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NEW MATHEMATICAL MODEL OF STRESS CONCENTRATION FACTOR IN TENSION OF RECTANGULAR BAR WITH OPPOSITE EDGE U-NOTCHES

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ABSTRACT
This paper describes a new mathematical model for stress concentration factor of rectangular bar with opposite edge U-notches subjected to tension. Determination of model parameters is achieved by combining the design-of-experiment technique implemented in MATLAB software and the numerical simulation using FEMAP software. The design-of-experiment technique is used to obtain a variety of combinations of geometrical parameters, ie. experimental points, whereas the numerical simulation is applied to thus obtained experimental points. Results agree well with the data given in literature. This procedure for determination of stress concentration factors could be applicable for other stress concentration cases.
Keywords: stress concentration, finite element analysis, design of experiment, tension, U-notch

1. INTRODUCTION
Development of mathematical and finite element analysis software together with improvements in computer technology are providing a possibility for improving and simplifying existing mathematical models of stress concentration factors. In this paper this possibility is shown on one stress concentration example (rectangular bar with opposite edge u-notches in tension). For this purpose FEMAP and MATLAB software are implemented. MATLAB is used for design of experiment and mathematical modeling while FEMAP is used for determination of stress concentration factors for variety of different geometries of investigated stress concentration case.

2. PROBLEM
The stress concentration factor K can be defined as the ratio of the peak stress in the body (or stress in the perturbed region) to some other stress (or stresslike quantity) taken as a reference stress:

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \qquad (1)$$

The subscript t indicates that the stress concentration factor is a theoretical factor.
In this paper the case of a stress concentration in the rectangular bar with opposite edge U-notches subjected to tension is investigated (figure 1).

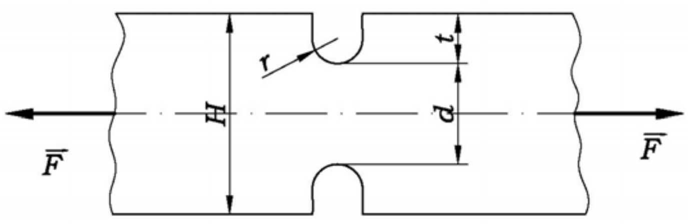


Figure 1. Rectangular bar with opposite edge U-notches in tension.

Nominal stress is:

$$\sigma_{nom} = \frac{F}{d \cdot \delta}, \ \delta - \text{plate thickness} \qquad (2)$$

The models given in the literature [1,2] define the stress concentration as function of the geometrical factors $\sqrt{\frac{t}{r}}$ and $\frac{t}{H}$.

$$K = C_1 + C_2 \cdot \left(\frac{2t}{H}\right) + C_3 \cdot \left(\frac{2t}{H}\right)^2 + C_4 \cdot \left(\frac{2t}{H}\right)^3 \qquad (3)$$

where C_1, C_2, C_3, C_4:

$$\begin{aligned} C_1 &= a_1 + a_2 \cdot \sqrt{\frac{t}{r}} + a_3 \cdot \frac{t}{r} \\ C_2 &= b_1 + b_2 \cdot \sqrt{\frac{t}{r}} + b_3 \cdot \frac{t}{r} \\ C_3 &= c_1 + c_2 \cdot \sqrt{\frac{t}{r}} + c_3 \cdot \frac{t}{r} \\ C_4 &= d_1 + d_2 \cdot \sqrt{\frac{t}{r}} + d_3 \cdot \frac{t}{r} \end{aligned} \qquad (4)$$

The constants a_i, b_i, c_i, d_i are different in the existing models in the literature [1,2], but in the both cases two models exist: the first one for the range $0{,}1 < t/r < 2$ and the second one for the range $0{,}1 < t/r < 2$. Comparison of the existing models shows large incoherence between the models and also between the models and the appropriate experimentally obtained chart (Chart 2.4, p. 84) given in the literature [1].

3. DESIGN OF EXPERIMENT

For the design of experiment the space filling design technique is implemented. Thirty points which cover well the investigated area were obtained by MATLAB software (Model based calibration toolbox). The investigated area is set in the range for $1 \le \frac{t}{r} \le 10$ and $0{,}1 < \frac{t}{H} < 0{,}25$. The design matrix is shown in the figure 2.

