12th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2008, Istanbul, Turkey, 26-30 August, 2008

SIMPLE APPROACH TO SOLUTION OF AUTONOMY AND INVARIANCE OF MIMO CONTROL LOOPS: SIMULATION CONTROL OF THREE-VARIABLE SYSTEM OF STEAM TURBINE

Pavel Navrátil, Jaroslav Balátě & Tomas Sysala
Tomas Bata University in Zlín, Faculty of Applied Informatics
Nad Stráněmi 4511, 760 05 Zlín
Czech Republic

ABSTRACT
This paper describes one of possible way to control of MIMO control loop with utilisation of the so called binding members and correction members. The designed method combinations classical way to ensure of autonomy of control loop via binding members and the use of the method of SISO branched control loops with measurement of dominant disturbance variables to ensure of invariance of control loop by means of correction members. Simulation verification of the method was carried out for three-variable control loop of a steam turbine.
Keywords: autonomy, invariance, MIMO control loop, synthesis, correction members, binding members

1. INTRODUCTION
In control applications, at large numbers of processes (air-conditioning plants, distillation columns, steam boilers, turbines, etc.) several variables have to be controlled at the same time. In this case there is not larger member of independent SISO (single-input single-output) control loop. These control loops are complex with several controlled variables where separate variables are not mutually independent. Mutual coupling of controlled variables is usually given by simultaneous action of each of input (manipulated and disturbance) variables of controlled plant to all controlled variables. These control loops are called MIMO (multi-input multi-output) control loops and they are a complex of mutually influencing simpler control loops [2].

2. CONTROL LOOP OF MIMO SYSTEM
We consider MIMO control loop with measurement of disturbance (see Figure 1). $\boldsymbol{G}_S(s)$, $\boldsymbol{G}_{SV}(s)$, $\boldsymbol{G}_R(s)$ and $\boldsymbol{G}_{KC}(s)$ are transfer matrixes of a controlled plant, disturbance variables, controller and correction members. Signal $\boldsymbol{Y}(s)$ $[n\times 1]$ is a vector of controlled variables, $\boldsymbol{U}(s)$ $[n\times 1]$ is a vector of manipulated variables and $\boldsymbol{V}(s)$ $[m\times 1]$ is a vector of disturbance variables.

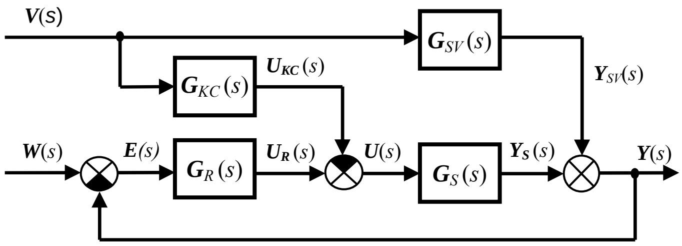

Figure 1. MIMO control loop with measurement of disturbance v

Transfer matrixes of controlled plant and of disturbance variables are considered in the following forms

$$\boldsymbol{G}_S(s)=\frac{\boldsymbol{Y}_S(s)}{\boldsymbol{U}(s)}=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{SV}(s)=\frac{\boldsymbol{Y}_{SV}(s)}{\boldsymbol{V}(s)}=\begin{bmatrix} S_{V11} & S_{V12} & \cdots & S_{V1m} \\ S_{V21} & S_{V22} & \cdots & S_{V2m} \\ \vdots & \vdots & \cdots & \vdots \\ S_{Vn1} & S_{Vn2} & \cdots & S_{Vnm} \end{bmatrix} \quad \text{(1a), (1b)}$$

Transfer matrixes of controller and of correction member are considered in these forms

$$\boldsymbol{G}_R(s)=\frac{\boldsymbol{U}_R(s)}{\boldsymbol{E}(s)}=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{KC}(s)=\frac{\boldsymbol{U}_{KC}(s)}{\boldsymbol{V}(s)}=\begin{bmatrix} KC_{11} & 0 & \cdots & 0 \\ 0 & KC_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & KC_{nn} \end{bmatrix} \quad \text{(2a), (2b)}$$

