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DESIGN OF STABILIZING CONTROLLERS FOR INTERVAL SYSTEMS

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ABSTRACT
The controllers of standard PI and PID type play the essential role in contemporary industrial practice. Therefore, the simple techniques of controller tuning are still demanded, especially in case that these algorithms are able to cope with various uncertain conditions. The contribution is focused on control design for interval systems using the computation of all possible stabilizing PI controllers in combination with the choice of the final one through an algebraic approach. The needful amount of theory is followed by an example.
Keywords: stabilizing controllers, interval systems, control system design

1. INTRODUCTION
The mathematical model containing interval parameters is a common tool for description of imprecisely known industrial processes. And despite the existence of many advanced control technologies, an easy and effective way of conventional PI or PID control design for these systems is still very topical.
This paper combines the method of determination of all possible stabilizing PI controllers with fixed parameters for interval plants [6] and an algebraic control synthesis [4]. The enclosed illustrative example presents the application of this technique during robust stabilization of fourth order interval plant with integrative behaviour.

2. COMPUTATION OF STABILIZING PI CONTROLLERS
A possible approach to calculation of all stabilizing PI controllers based on plotting the stability boundary locus is proposed in [6]. The method supposes the classical closed-loop control system with controlled plant:

$$G(s) = \frac{B(s)}{A(s)} \tag{1}$$

and PI controller:

$$C(s) = k_P + \frac{k_I}{s} = \frac{k_P s + k_I}{s} \tag{2}$$

First, one needs to use the substitution $s = j\omega$ in the plant (1) and subsequently to decompose the numerator and denominator of this transfer function into their even and odd parts:

$$G(j\omega) = \frac{B_E(-\omega^2) + j\omega B_O(-\omega^2)}{A_E(-\omega^2) + j\omega A_O(-\omega^2)} \tag{3}$$

Then, the expression of closed-loop characteristic polynomial and setting the real and imaginary parts to zero lead to the equations:

$$
\begin{aligned}
k_P &= \frac{\omega^2 A_O(-\omega^2)B_O(-\omega^2) - \left(-A_E(-\omega^2)\right)B_E(-\omega^2)}{-\omega^2 B_O^2(-\omega^2) - B_E^2(-\omega^2)} \\
k_I &= \frac{\left(-A_E(-\omega^2)\right)\left(-\omega^2 B_O(-\omega^2)\right) - \omega^2 A_O(-\omega^2)B_E(-\omega^2)}{-\omega^2 B_O^2(-\omega^2) - B_E^2(-\omega^2)}
\end{aligned}
\tag{4}
$$

Simultaneous solving of these relations and plotting the obtained values into the (k_P, k_I) plane result in the stability boundary locus, which splits the (k_P, k_I) plane up to the stable and unstable regions. The determination of the stabilizing one(s) can be done via a test point within each region. Furthermore, this technique can be embellished with the Nyquist plot based approach from [5] to avoid potential problems with proper frequency gridding. In this refinement, the frequency axis can be divided into several intervals by the real values of ω which fulfill:

$$
\mathrm{Im}\left[G(s)\right] = 0 \tag{5}
$$

Such intervals are then sufficient for testing.

3. IMPROVEMENT OF THE METHOD FOR INTERVAL SYSTEMS

So far, the area of stabilizing controller coefficients for a given plant with only fixed parameters can be computed. However, the paper by Tan and Kaya [6] has improved the stabilization also for interval plants using the simple idea of its combination with the sixteen plant theorem [1]. In compliance with this principle, a first order controller robustly stabilizes an interval plant if and only if it stabilizes its 16 Kharitonov plants, which are defined as:

$$
G_{i_1,i_2}(s) = \frac{B_{i_1}(s)}{A_{i_2}(s)} \tag{6}
$$

where $i_1, i_2 \in \{1,2,3,4\}$; and $B_1(s)$ to $B_4(s)$ and $A_1(s)$ to $A_4(s)$ are the Kharitonov polynomials for the numerator and denominator of the interval system (6), respectively.
Remind that the Kharitonov polynomials for an interval polynomial:

$$
A(s,a) = \sum_{i=0}^{n} \left[a_i^-; a_i^+\right] s^i \tag{7}
$$

can be constructed using the upper and lower bounds of interval parameters according to the rule [2]:

$$
\begin{aligned}
A_1(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + \cdots \\
A_2(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + \cdots \\
A_3(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + \cdots \\
A_4(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + \cdots
\end{aligned}
\tag{8}
$$

The stabilization of an interval plant is grounded in the stabilization of all 16 fixed Kharitonov plants together, and so the final stability region is given by intersection of all partial regions.

4. THE CHOICE OF A CONTROLLER

The suitable choice of a controller from the set of stabilizing ones is an important issue. In this paper, the tuning method based on an algebraic approach in the ring of proper and stable rational functions is used [3, 7]. It takes advantage of Youla-Kučera parameterization and conditions of divisibility. More details about the methodology can be found e.g. in [4].
Under assumption of controlled system (1) in the form of first order plant:

$$G(s) = \frac{b_0}{s + a_0} \tag{9}$$

the parameters of PI controller (2) can be calculated from relations:

$$k_P = \frac{2m - a_0}{b_0}; \quad k_I = \frac{m^2}{b_0} \tag{10}$$

where $m > 0$ is a tuning parameter.

