12th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2008, Istanbul, Turkey, 26-30 August, 2008

# SLIP ANGLE OBSERVER OF THE VEHICLE BODY SIDE

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## ABSTRACT
*Usually, a reduced car model contains only the state variables ( speed, body side slip angle and yaw rate), which are essential for the vehicle dynamic and ABS control. When certain of these state variables cannot be measured, observers are implemented without unacceptable expenses. Since in this case the system is a non-linear one, a Luenberger observer is not possible directly to be used. In this paper, a non-linear observer based on a reduced track model has been designed and used to estimate the vehicle body side slip angle.*

## 1. INTRODUCTION
During our days, the equipments within vehicles include a lot of electrical and electronic subsystems. On top of these there are the safety relevant vehicle subsystems, such as drive dynamics control and anti-lock brake systems. To decrease the design effort for such systems, the computer modeling and simulation are used. The aim of computer models, such as the one shown schematically in Figure 1 is to reveal, as early as possible in the design phase, the effect on the dynamic vehicle behaviour of new components operating in conjunction with the existing subsystems.

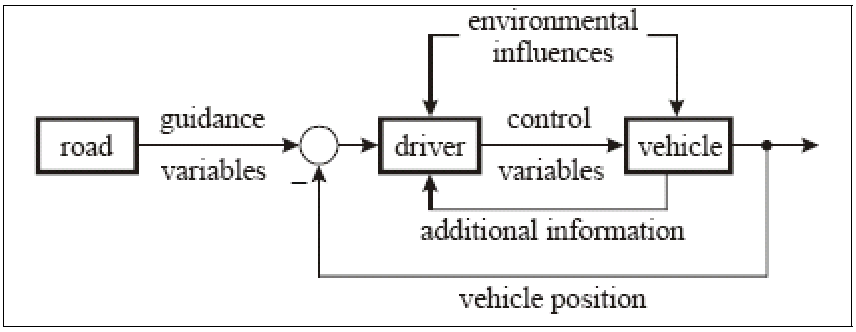


*Figure 1. The standard vehicle-driver-road control loop.*

The modelling approach has the following aims:
• reduction of the model complexity to a level sufficient for vehicle dynamics;
• implementation on a PC in a common programming language, so that usage is widely spread instead of being constrained to a few specialised departments;
• interaction between sub-models, whereby the design time is concentrated upon the subsystem currently under investigation;
• only necessary accuracy, so that time consuming tests with experimental vehicles can be reduced.
In order to obtain these aims, the system will be decomposed into its individual components. When carrying out such a partitioning, it is important to ensure that meaningful variables are chosen for the interfaces between the different sub-models. It is thus sensible to choose, insofar as it is possible,

torques or forces and angular or longitudinal velocities. The reduced model (reduced non-linear two-track model) should contain only those state variables which are essential for vehicle dynamic and ABS control. These are the vehicle speed, the vehicle body side slip angle and the yaw rate. In general, observers are implemented when certain state variables cannot be measured without unacceptable expense. If, as is here the case, the system is in non-linear form, the direct application of a Luenberger observer is not possible. To overcome this problem in the paper a non-linear observer is designed and used for the observation of the vehicle body side slip angle, based on reduced non-linear two-track model.

## 2. A NON-LINEAR OBSERVER BASIC THEORY

The vehicle body side slip angle is estimated using a non-linear observer. Andreescu [1] and Zeitz [5] propose, for example, an observer design using linearization which is based on the Luenberger observer. Figure 2 shows the structure of the non-linear observer from Zeitz.

