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GENERAL DYNAMIC MODEL OF THE TURBOGENERATOR RELATED TO THE TYPE OF SUPPORT

Dr. Sc. Shaban A. Buza, Dr.Sc. Xhevat Perjuci,
Dr. Sc. Kastriot A. Buza, Dr. Sc. Fevzi Radoniqi
Faculty of Mechanical Engineering
Prishtina, Kosova

Dr. Sc. Nysret Avdiu
Faculty of Electrical and Computing Engineering
Prishtina, Kosova

ABSTRACT
Turbogenerator as a set of turbines and electric generator presents a very complicated mechanical system. Its design must take into consideration the rotordynamics of rotor, which undergoes bending and torsional oscillation.
The oscillation of the rotor depends upon its geometry and type of support.
In this paper the dynamic models for the turbogenerator with three turbines and an electric generator supported in different types of bearings has been analysed.
The mathematic model was extended with electromagnetic moment and the resistant moment of the working machine.
The adopted models – mechanical and its mathematical - are a good base for dynamic analysis at the different types of turbogenerators.
Key words: Turbogenerator, Turbine, Electric Generator, Bending and Torsional Oscillation

1. INTRODUCTION

The engineering components concerned with the subject of rotordynamics are the rotors of machines, especially turbines and generators known as turbogenerators. Turbogenerators as complicated machines that are used to develop power, consist of several turbines (gas and/or steam) coupled to the electric generator. During the rotating machines design analysis of the torsional oscillations are of vital importance, but in this paper a general model was built including also bending oscillations.
Adopted model includes oscillations of the elastic shaft depending upon its geometry, number of disks assembled on it, the type of support (bearings and foundation) and as well as on excitation forces.
Gyroscopic effect as the inertia-related moment in an angular lateral motion of a rotor which tends to maintain parallel the skewed axis of vibration and rotational axis, influences on the speed, sometimes doubling its number.
For analysis of the general dynamic model has been built based on a model with a single non-central disk with necessary approximations for two types of support – rigid and elastic bearings and as well as for rigid and elastic foundation. Based on adopted dynamic model the mathematical model was built.
The model includes the dynamic analysis of the transient process involving mechanical, thermal and electromagnetic processes. Such processes occur at rotating machines with instantaneous changes in machine conditions as speed, load, usually during start-up or shutdown.

2. BUILDING DYNAMIC AND MATHEMATICAL MODEL

For dynamic analysis, the turbogenerator is expressed through a model representing an elastic shaft with several disks supported i several bearings and foundation. The general dynamic model of elastic shaft with *n-disks* supported in *(n+1)* bearings-foundation system is given in *Figure 1.* Its mathematical model is made based in simpler model of the elastic shaft (i-th disk supported in two bearings) with non-central disk enabling analysis of the oscillations under torsion and bending at the same time, as well as elastic line deflection and rotation as a result of disk des-equilibration known as precession and nutation that characterizes the gyroscopic effect.

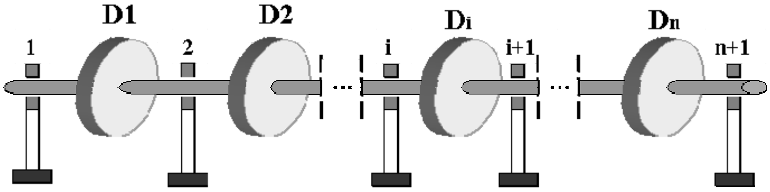
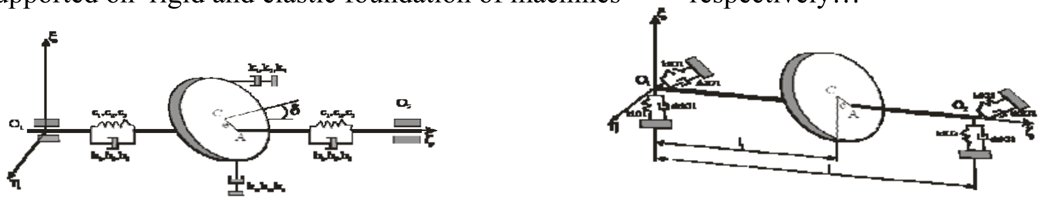


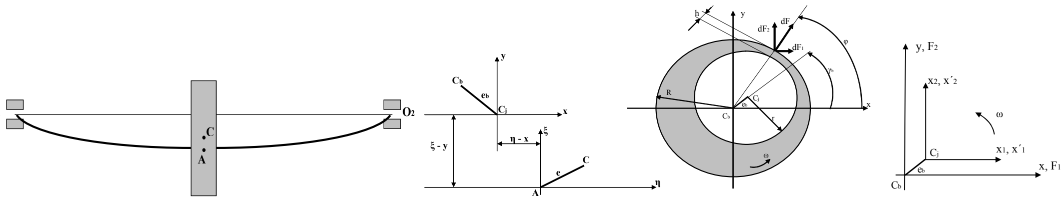
Figure 1. Dynamic model of an elastic shaft with n-disks supported in (n+1) bearings-foundations system

