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STRAIN GRADIENT EFFECT IN THE FAILURE OF STRETCH-BEND METAL SHEETS

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ABSTRACT
This paper analyzes the effect of bending in the formability of metal sheets. Depending on the severity of the gradient, two kinds of failure are expected: a Necking-controlled Failure, which produces a neck in the entire sheet thickness, and a Fracture-controlled Failure, in which a ductile fracture initiates at the outer surface of the sheet. Both types of failures under plane strain conditions are modelled and discussed in the present work.
Keywords: Sheet-metal forming, Bending effect, Forming Limit Diagram

1. INTRODUCTION

Traditionally, sheet metal failure criteria have been formulated assuming a uniform strain/stress field through the thickness of the sheet. The stress/strain gradients through the sheet thickness are seldom taken into account explicitly. This is usually considered in the analysis by assuming that the mid-plane strain characterizes the formability of the sheet. This simple approximation, sometimes referred to as the Mid-Plane Rule, is found to yield consistent and effective results, although it is sometimes too conservative [1].

In the absence of strain gradient, *e.g.* in-plane stretching, all fibres in thickness of the sheet undergo the same strain. The sheet failure is essentially controlled by the initiation of necking or the initiation of ductile fracture in one of these fibres. In fact, when this occurs, all fibres in the thickness will presumably neck or fracture too, yielding the failure of the entire sheet.

However, when a strain gradient is present in the sheet, *e.g.* in stretch-bending processes, the picture may change drastically, especially if necking controls the failure. In this situation, necking of the more deformed fibre (outer fibre) does not cause sheet instability. In fact, when the outer fibre attains the necking strain, the underlying layers of less deformed materials postpone the necking of the sheet. Given that necking is a matter of plastic instability, it is reasonable to assume that the instability of a sheet is achieved only when all fibres in the sheet thickness are already necked. Thus, a simple approach to model this kind of failure is to assume that sheet necks when the less strained fibre (inner fibre) reaches the necking strain. This approach is consistent with Tharrett and Stoughton’s work [1], where they pointed out that sheet necking is observed when the strain on the concave side of the sheet reaches a limit strain. This failure criterion is called the Concave-Side Rule.

As bending becomes more important, *e.g.* in air bending or flanging, the fibres on the concave side of the sheet may undergo compression and thicken. As a consequence the necking will not govern the sheet failure any longer, giving way to ductile fracture as the dominant failure mechanism. The ductile fracture initiates with the appearance of a crack at the more stretched layer (outer fibre) in the sheet. Therefore, a conservative failure criterion might be to assume that the sheet fails (or it can be considered unusable) when the outer fibre fractures. This criterion is obviously not new and has been successfully checked in a variety of bending operations [2].

According to the above ideas, two independent kinds of failures can be expected in a sheet under a strain gradient: (1) a *Necking-controlled Failure*, which takes place when all layers in the sheet thickness neck, and (2) a *Fracture-controlled Failure*, which arises when the outer fibre fractures.

2. FAILURE CRITERIA FOR STRETCH-BENDING

Let us assume a stretch-bend sheet of initial thickness t_0, under a plane-strain state, as shown in Fig. 1. A major true strain distribution across the sheet thickness is introduced, $\varepsilon_1(r) = \ln(r / R_n)$, where R_n is the radius of the neutral surface. Let $\varepsilon_{1,out}$ and $\varepsilon_{1,in}$ be the major true strains measured in the outer (convex) and inner (concave) surfaces, respectively. The relationship between both external strains is given by

$$\varepsilon_{1,out} = \ln\left(\frac{R+t}{R_n}\right) = \varepsilon_{1,in} + \ln\left(1 + \frac{t}{R}\right) \tag{1}$$

where t is the current sheet thickness and R is the inner radius of the sheet. Two levels of approximation can be considered to evaluate the Eq. (1). The first one is to assume that no thickness change for small bend radius occurs during the stretch-bending process, that is, $t = t_0$

$$\varepsilon_{1,out} = \varepsilon_{1,in} + \ln\left(1 + \frac{t_0}{R}\right) \tag{2}$$

Generally speaking, this assumption may be valid when the stretching is small. The second approach considers that the thickness changes during deformation. This is done by assuming that the volume of an elementary portion of the sheet remains constant. Thus, Eq. (1) yields the expression

$$\varepsilon_{1,out} = \varepsilon_{1,in} + \frac{1}{2}\ln\left(1 + 2\frac{t_0}{R}e^{-\varepsilon_{1,in}}\right) \tag{3}$$