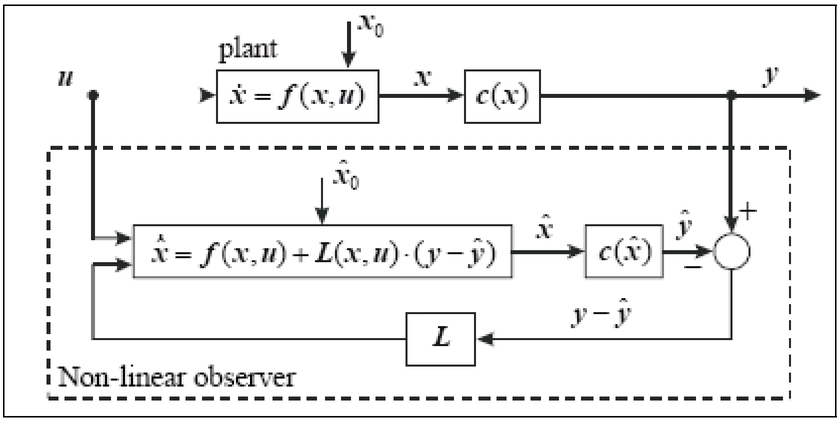


*Figure 2. Structure of the non-linear observer.*

As seen in Figure 2, for the non-linear observer, two major problems must be solved: the calculus and choice of the observer gain matrix $\boldsymbol{L}$ and the observer design. Using the liniarization approach [1,3,6], we obtain a formal linear error differential equation whose solution can be reduced to zero with a suitable choice of the observer gain matrix $\boldsymbol{L}$. This matrix must be found such that the observer dynamics matrix $\boldsymbol{F}(\hat{\mathbf{x}},\mathbf{u})$ is constant and its eigenvalues lie to the left of the **j**-axis, so that the solution $\tilde{\mathbf{x}}\,(\mathbf{t})$ of the estimation error differential equation tends to zero for $t\to\infty$ for any initial conditions. The suitable choice of $\boldsymbol{L}(\hat{\mathbf{x}},\mathbf{u})$ is given by equating $\boldsymbol{F}(\hat{\mathbf{x}},\mathbf{u})$ with a constant matrix $\mathbf{G}$, whose values are predefined according to the desired dynamics (pole placement):

$$\boldsymbol{F}(\hat{\boldsymbol{x}},\boldsymbol{u}) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}},\boldsymbol{u}) - \boldsymbol{L}(\hat{\boldsymbol{x}},\boldsymbol{u})\cdot\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}}) \overset{!}{=} \boldsymbol{G}\,. \tag{1}$$

Thus, the observer gain matrix $\boldsymbol{L}(\hat{\mathbf{x}},\mathbf{u})$, is calculated as:

$$\boldsymbol{L}(\hat{\boldsymbol{x}},\boldsymbol{u}) = \left[\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}},\boldsymbol{u}) - \boldsymbol{G}\right]\cdot\left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}})\right]^{+}. \tag{2}$$

The matrix $\frac{\partial \mathbf{c}}{\partial \hat{\mathbf{x}}}(\hat{\mathbf{x}})$ is in general non square. For the inversion, it used the Moore-Penrose pseudo-inversion, as follows:

$$\left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}})\right]^{+} = \left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}})\right]^{\mathrm{T}}\cdot\left(\left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}})\right]\cdot\left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}})\right]^{\mathrm{T}}\right)^{-1}. \tag{3}$$

## 3. SLIP ANGLE OBSERVER DESIGN

To observe the vehicle body side slip angle $\boldsymbol{\beta}$, a non-linear observer is used. The whole modeling is based on the reduced non-linear two-track model [3]. According it, we consider the following notations: $F_{Lij}$ – the longitudinal wheel forces (where $ij=FL$ – front left wheel), $FR$ - front right wheel, $RL$ – rear left wheel, $RR$ – rear right wheel and $\delta_W$ – wheel turn angle.
All these are inputs $\boldsymbol{u}$ for the observer. The wheel forces in $\boldsymbol{L}$ – directions are thus explicitly defined as inputs, and the wheel forces in the $\boldsymbol{S}$ – directions appear implicitly in the front and rear tire side slip constants $c_F$ and $c_R$ [3].
In accordance with all these considerations, the reduced non-linear two-track model equation is:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) \Leftrightarrow \begin{bmatrix} \dot{v}_{CoG} \\ \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} f_1(v_{CoG},\beta,\dot{\psi},F_{LFL},F_{LFR},F_{LRL},F_{LRR},\delta_W) \\ f_2(v_{CoG},\beta,\dot{\psi},F_{LFL},F_{LFR},F_{LRL},F_{LRR},\delta_W) \\ f_3(v_{CoG},\beta,\dot{\psi},F_{LFL},F_{LFR},F_{LRL},F_{LRR},\delta_W) \end{bmatrix}, \tag{4}$$

where $v_{CoG}$ is the vehicle $CoG$ (Center of Gravity) velocity, and $\psi$ is the yaw angle.
The states, inputs and measurements are following defined:

