12th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2008, Istanbul, Turkey, 26-30 August, 2008

OBJECT LENGTH AND AREA CALCULATIONS ON THE DIGITAL IMAGE

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ABSTRACT
The parts of structure elements and other products can be photographed with a digital camera. The photo is then analyzed and the appreciation of quality of the product relates to the magnitude and shape of irregularities, i.e. their area and extent and the relationship between them. The paper introduces a sample-anchoring method for estimating the arc length and the area of a closed planar curve consisting of the geometrically ordered data points. An adjunctive pixel is appended to each pixel. The adjunctive pixel is computed by the use of a changeable convolution mask, where the pixel anchoring into the neighborhood of the original pixel is taken into account. The curve is approximated by two polygons. Vertices of the first one are adjunctive pixels and the vertices of the second one are convolve pixels. The sample anchoring method uses three parameters. The first one models the convolution mask, the second and the third one model the neighborhood of the pixel and the stop criterion of the convolution process.

Keywords: sample-anchoring method, adjunctive pixel, convolution process

1. INTRODUCTION
The paper discusses the methods for approximating the arc length of a planar curve and the area of a region bounded with a closed curve where the data are taken from the rectangular lattice of the gray scale values. The method commonly used for that is the sample-count method [1]. The sample-distance method takes into account the length of the diagonal in 8-connected path [1]. The sample-normal method requires the construction of the unit normal vectors for the sampled curve points. The paper introduces a sample-anchoring method for estimating the arc length and the area of a closed planar curve consisting of the geometrically ordered data points [2].

2. IMAGE AND CURVE
A digital image *I* is a rectangular $M \times N$ grid of pixels [2]. A pixel is represented by a square centered on the point (X_i,Y_j), the coordinates of the point $\mathbf{P}_{ij}$, and a pixel value $f(x,y)$, therefore $P_{ij} = \{(x,y), f(x,y); X_i - \Delta_x < x < X_i + \Delta_x, Y_j - \Delta_y < y < Y_j + \Delta_y, X_i = (2i+1)\Delta_x, Y_j = (2j+1)\Delta_y\}$, where *i* and *j* are integers, Fig. 1. (a). A digital image is a set of pixels $I = \{P_{ij}; 1 \le i \le M, 1 \le j \le N\}$. The object on the image is a set of such pixels where $f(x,y) \approx c$ for a given constant *c*. We can construct the object boundary closed curve as a set of *n* pixels $\Gamma = \{P_k\}_{k=0}^{k=n-1}$. A digital curve Γ generally comes from the digitalization of a real curve *C*. If the digital curve Γ is the result of an

edge detection algorithm there is no information about the original *C* and therefore geometrical characteristics are not well defined. The set $\bigcup_{k=0}^{k=n-1} P_k$ includes the actual unknown curve *C*.

3. SAMPLE-ANCHORING METHOD

By the here introduced sample-anchoring method, every pixel its adjunctive pixel $\tilde{P}$ is added as a closed square centered on **P**, $\tilde{P} = \{(x,y), f(x,y); X-\delta_x \le x \le X+\delta_x, Y-\delta_y \le y \le Y+\delta_y\}$ where $\delta_x = \varepsilon_x \Delta_x, \delta_y = \varepsilon_y \Delta_y$ and $\varepsilon_x > 0, \varepsilon_y > 0$. If $\varepsilon_x > 1, \varepsilon_y > 1$ and if the digital contour Γ is dense [1] the adjunctive pixels build up a set including Γ and probably the curve *C*. In the case where $\varepsilon_x = \varepsilon_y$ we denote $\varepsilon_x = \varepsilon_y = \varepsilon$. The basic idea of the sample-anchoring method is to move each point **P** from its initial position in order to compute a smoother polygonal approximation of the curve