2.1. Autonomy and invariance of control loop

At control of MIMO control loop, beside stability and quality of control, it is often required for control loop to be autonomous and invariant. In order to determine the conditions for autonomy and invariance we start from a command transfer matrix $\boldsymbol{G_{W/Y}}(s)$ and disturbance transfer matrix $\boldsymbol{G_{V/Y}}(s)$ [2], therefore

$$\boldsymbol{G}_{W/Y}(s)=[\boldsymbol{I}+\boldsymbol{G}_S(s)\boldsymbol{G}_R(s)]^{-1}\boldsymbol{G}_S(s)\boldsymbol{G}_R(s) \tag{3}$$

$$\boldsymbol{G}_{V/Y}(s)=[\boldsymbol{I}+\boldsymbol{G}_S(s)\boldsymbol{G}_R(s)]^{-1}[\boldsymbol{G}_{SV}(s)-\boldsymbol{G}_S(s)\boldsymbol{G}_{KC}(s)] \tag{4}$$

Autonomy of control loop

It is resulted from the equation (3) that the control loop is autonomous if it is ensured that the matrix $\boldsymbol{G}_S(s)\ \boldsymbol{G}_R(s)$ is diagonal. On the base of the condition it is possible to derive the following relation

$$\frac{R_{kl}}{R_{ml}}=\frac{s_{lk}}{s_{lm}} \qquad k,l,m=<1,\ldots,n>, s_{lm}\neq 0 \tag{5}$$

s_{lk}, s_{lm} - algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$
R_{kl}, R_{ml} - separate members (binding members) of a transfer matrix of controller $\boldsymbol{G_R}(s)$

Main (diagonal) controllers R_{11}, R_{22}, R_{33} etc. are usually known already from the first design of conception of control. The design of main controllers is carried out by any SISO method of synthesis. The above mentioned relation (5) is therefore used for calculation of all remaining members of matrix controller $\boldsymbol{G_R}(s)$, i.e. for calculation of transfers of binding members.

Invariance of control loop

For ensuring absolute invariance it is necessary that the disturbance transfer matrix $\boldsymbol{G}_V(s)$ (4) is zero. This is possible if the following relation is valid

$$\boldsymbol{G}_{\mathrm{KC}}(\mathrm{s})=\boldsymbol{G}_S^{-1}(\mathrm{s})\boldsymbol{G}_{\mathrm{SV}}(\mathrm{s}) \tag{6}$$

At design of correction members, internal couplings are omitted at MIMO control loop and thus n SISO branched control loop with measuring of a disturbance variable are gained. Connection of all these SISO control loops is the same and they differ only in separate transfers of controlled plants, controllers, correction members and disturbance variables [2]. Common connection of these control loops is presented on the following Figure 2. Correction members KC are determined on the base of the condition (6). The invariance of the control loop is ensured, according to the above mentioned method, by using analogy of SISO branched control loops with measuring of disturbance variable v.

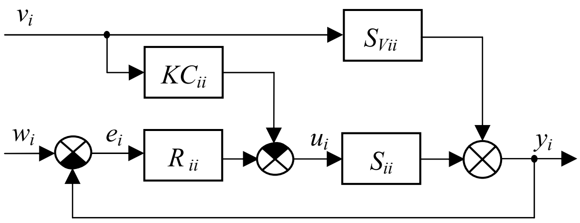

Figure 2. Block diagram of SISO control loop with measuring of disturbance variable v_i

$$KC_{ii}=\frac{S_{Vii}}{S_{ii}} \qquad i=<1,\ldots,n>, S_{ii}\neq 0 \tag{7}$$

S_{Vii}- separate members of transfer matrix of disturbance variables $\boldsymbol{G_{SV}}(\mathrm{s})$
S_{ii} - separate members of transfer matrix of controlled plant $\boldsymbol{G_S}(\mathrm{s})$