State Vector:

$$\boldsymbol{x} = \begin{bmatrix} v_{CoG} \\ \beta \\ \dot{\psi} \end{bmatrix}, \tag{5}$$

Input Vector

$$\boldsymbol{u} = [F_{LFL}, F_{LFR}, F_{LRL}, F_{LRR}, \delta_W]^T, \tag{6}$$

Measurement

$$\boldsymbol{y} = \begin{bmatrix} v_{CoG} \\ \dot{\psi} \end{bmatrix}. \tag{7}$$

Note that the angle $\delta_W$, included in the input vector $\boldsymbol{u}$, is handled by the driver.
By steering, the vehicle body side slip angle $\boldsymbol{\beta}$ is controlled. In dangerous driving situations, the driver may be supported by a vehicle dynamic control system, which uses the longitudinal wheel forces $F_{LFL}$ , $F_{LFR}$ , $F_{LRL}$ and $F_{LRR}$ as additional input variables. The elements of the Jacobian matrix are presented in [3]. To determine the observer gain matrix $\boldsymbol{L}(\hat{\mathbf{x}},\mathbf{u})$, a suitable desired observer dynamic matrix $\boldsymbol{G}$ must be chosen in Eq. (1).
To determine the eigenvalues of the observer dynamic matrix $\boldsymbol{F}$ it must proceed too a diagonal form for matrix $\boldsymbol{G}$, because in this case these are directly read from the main diagonal elements.
Thus, the desired matrix $\boldsymbol{G}$ is chosen as a diagonal matrix, whose main diagonal elements are the desired eigenvalues for the matrix $\boldsymbol{F}(\hat{\mathbf{x}},\mathbf{u})$, as follows:

$$\boldsymbol{G} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}. \tag{8}$$

The three eigenvalues $\lambda_1$ , $\lambda_2$ , $\lambda_3$ , must be carefully chosen in order to influence the dynamics of the observer gain matrix $L(\hat{\mathbf{x}},\mathbf{u})$. With $\mathbf{y} = \mathbf{c}(\mathbf{x}) = \begin{bmatrix} \mathbf{v}_{CoG} \\ \dot{\psi} \end{bmatrix}$, **c** is a constant 2x3 matrix. The pseudo-inverse is obtained using the Eq. 3, as follows:

$$\left[\frac{\mathbf{dc}}{\mathbf{dx}}(\hat{x})\right]^{+} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{9}$$

The Jacobian matrix $\partial \mathbf{f} / \partial \mathbf{x}$ , the matrix $G$ and the pseudo-inverse are substituted into Eq.(2). Thus, the matrix $L(\hat{\mathbf{x}},\mathbf{u})$ elements are obtained as function of the Jacobian matrix and the desired eigenvalues, as:

$$L(\hat{x}, u, \lambda_1, \lambda_3) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} - \lambda_1 & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_3} - \lambda_3 \end{bmatrix}. \tag{10}$$

Note that only two of the chosen eigenvalues appear in the above calculation, thus only two of the desired eigenvalues of the matrix $F(\hat{\mathbf{x}},\mathbf{u})$ can be placed in the chosen position. There remains a time - variant eigenvalue $\lambda_2$ . It can be seen, that also $\lambda_2$ remains in the stable region.

## 4. CONCLUSIONS

Starting from the theory of Luenberger observers, has been designed a non-linear observer to estimate the vehicle body side slip angle. This new observer is based on a reduced non-linear two-track model using a linearization approach to obtain a formal linear error differential equation whose solution has been reduced to zero bt using a suitable choice of the observer gain matrix.

## 5. REFERENCES


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