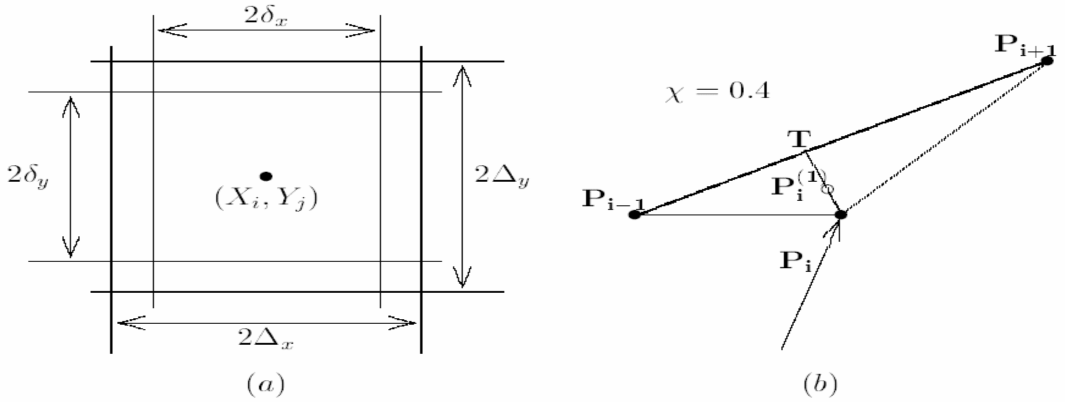

Figure 1. (a) Pixel (bound with thick line) and the adjunct pixel (bound with thin line). (b) First step of the sample-distance method by using the formulae (2)

C. In order to smoothen the digital curve Γ, a convolution of the point and its neighborhood is applied. If Δ_i denotes the distance between the points $\Delta_i = |\mathbf{P}_{i+1} - \mathbf{P}_i|$ and $\lambda_i = \frac{\Delta_{i-1}}{\Delta_{i-1} + \Delta_i}$ one can introduce a changeable convoluting mask and use De Casteljau algorithm

$$\mathbf{P}_i^{(k+1)} = (1-\lambda_i)\mathbf{P}_{i-1}^{(k)} + \lambda_i \mathbf{P}_{i+1}^{(k)}, \quad i = 0,1,2,\cdots,n. \tag{1}$$

A transformation of the upper equation to $\mathbf{P}_i^{(k+1)} = \mathbf{P}_{i-1}^{(k)} + \lambda_i(\mathbf{P}_{i+1}^{(k)} - \mathbf{P}_{i-1}^{(k)})$ shows explicitly that the point $\mathbf{P}_i^{(k+1)}$ lies on the edge with vertices $\mathbf{P}_{i-1}^{(k)}$ and $\mathbf{P}_{i+1}^{(k)}$ and retains the ratio of the distances between the successive three points in each step of the convolution.

With some help of the elementary geometry we can deduce the following changeable convolution mask too, Fig. 1. (b),

$$\mathbf{P}_i^{(k+1)} = \chi(1-\lambda_i)\mathbf{P}_{i-1}^{(k)} + (1-\chi)\mathbf{P}_i^{(k)} + \lambda_i \chi \mathbf{P}_{i+1}^{(k)}, \quad i = 0,1,2,\cdots,n. \tag{2}$$

If $\chi = 1$, the point $\mathbf{P}_i^{(k+1)}$ divides the edge $\mathbf{P}_{i-1}^{(k)}\mathbf{P}_{i+1}^{(k)}$ at the ratio $\frac{\Delta_{i-1}}{\Delta_i}$. The point $\mathbf{P}_i^{(k+1)}$ lies inside the triangle $\mathbf{P}_{i-1}^{(k)}\mathbf{P}_i^{(k)}\mathbf{P}_{i+1}^{(k)}$, if the parameter χ has values $0 < \chi < 1$. We introduce anchoring as follows: during the iterations every point $\mathbf{P}_i^{(k)}$ has to be a point of the $\tilde{P}_i$. If the point $\mathbf{P}_i^{(k)}$ is not a point of the $\tilde{P}_i$ we put it on the boundary of the $\tilde{P}_i$ between point $\mathbf{P}_i$ and $\mathbf{P}_i^{(k)}$ and name adjunct to the point $\mathbf{P}_i$ and denote as $\overline{\mathbf{P}}_i$. Over the adjunct points, we do not execute the convolutions.