2.2. MIMO control loops synthesis

In practice the possible approximate solution of MIMO control loop is applied from analysis of MIMO control loop and really used control schemes in particular technological equipments [2]. One of the possible methods of solution of MIMO control loops synthesis is described in the following part of this paper. Generally it is possible to divide this solution into three parts

- Design of **main (diagonal) controllers** by any synthesis method of SISO control loops, i.e. design of parameters of main controllers for n SISO control loops ($R_1, R_2, \ldots, R_n$),
  Here, it is considered that original diagonal transfer functions S_{ii} ($i = 1, \ldots, n$) of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$ are modified to diagonal transfer functions $S_{ii,x}$ ($i = 1,\ldots, n$). In these modified transfer functions influences of aside-from diagonal transfer functions of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$, i.e. S_{ij} ($i\neq j,\ i, j = 1, \ldots, n$) on original diagonal transfer functions S_{ii} ($i = 1, \ldots, n$) are included. Transfer functions $S_{ii,x}$, i.e $S_{11,x}$, $S_{22,x}$, $S_{33,x}$ etc. are determined from equation (8) by using relations (3) and (5).

$$S_{ii,x}=\sum_{j=1}^{n}S_{ij}\frac{s_{ij}}{s_{ii}} \quad i,j=<1,\ldots,n>, s_{ii}\neq 0 \tag{8}$$

  s_{ii},s_{ij} - algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$
  S_{ij} - separate members of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$
- Ensuring autonomy of control loop by means of **binding members** of transfer matrix of controller

$\boldsymbol{G}_R$(s). It is ensured by using (5), whereas original diagonal transfer functions of transfer matrix of controlled plant $\boldsymbol{G_S}(s)$ (1a), i.e. S_{ii} are considered.

- Ensuring of invariance control loop via **correction members** *KC* by using *n* SISO control loops with measuring of disturbance variables. It is ensured by using (7), whereas original diagonal transfer functions of transfer matrix of controlled plant $\boldsymbol{G_S}(s)$ (1a), i.e. S_{ii} are considered.

3. SIMULATION EXAMPLE

3.1. MIMO controlled plant

Steam turbine is a typical example of MIMO controlled plant. In this case is considered the turbine with two controlled withdrawals which drives electric generator supplying determined part of electric network (it means the turbine operates without phasing into power network). Here, the turbine represent three-variable control loop. The scheme of three-variable control loop of steam turbine is presented on the Figure 3.

Denominations on the figure mean: Δy_{VT}, Δy_{ST}, Δy_{NT} - change of opening position of control valves of high-pressure, medium-pressure and low-pressure part of turbine, $\Delta m'_{01}$, $\Delta m'_{02}$ - change of mass flow of withdrawn steam, Δp_{01}, Δp_{02} - change of steam pressure in corresponding withdrawals, $\Delta\omega$ - change of angular speed of turbo-generator, ΔM_G - change of electric load of turbo-generator.

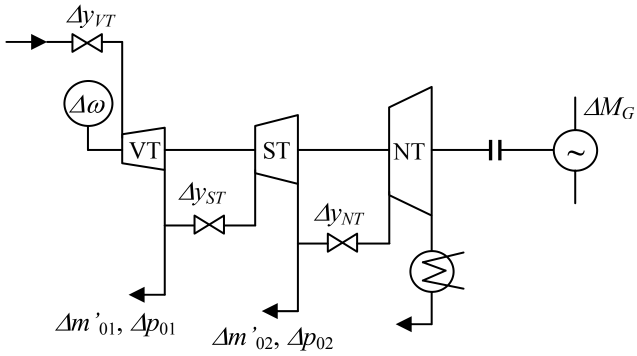

Figure 3. Three-variable control loop of steam

Controlled variables are $\Delta\omega$, Δp_{01}, Δp_{02}, disturbance variables ΔM_G, $\Delta m'_{01}$, $\Delta m'_{02}$ and manipulated variables are Δy_{VT}, Δy_{ST}, Δy_{NT}