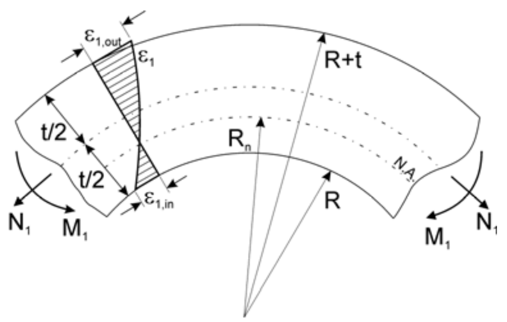

Figure 1. Variables in the stretch-bending.

Table 1. Mechanical properties and material constants for 1008 AK steel sheets.

t_0 (mm)	0.690	0.920	1.040
K (MPa)	549.1	552.0	556.8
n	0.230	0.230	0.240
r_0	1.985	1.660	1.740
r_{90}	1.860	2.120	1.800
FLD_0	0.278	0.293	0.358
a (Hosford's exponent)	6	6	6
C_1 (Freudenthal)	277.8	279.3	278.3
C_2 (Cockcroft & Latham)	277.8	279.3	278.3

A simple *Necking-controlled Failure Criterion* can be formulated assuming that the necking of the sheet arises when the inner fibre ($\varepsilon_{1,in}$) reaches the necking strain in plane stretching, that is, $\varepsilon_{1,in} = \varepsilon_{1,necking}$. In practice the value of $\varepsilon_{1,necking}$ when no effect of bending is present is estimated as the value of FLD_0. Introducing $\varepsilon_{1,in} = \varepsilon_{1,necking}$ in Eqs. (2) and (3), the strain at the outer fiber can be readly obtained as a function of the ratio t_0/R.

As mentioned before, when bending becomes sufficiently high, fracture dominates the sheet failure. Thus, a conservative *Fracture-controlled Failure Criterion* may be express assuming that fracture arises when the major strain at the outer fibre reaches a critical fracture strain, *i.e.* $\varepsilon_{1,out} = \varepsilon_{1,\,fracture}$. The fracture strain $\varepsilon_{1,\,fracture}$ in plane-strain conditions can be obtained from experimental data or mathematical models. A number of theoretical fracture criteria have been analyzed by Jain *et al.* [3]. It is worthy to note that the selection of the appropriate criterion is essential, and it will depend on the material and the forming conditions.

3. PRACTICAL APPLICATION. DISCUSSION

To explore the capability of the failure criteria to predict the effect of bending in the sheet failure, the experimental results reported by Tharrett and Stoughton [1] have been analyzed. Briefly, a series of stretch-bend tests in plane strain conditions were carried out. Three different sheet thicknesses (0.69, 0.92 and 1.04 mm) of 1008 AK steel were tested. The mechanical properties and material constants for this material are given in Table 1.

The blanks were deformed until failure over a series of punches of different radii that varied from 12.7 to 0.507 mm. The strains were measured on both sides of the sheets. The experimental results are show in Fig. 2. It represents the major strains measured on the convex (outer) side for different t_0 / R ratios. The higher the t_0 / R ratio, the higher the severity of bending. As designated by the authors, the tests where a visible neck at the concave (inner) side was observed are represented as solid circles. Otherwise, open circles are plotted.

