameter of the disc.

Now is necessary determining the plasticity criterion.

2.2. Yield Criterion

Normal stresses and shear stresses in a differential element relate through the von Misses yield criterion.
Assuming the hypothesis, and supposing that the increase in the plastic deformation depends on the stress deviatory tensor, the von Misses criterion (6) is used as a yield or plastic discontinuity criterion.

$$\sigma_z - \sigma_r = \gamma\sqrt{1 - 3\left(\frac{\tau_{rz}}{\gamma}\right)^2} \quad \text{where (yield strength) is } \gamma = 3\tau_{\max}^2 \qquad (6)$$

Where *k* is the yield limit for pure shear.
Three approximations to determine the yield criterion are described below.

2.2.1. ***First yield criterion (first solution adopted)***

The upsetting pressure *P* is related to the stress σ_r through the equation (7), assuming that the shear stress τ is negligible.

$$P = A - B\sigma_r \tag{7}$$

Then *B*=1, replacing this constant in the general solution (5), the first approximation for the dimensionless average pressure is obtained.

$$\frac{P_a}{2k} = \frac{2h}{\mu D}\left[\frac{h}{\mu D}\left(e^{\frac{\mu D}{h}} - 1\right) - 1\right] \tag{8}$$

2.2.2. ***Second yield criterion (second solution adopted)***

In this second approximation, the shear stress τ is considered different from zero in the equation (6), assuming that the shear stress τ is not cero, thus obtaining:

$$P = \gamma\sqrt{1 - 3(\mu P)^2} - \sigma_r \tag{9}$$

Finding the value of the pressure *P* from (10):

$$P = \frac{\gamma}{\sqrt{1 - 3(\mu P)^2}} - \frac{1}{1 + 3\mu^2}\sigma_r \tag{10}$$

Where the term multiplying σ_r in (11) is equal to *B*. Replacing this constant in the general solution (5), a new approximation for the average pressure is obtained.

$$P_a = \frac{\gamma 2h\left(1 + 3\mu^2\right)}{\mu D\sqrt{1 + 3\mu^2}}\left[\frac{h\left(1 + 3\mu^2\right)}{\mu D}\left(e^{\frac{\mu D\left(\frac{1}{1 - 3\mu^2}\right)}{h}} - 1\right) - 1\right] \tag{11}$$

2.2.3. Third solution adopted

It takes as another solution (12), the division between the square of the first yield criterion between the second criterion. We do this to correct the effect of shear tension introduced in the second yield criterion. This solution, as will be seen later works best, which is a good contribution. Is the one that comes closest, as can be seen in figure 1, to the results of experiments developed.

$$P_a = \frac{P_{a\,1}^2}{P_{a\,2}} \tag{12}$$

3. EXPERIMENTS

In a press, controlling the values of pressures that were printed in every moment on the work piece as shown in figure 3, were measured relations D/h which were obtained in each case. It used work pieces of Pb with initial diameter and height 40 mm. It was subsequently graphed the relations between the average pressures applied and the slenderness relations gained (figure 1).

4. RESULTS

This section shows the results of the simulations. Figure 1 shows the graphs of the equations (8) and (11) and the third solution, which are compared to the experimental data obtained, where the broken line corresponds to the simulation made with the first yield criterion, the continuous fine line corresponds to the second yield criterion, dots correspond to the experimental data, and the continuous width line correspond to our experimental data.

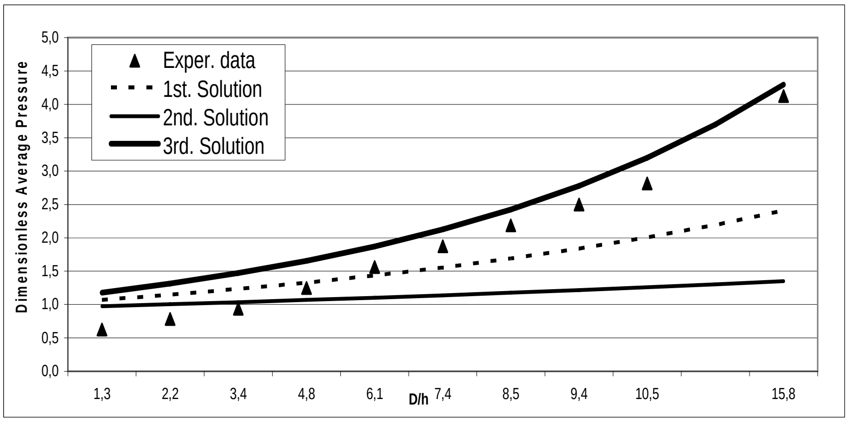

Figure 1. Three solutions to the average pressure for different values of D/h, in comparison with the experimental data.

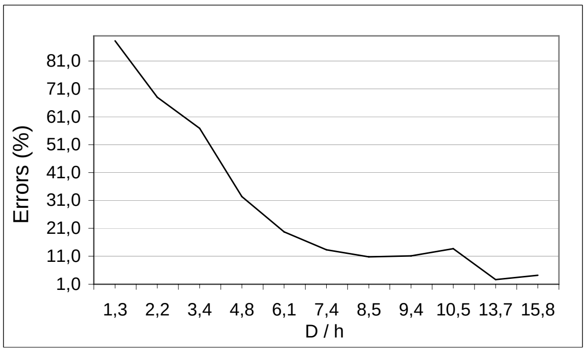

Figure 2. Errors for the 3rd. solution.

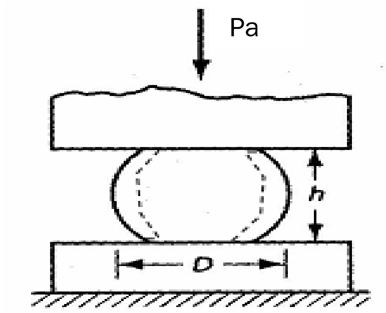

Figure 3. Process of deformation of a cylindrical body.

From the graph in Figure 1 itself, commenting on various aspects:

- The third solution is the one that comes closest to the results of experiments.
- The trend of the solution is to be above the experimental results.
- While the relationship D/h grows, the solution is closer to the real values, which was expected, because of the problem is being treated as one-dimensional one when it is really two-dimensional.

Analysing the errors graph in Figure it is possible to observe that:

- Mistakes are moderated especially from slenderness relationship bigger than 5, so that the solution can be considered adequate.
- The error is smaller when the slenderness relationship is greater, because of is a one-dimensional solution for a two-dimensional problem.

5. CONCLUSIONS

A new predictive model of the deformation of a cylindrical body stress was successfully developed. This is an analytical model with an easy numerical implementation that considers the process as a plane tension case.

Limitations of the model are also shown, that's why we can conclude that the process of deformation of a cylindrical body is not a one-dimensional problem.

The analytical model includes the shear stresses in the von Misses criterion, thus improving the forecasts made by other analytical methods of simulation.

6. REFERENCES

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