3.2. Mathematical model of the MIMO control loop

Resulting differential equations for creating mathematical model of the plant were gained already after deriving and using linearization from the project OTROKOVICE elaborated by the firm ALSTOM Power [1]. Differential equations were re-write into other form by introducing relative values (with regard to starting stable state-operational, i.e. to calculated point) at which relation of values can be generally written in the form $\varphi_X = \Delta X / (X)_0$. Then, the Laplace transform of an output (controlled) variable was given by the following relation

$$\boldsymbol{Y}(s)=\boldsymbol{G}_S(s)\boldsymbol{U}(s)+\boldsymbol{G}_{SV}(s)\boldsymbol{V}(s)\Rightarrow\begin{bmatrix}\varphi_\omega(s)\\ \varphi_{p_{01}}(s)\\ \varphi_{p_{02}}(s)\end{bmatrix}=\boldsymbol{G}_S(s)\begin{bmatrix}\varphi_{y_{VT}}(s)\\ \varphi_{y_{ST}}(s)\\ \varphi_{y_{NT}}(s)\end{bmatrix}+\boldsymbol{G}_{SV}(s)\begin{bmatrix}\varphi_{M_G}(s)\\ \varphi_{m'_{01}}(s)\\ \varphi_{m'_{02}}(s)\end{bmatrix} \tag{9}$$

where

$$\boldsymbol{G}_S(s)=\begin{bmatrix}\dfrac{0.73s^2+1.59s+1.106}{12.32s^3+21.78s^2+10.67s+1} & \dfrac{0.455s^2+0.740s+0.087}{12.32s^3+21.78s^2+10.67s+1} & \dfrac{0.321s^2+0.327s+0.037}{12.32s^3+21.78s^2+10.67s+1}\\ \dfrac{1.681s+1.314}{1.505s^2+2.475s+1} & \dfrac{-1.246s-0.959}{1.505s^2+2.475s+1} & \dfrac{-0.011}{1.505s^2+2.475s+1}\\ \dfrac{1.764}{1.505s^2+2.475s+1} & \dfrac{1.561s+0.039}{1.505s^2+2.475s+1} & \dfrac{-1.118s-0.966}{1.505s^2+2.475s+1}\end{bmatrix} \tag{10}$$

$$\boldsymbol{G}_{SV}(s)=\begin{bmatrix}\dfrac{-1.505s^2-2.477s-1}{12.32s^3+21.78s^2+10.67s+1} & \dfrac{-0.092s-0.148}{12.32s^3+21.78s^2+10.67s+1} & \dfrac{-0.090s-0.079}{12.32s^3+21.78s^2+10.67s+1}\\ 0 & \dfrac{-0.400s-0313}{1.505s^2+2.475s+1} & \dfrac{-0,005}{1.505s^2+2.475s+1}\\ 0 & \dfrac{-0.420}{1.505s^2+2.475s+1} & \dfrac{-0.501s-0.432}{1.505s^2+2.475s+1}\end{bmatrix} \tag{11}$$

3.3. Synthesis of three-variable control loop of a steam turbine

The principal described in the paragraph 2.2 is used at solution of synthesis of the three-variable control loop. First transfers of main controllers R_{11}, R_{22}, R_{33} are determined for transfer functions $S_{11,x}$, $S_{22,x}$ a $S_{33,x}$ (equation (8)) then autonomy of control loop by using relation (5) is being solved and in the end fulfilment of the condition of invariance (approximate invariance) of control loop is ensured by using equation (7). At design of parameters of main controllers the following methods were used:

Ziegler Nichols step response method [2], method of desired model (method of dynamics inversion) [5], polynomial method of synthesis - 1DOF (1 degree of freedom) configuration [4]. In the next part of this paper one chosen method for design of parameters of main controllers, i.e. a method of desired model, is used. Transfer matrix of controllers $\boldsymbol{G}_R(s)$ with utilization of method of desired model and transfer matrix of correction members $\boldsymbol{G}_{KC}(s)$ are given by the equation (11).