During the iterative convolutions we compute the polygons which become smoother between the successive adjunct points. If we connect only the successive adjunct points, we get an approximation of digital contour Γ, whose length may be close to the lowest possible length of all continuous contours that approximate the given digital contour Γ. Therefore it is not necessary that two successive adjunct points connect a very smooth polygon. This is the reason for the introduction of a cylinder with a radius $R = \rho \frac{\Delta_x + \Delta_y}{2}$, $\rho > 0$, the cylinder axis being a line between two successive adjunct points. We interrupt the iteration when all convolved points between the successive adjunct points are located within the cylinder. The successive adjunct points have indexes $i_0 < i_1 < \cdots < i_m$. We can interrupt the convolution process in the segment between the successive adjunct points $\overline{\mathbf{P}}_{i_l}$ and $\overline{\mathbf{P}}_{i_{l+1}}$ when the following condition is fulfilled

$$\frac{|(\mathbf{P}_i^{(k)} - \overline{\mathbf{P}}_{i_l}) \times (\overline{\mathbf{P}}_{i_{l+1}} - \overline{\mathbf{P}}_{i_l})|}{|\overline{\mathbf{P}}_{i_{l+1}} - \overline{\mathbf{P}}_{i_l}|} < R, \quad i = i_l + 1, \cdots, i_{l+1} - 1, \tag{3}$$

where $\times$ denotes vector cross-product operation. When the condition (3) is fulfilled for all pairs of the successive adjunct points $(\overline{\mathbf{P}}_{i_l}, \overline{\mathbf{P}}_{i_{l+1}})$, $l = 1, \cdots, m-1$, two approximations of the unknown curve C have been computed. The first one is the polygon whose vertices are the adjunct points $\overline{\mathbf{P}}_{i_0}, \overline{\mathbf{P}}_{i_1}, \cdots, \overline{\mathbf{P}}_{i_m}$ and the second one is the polygon whose vertices are $\mathbf{P}_0^{(e)}, \mathbf{P}_1^{(e)}, \cdots, \mathbf{P}_n^{(e)}$, where the superscript *(e)* denotes the point $\mathbf{P}_i^{(e)}$ computed during the convolution process from point $\mathbf{P}_i$.

Thereafter we can compute two approximations of the length of C from the digital curve Γ. We get the first one as a sum of all distances between successive adjunct points

$$\overline{L} = \sum_{j=0}^{m-1} |\overline{\mathbf{P}}_{i_{j+1}} - \overline{\mathbf{P}}_{i_j}|. \tag{4}$$

If the curve C is closed we have to add the term $|\overline{\mathbf{P}}_{i_0} - \overline{\mathbf{P}}_{i_m}|$ to the sum (4). The second one is intuitively much more realistic

$$L = \sum_{i=0}^{n-1} |\mathbf{P}_{i+1}^{(e)} - \mathbf{P}_i^{(e)}|. \tag{5}$$

If curve C is closed too, we have to add the term $|\mathbf{P}_0^{(e)} - \mathbf{P}_n^{(e)}|$ to the sum (5). We can approximate the area of an object, whose boundary is a closed planar digital contour Γ, with the area of a closed polygon whose vertices are adjunct points as follows

$$\overline{A} = \frac{1}{2} |\sum_{j=1}^{m-1} (\overline{\mathbf{P}}_{i_{j+1}} - \overline{\mathbf{P}}_{i_0}) \times (\overline{\mathbf{P}}_{i_j} - \overline{\mathbf{P}}_{i_0}) \tag{6}$$

and in the second case where the vertices are $\mathbf{P}_0^{(e)}, \mathbf{P}_1^{(e)}, \cdots, \mathbf{P}_n^{(e)}$

$$A = \frac{1}{2} |\sum_{i=1}^{n-1} (\mathbf{P}_{i+1}^{(e)} - \mathbf{P}_0^{(e)}) \times (\mathbf{P}_i^{(e)} - \mathbf{P}_0^{(e)})|. \tag{7}$$

Intuitively, we expect that the estimated area $\overline{A}$ of an object is smaller than A omputed from (7).