5. AN ILLUSTRATIVE EXAMPLE

Suppose that controlled process is described by interval transfer function adopted from [6]:

$$G(s) = \frac{[10; 30]}{s^4 + [85; 95]s^3 + [1900; 2000]s^2 + [3450; 3750]s} \tag{11}$$

The goal is to find all robustly stabilizing PI controllers (see sections 3 and 4), choose one of them via algebraic approach (section 5) and verify stability of closed control loop by simulation.
First, consider e.g.:

$$G_{1,1}(s) = \frac{B_1(s)}{A_1(s)} = \frac{10}{s^4 + 95s^3 + 2000s^2 + 3450s} \tag{12}$$

as the first of sixteen Kharitonov plants. The equation (4) takes the concrete form:

$$k_P = -0.1\omega^4 + 200\omega^2; \quad k_I = -9.5\omega^4 + 345\omega^2 \tag{13}$$

Using of (5) and consequent stability test for two obtained intervals lead to the range of the frequency $\omega \in (0; 6.0263)$, which is necessary for computing/plotting the stability boundary locus. The analogical procedure has been done for all 16 Kharitonov plants. However, in this specific case, the locus only 8 systems is enough to investigate, because the nominator of (12) takes only two extreme values and the construction of Kharitonov polynomials would be redundant here.
The figure 1 provides the graphical representation of the stability boundary locus for 8 Kharitonov plants, while the figure 2 brings closer look to the intersection, which constitutes final stability region for the interval plant (11).

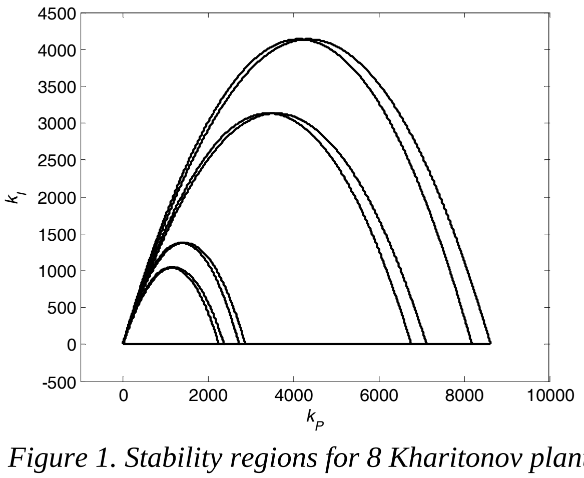

Figure 1. Stability regions for 8 Kharitonov plants

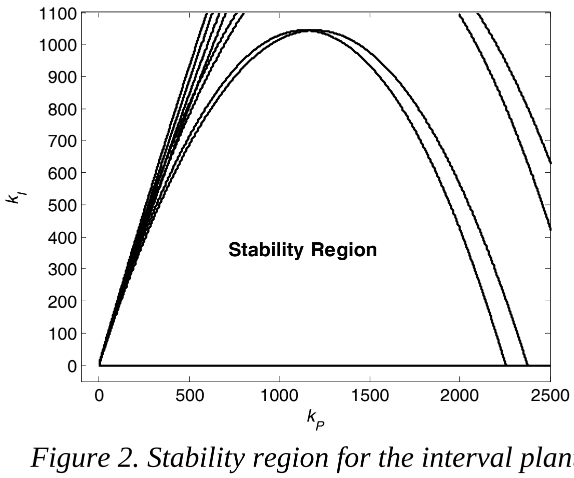

Figure 2. Stability region for the interval plant

Now, a certain controller must be chosen from the possible ones. The outlined algebraic approach is employed for this purpose. However, control quality is not the matter of research in this contribution. The controller has been tuned just to demonstrate robust stability of the closed control loop.
The nominal system has been obtained using the middle values of interval coefficients in (11) and then via the very simple approximation, i.e.:

$$\frac{20}{s^4+90s^3+1950s^2+3600s} \approx \frac{20}{3600s} = \frac{0.00\overline{5}}{s} = G_N(s) \quad (14)$$

For example, the tuning parameter $m=1$ gives the controller parameters (10):

$$k_P = 360; \quad k_I = 180 \quad (15)$$

which lie in the stability region from the figure 2.
Finally, the figure 3 shows the control responses of the loop with this PI controller and 4096 “representative” systems from the interval family (11). Each interval parameter has been divided into 7 subintervals and thus these 8 values and 4 parameters result in $8^4 = 4096$ systems for simulation. As can be seen, the controller (2) with parameters (15) really stabilizes the interval plant (11).

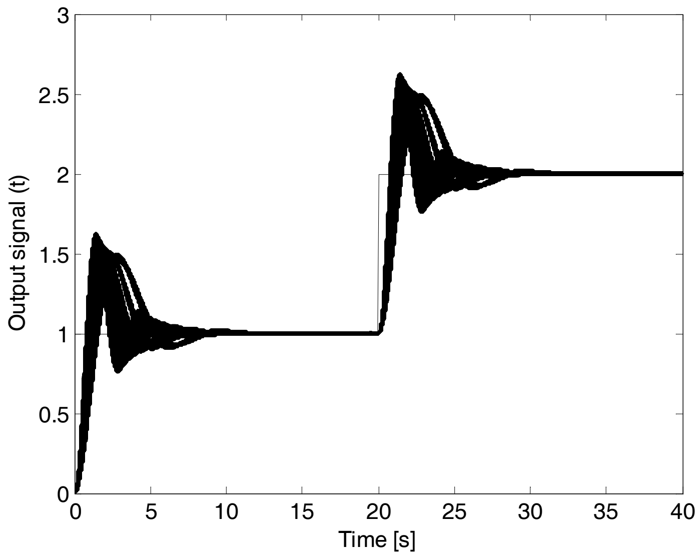

Figure 3. The output signals of 4096 “representative” plants from the interval family

6. CONCLUSION

The contribution has presented a possible PI controller design for interval systems based on the stability region determination and the selection of the proper regulator through the algebraic methodology. The fourth order integrative interval plant has been successfully stabilized in the illustrative example.

7. ACKNOWLEDGEMENT

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