$$\boldsymbol{G}_R(s)=\begin{bmatrix} \frac{0.904s+0.110}{s} & \frac{0.080s+0.010}{s} & \frac{0.032s^2+0.038s+0.0044}{s(s+1.057)} \\ \frac{1.22s+0.149}{s} & \frac{-0.065s-0.135}{s} & \frac{0.044s^2+0.053s+0.0077}{s(s+1.057)} \\ \frac{1.702s+0.208}{s} & \frac{-0.090s+0.013}{s} & \frac{-0.135s-0.140}{s} \end{bmatrix} \quad \boldsymbol{G}_{KC}(s)=\begin{bmatrix} \frac{-2.063s^2-3.394s-1.371}{s^2+2.178s+1.516} & 0 & 0 \\ 0 & \frac{0.321s+0.251}{s+0.769} & 0 \\ 0 & 0 & 0.448 \end{bmatrix} \qquad (12)$$

Simulation results

Simulation of three-variable control loop of a steam turbine with utilization of one chosen SISO synthesis method is presented on the following figure (see Figure 4) [3].

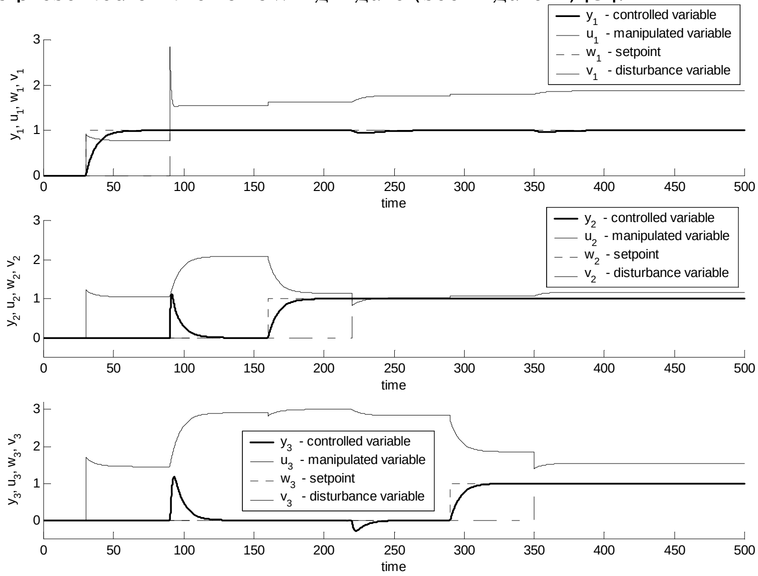

Figure 4. Simulation of control loop with utilization of method of desired model

Variables on figure corresponds to definite variables existing in the three-variable control loop of steam turbine, i.e. $y_1 \to \varphi_\omega$, $y_2 \to \varphi_{P01}$, $y_3 \to \varphi_{P02}$, $v_1 \to \varphi_{MG}$, $v_2 \to \varphi_{m'01}$, $v_3 \to \varphi_{m'02}$, $u_1 \to \varphi_{yVT}$, $u_2 \to \varphi_{yST}$, $u_3 \to \varphi_{yNT}$

3.4. Evaluation of simulation experiments

It is obvious from the simulation of control process presented above (see) and from other simulation experiments that the condition of autonomy was fulfilled. From the simulation process control is also obvious that the control loop is invariant or let us say approximately invariant, i.e. that influence of disturbance variables is eliminated only at steady state.

4. CONCLUSION

It was described possible procedure to control of MIMO control loop. Simulation verification of this method of control is presented on three-variable control loop of steam turbine. At first our method deals with setting-up of main (diagonal) controllers, then determination of binding members for ensuring autonomy and in the end calculation of correction members for ensuring invariance.

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Acknowledgments: This work was supported in part by the Grant agency of Czech Republic under grant No: 101/06/0920 and in part by the Ministry of Education of the Czech Republic under grant No. MSM 7088352102.