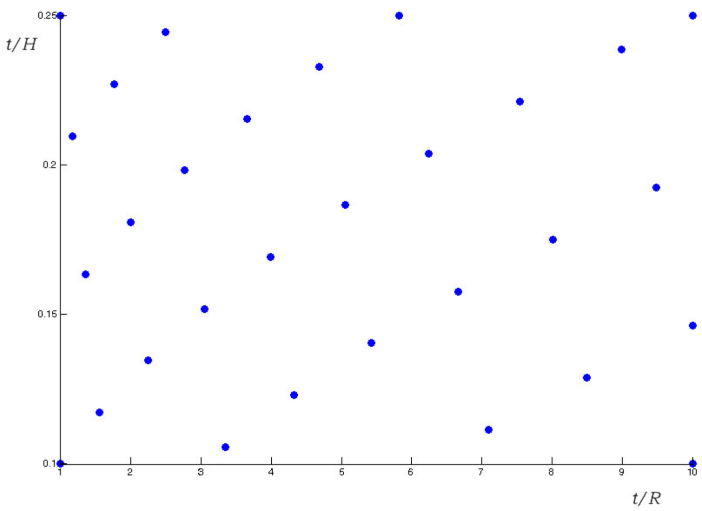
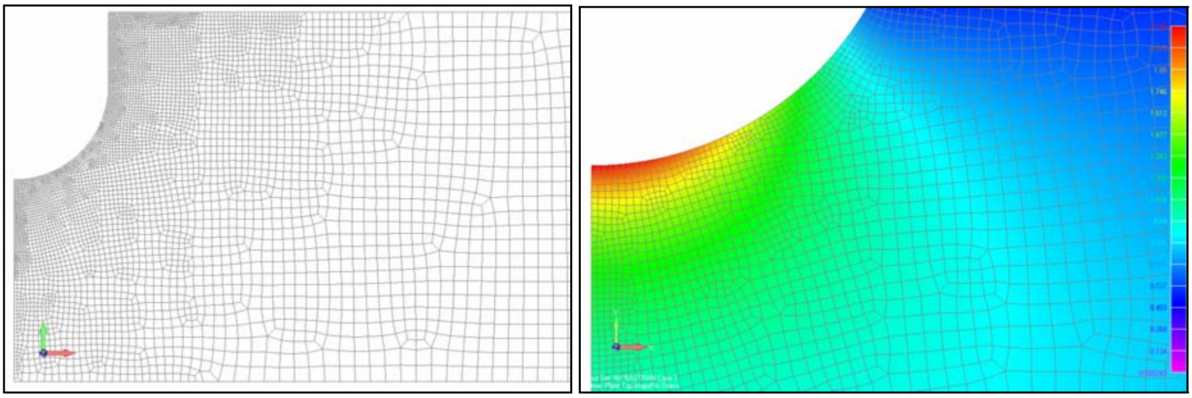


Figure 2. Design matrix.

4. EXPERIMENT - FINITE ELEMENT ANALYSIS

Finite element analysis is applied to the combinations of geometrical parameters, ie. the experimental points obtain by the design of experiment. Numerical simulation is done with FEMAP software. The investigated stress concentration case is analyzed as a 2D problem with two symmetry planes so only a quarter of the plate is modeled. During the models meshing, effort was made to obtain a fine mesh in the locations of stress concentration in order to get valid results. For that purpose four different element size were used, the smallest one at the location of the stress concentration, with the gradual mesh size increase to the end of a model. Quad shape elements are implemented. Example of a mesh and a FEM analysis result for one experimental point is shown in the figure 3.



a) mesh *b) stress concentration location*

Figure 3. Finite element analysis.

5. MATHEMATICAL MODEL

Based on the results obtained from the FEM analysis, related to the combinations of geometrical factors $\left(\sqrt{\frac{t}{r}}, \frac{t}{H}\right)$, a 4th order linear polynomial response model is computerized by MATLAB software.

$$K_t = 0{,}756889 + 2{,}92489\,x - 2{,}79324\,y + 0{,}0558271\,x^2 - 10{,}4138\,x\,y + 6{,}65308\,y^2 - 0{,}467562\,x^2y + 13{,}3184\,xy^2 + 1{,}1454\,x^2y^2 \quad \text{......... (5)}$$

where:

$$x = \sqrt{\frac{t}{r}},\ y = \frac{t}{h} \quad \text{........................ (6)}$$

The obtained model shows a excellent fit with the experimental data, with PRESS RMSE[1] – 0,03307 and RMSE[2] – 0,02127.

The expression (4) can be transformed into the form (3,4) used in the literature [1,2]:

$$K = C_1 + C_2 \cdot \left(\frac{2t}{H}\right) + C_3 \cdot \left(\frac{2t}{H}\right)^2 \quad \text{........................ (7)}$$

where C1, C2, C3:

$$C_1 = 0{,}756889 + 2{,}92489 \cdot \sqrt{\frac{t}{r}} + 0{,}0558271 \cdot \frac{t}{r}$$

$$C_2 = -1{,}39662 - 5{,}2069 \cdot \sqrt{\frac{t}{r}} - 0{,}233781 \cdot \frac{t}{r} \quad \text{........................ (8)}$$

$$C_3 = 1{,}66327 + 3{,}3296 \cdot \sqrt{\frac{t}{r}} + 0{,}28635 \cdot \frac{t}{r}$$

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1 Root mean squared error of predicted errors

2 Root mean squared error

5. MODEL EVALUATION

The obtained model (5) is very smooth and without any picks, what is required. The 3D model (surface) and the 2D chart for different values of the factor t/h in the investigated range are shown in the figure 4.