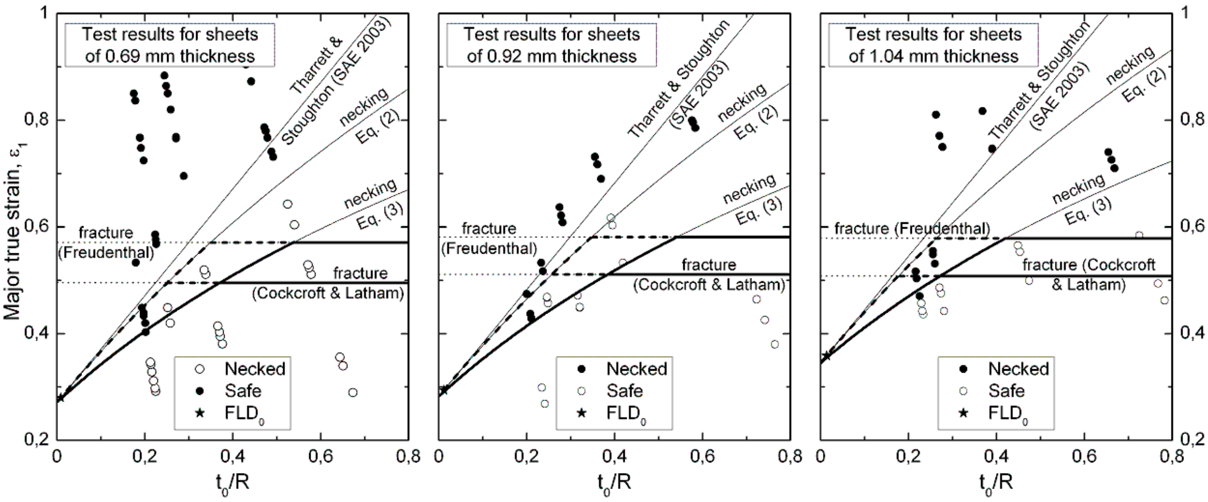

Figure 2. Major true strain on the convex (outer) side versus t_0 / R ratio for 1008 AK steel sheets (Tharrett & Stoughton [1]). Theoretical Necking criteria and Fracture criteria are also plotted.

The necking-controlled failure criteria given by Eqs. (2) and (3), along with the Tharrett and Stoughton's approach, are shown in Fig. 2. The value of necking strain $\varepsilon_{1,necking} \approx FLD_0$ for the three sheet thickness analysed are given in Table 1.

As discussed before, when t_0 / R becomes sufficiently high, the fracture tends to control the failure of the sheet. As a consequence, the necking lines in Fig. 2 must be truncated by a certain fracture line for high t_0 / R. In this work, a failure criterion based on the fracture at the convex side is assumed, where the in-plane stretching fracture strain $\varepsilon_{1,\,fracture}$ is estimated through two arbitrary classical ductile fracture criteria: Freudenthal's and Cockcroft and Latham's criteria (see [3]).

$$\int_0^{\varepsilon^P_{eq,f}} \sigma_{eq} d\varepsilon^P_{eq} = C_1 \quad \textit{(Freudenthal)} \qquad \int_0^{\varepsilon^P_{eq,f}} \sigma_1 d\varepsilon^P_{eq} = C_2 \quad \textit{(Cockcroft \& Latham)} \qquad (4)$$

where $\varepsilon^P_{eq,f}$ is the effective plastic strain at fracture and C_1 and C_2 are material constants. The value of C_1 and C_2 have been estimated assuming a fracture strain for pure tensile tests of 0.67 [4]. A Hollomon's law is assumed to model the strain hardening $\sigma_{eq} = K\varepsilon^n_{eq}$, and the anisotropic non-quadratic yield function proposed by Hosford [5] is used, $r_{90}\left|\sigma_1\right|^a + r_0\left|\sigma_2\right|^a + r_0 r_{90}\left|\sigma_1 - \sigma_2\right|^a = r_{90}(1+r_0)\sigma^a_{eq}$, where r_0 and r_{90} are the Lankford coefficients along the rolling (0º) and the transversal (90º) directions respectively (see material properties in Table 1). The predicted fracture strains are plotted in Fig. 2.

As can be seen in Fig. 2, the failure curve is formed by two parts: an initial segment that corresponds to the necking line, for low t_0 / R values, which is truncated by a second segment corresponding to a fracture line, for high t_0 / R values. The approaches with (dashed-line) and without (solid-line) thickness corrections are respectively shown in the figure. It can be seen that both approaches reproduce reasonably well the trends of the experimental data. Generally speaking, the thickness correction seems to be more effective for thicker sheets. In particular, the predictions obtained for the sheets of 1.04 mm thickness, using Freudenthal's fracture criterion, are remarkably good.

4. CONCLUSIONS

The effect of strain gradient on the formability of stretch-bend metal sheets has been analyzed. Two independent types of failures can be expected depending on the severity of the strain gradient: (1) a Necking-controlled Failure, which takes place when all layers in the sheet thickness become instable and neck, and (2) a Fracture-controlled Failure, which arises when the outer fibres of the sheet in the process zone fracture. Experimental stretch-bend data reported in the literature have been successfully analyzed with the present description. The simple criteria analysed here assume that failure depends essentially on what occurs in a single fibre, inner or outer fibre. This assumption can be quite restrictive in practice, and a more realistic hypothesis would be to consider that failure is controlled by the damage not in a single fibre but in a certain material volume. However, further experimental work needs to be done to support this idea.

5. REFERENCES

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