4. NUMERICAL EXPERIMENT

The boundary of an artificially created digital image of the ellipse is represented as a connected path, Fig. 2. If we first take into account only four Freeman directions, we get a closed curve denoted by Γ_4, pixels assigned by +. Secondly, if we take into account eight Freeman directions, we get a closed curve denoted by Γ_8 and pixels assigned by $\circ$. We apply the sample-anchoring method to the closed curve Γ_4, made from 60 gemetrically ordered points.

Using the formulae (2) the sample-anchoring method can be modeled with two additional independent

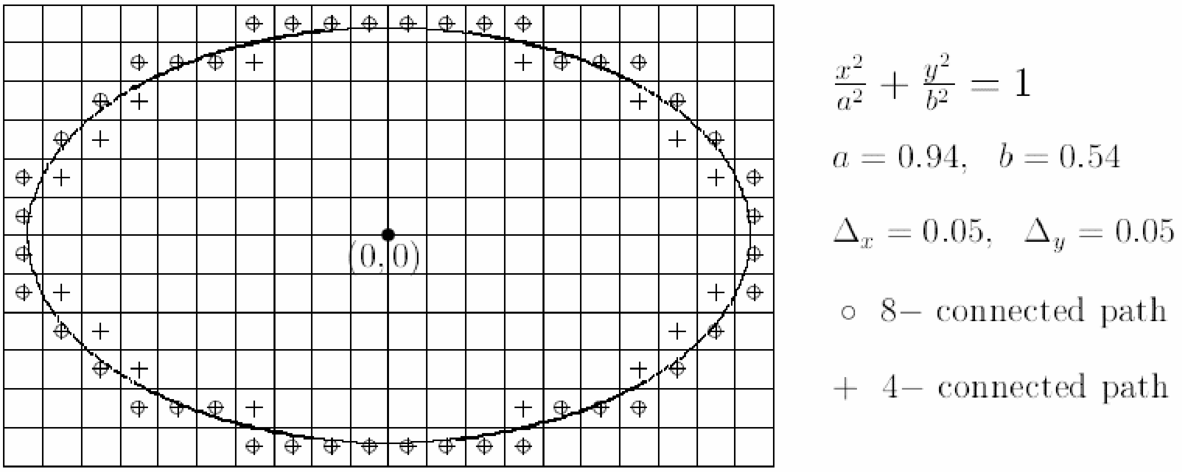

Figure 2. Arc length for the actual ellipse $L = 4.734860$ *and the area* $A = 1.564672$

parameters ε and ρ. Their influence on the approximate length and the area of digital curve Γ_4 is shown in table 1. It is seen from table 1 that the most important parameter is ε. The parameter ρ does not have any important influence on the computational results. If the ellipse has been approximated by the adjunct points, ρ has no influence at all. In the studied cases, the results were computed after three to five steps. We have tested the sample-anchoring method in order to find the optimal parameters. If we compute the geometrical parameters *L* and *A* from the adjunct points, we get the accurate value for the length at $\varepsilon = 0.6003$ and for the area at $\varepsilon = 0.1557$.

Table 1. The arc length $L(\varepsilon, \rho)$ *and the area* $A(\varepsilon, \rho)$ *have been computed by various values of the parameters* ε *and* ρ. *For (a) formulas (5) and (7) have been used but in case (b) the formulas (4) and (6) have been used*

	$L(\epsilon,\rho)/L$				$A(\epsilon,\rho)/A$			
(a)	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 0.5$	$\epsilon = 1.$	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 0.5$	$\epsilon = 1.$
$\rho = 0.1$	1.1948	1.0772	1.0142	0.9654	1.0074	0.9949	0.9830	0.9138
$\rho = 0.3$	1.1949	1.0795	1.0158	0.9675	1.0084	1.0051	0.9898	0.9269
$\rho = 0.5$	1.1949	1.0800	1.0182	0.9704	1.0084	1.0059	0.9988	0.9385
$\rho = 1.0$	1.1949	1.0800	1.0194	0.9724	1.0084	1.0059	1.0011	0.9456
(b)								
	1.1946	1.0769	1.0137	0.9651	1.0034	0.9913	0.9798	0.9054

5. REFERENCES

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