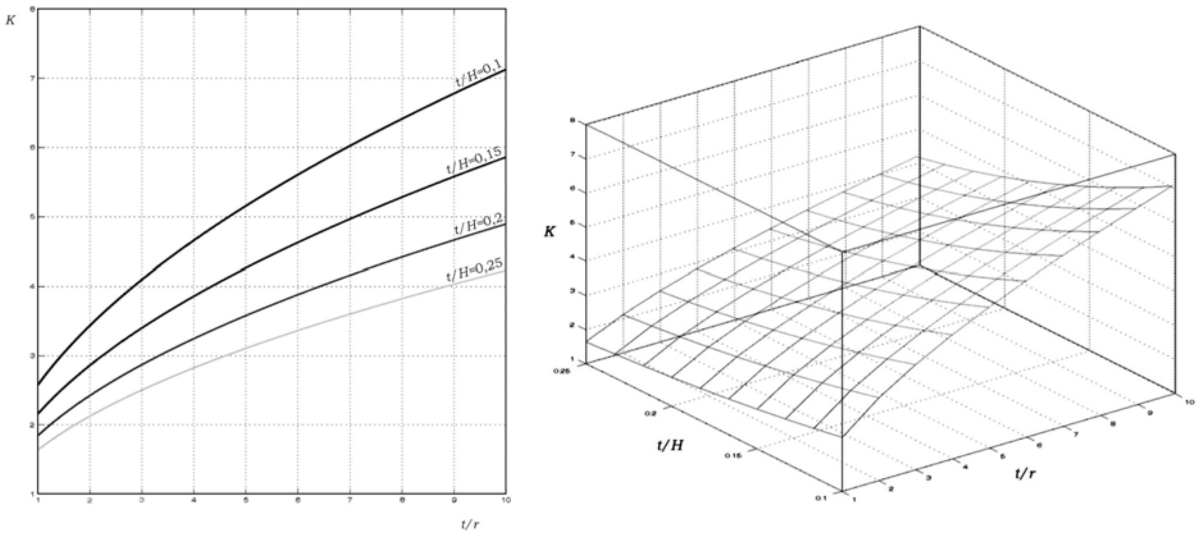

Figure 4. Stress concentration factor model (2D and 3D)

To compare the model (5) with chart given in [1] which is in dependence of the geometrical factors H/d and r/d four curves for H/d ={1,2; 1,3; 1,5; 2} in the range 0,05<r/d<0,3 are derived. Although the curves even cover the range wider than the investigated, the curves show good agreement with the literature. The chart is cut at the value of three of the stress concentration factor as it is done in [1].

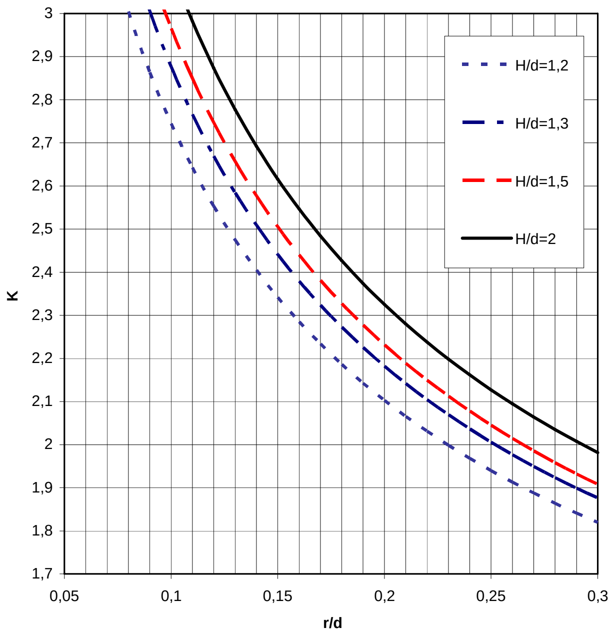

Figure 5. Stress concentration factor model (dependence on r/d and H/d)

6. CONCLUSIONS

The obtained model shows a good agreement with the data for stress concentration factors in the literature. The procedure and the software implemented in this paper could be used for other cases of stress concentration in order to simplify and improve existing models. Also, it could be possible to make a model for a wider range $\left(0{,}1 \le \frac{t}{r} \le 50\right)$ than it is done in this paper.

7. REFERENCES

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