For the dynamic model (*Figure 2 and Figure 3*) the following approximations has been adopted: non-mass shaft is elastic and disk is rigidly assembled; disk is statically and dynamically des-equilibrated; elastic shaft behaves as spring element; bearings are rigid at model in *Figure 2a* and flexible at model in *Figure 2b*; the position of the disk in relation to bearing axis is shown in *Figure 3a*, while lubricant hydrodynamics of bearings with journal position is shown in *Figure 3b*; the elastic system is supported on rigid and elastic foundation of machines respectively…



a) *b)*

Figure 2. Dynamic model of an elastic shaft with one disk supported on rigid (a) and elastic (b) bearings



a) *b)*

Figure 3.Elastic shaft with one disk supported on bearings with lubricant

Initially, the mathematical model for *i-th element* of the dynamic model *(Figure 1)* will be analysed. The bending and torsion matrix of the des-equilibrated disk and bearing-foundation system has form:

$$\overline{M}_i \cdot \ddot{q}_{d,i} = \begin{bmatrix} m_i & 0 & 0 & 0 & 0 \\ 0 & m_i & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & A_i & 0 \\ 0 & 0 & 0 & 0 & J_i \end{bmatrix} \cdot \begin{bmatrix} \ddot{\xi}_i \\ \ddot{\eta}_i \\ \ddot{\alpha}_i^* \\ \ddot{\beta}_i^* \\ \ddot{\varphi}_i \end{bmatrix}, \quad \overline{M}_{b,j} \cdot \ddot{q}_{b,j} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_{b,j} & 0 \\ 0 & 0 & 0 & m_{b,j} \end{bmatrix} \cdot \begin{bmatrix} \ddot{x}_j \\ \ddot{y}_j \\ \ddot{x}_{b,j} \\ \ddot{y}_{b,j} \end{bmatrix} \tag{1}$$

The matrix relation that characterizes gyroscopic effect is given by:

$$G_i \cdot \dot{q}_{d,i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_i \cdot \dot{\varphi}_i & 0 \\ 0 & 0 & -J_i \cdot \dot{\varphi}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\xi}_i \\ \dot{\eta}_i \\ \dot{\alpha}_i^* \\ \dot{\beta}_i^* \\ \dot{\varphi}_i \end{bmatrix} \quad G_i' \cdot q_{d,i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_i \cdot \ddot{\varphi}_i & 0 \\ 0 & 0 & -J_i \cdot \ddot{\varphi}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \xi_i \\ \eta_i \\ \alpha_i^* \\ \beta_i^* \\ \varphi_i \end{bmatrix} \tag{2}$$

Where *i=1,2,…,n* is number of disks and *j=1,2,…,nb* is number of bearings with foundations.

The matrix for influence coefficients of the rotating angle of one disk to another related to bending and torsion respectively, is given in a form:

$$\sum_{l=1}^{L}\left(\begin{bmatrix} b_{(i-1)i,l} & -b_{i(i+1),l}\end{bmatrix}\cdot\begin{bmatrix}(\dot{\varphi}_i-\dot{\varphi}_{i-1})^l\\(\dot{\varphi}_{i+1}-\dot{\varphi}_i)^l\end{bmatrix}\right), i=2,3,...,n;\ \sum_{l=1}^{L}\left(\begin{bmatrix} c_{(i-1)i,l} & -c_{i(i+1),l}\end{bmatrix}\cdot\begin{bmatrix}(\varphi_i-\varphi_{i-1})^l\\(\varphi_{i+1}-\varphi_i)^l\end{bmatrix}\right), i=2,3,...,n \quad (3)$$

The matrix expressing stiffness coefficients for bearing-foundation system and lubricant coefficients for bearing:

$$P_{p,j}\cdot q_{p,j}=\begin{bmatrix}p_{11,j} & 0\\ 0 & p_{22,j}\end{bmatrix}\cdot\begin{bmatrix}y_j-\xi\\ x_j-\eta\end{bmatrix}; P_{b,j}\cdot q_{b,j}=\begin{bmatrix}k_{b,j} & 0\\ 0 & k_{b,j}\end{bmatrix}\cdot\begin{bmatrix}x_{b,j}\\ y_{b,j}\end{bmatrix}; P_{l,j}\cdot q_{l,j}=\begin{bmatrix}p_{l12,j} & p_{l11,j}\\ p_{l22,j} & p_{l21,j}\end{bmatrix}\cdot\begin{bmatrix}x_j-x_{b,j}\\ y_j-y_{b,j}\end{bmatrix} \quad (4)$$

The damping coefficients of lubricant and for bearing-foundation system are shown in the matrix:

$$C_{b,j}\cdot\dot{q}_{b,j}=\begin{bmatrix}c_{b,j} & 0\\ 0 & c_{b,j}\end{bmatrix}\cdot\begin{bmatrix}\dot{x}_{b,j}\\ \dot{y}_{b,j}\end{bmatrix}; C_{l,j}\cdot\dot{q}_{l,j}=\begin{bmatrix}c_{l12,j} & c_{l11,j}\\ c_{l22,j} & c_{l21,j}\end{bmatrix}\cdot\begin{bmatrix}\dot{x}_j-\dot{x}_{b,j}\\ \dot{y}_j-\dot{y}_{b,j}\end{bmatrix} \quad (5)$$

After building matrixes related to the coordinates and its derivates, the excitation matrix for the dynamic model has form:

$$Q_i=\begin{bmatrix} m_i e_i(\ddot{\varphi}_i\sin\varphi_i+\dot{\varphi}_i^2\cos\varphi_i)\\ m_i e_i(-\ddot{\varphi}_i\cos\varphi_i+\dot{\varphi}_i^2\sin\varphi_i)\\ \delta_i(A_i-J_i)[\ddot{\varphi}_i\cos(\varphi_i-\varepsilon_i)-\dot{\varphi}_i^2\sin(\varphi_i-\varepsilon\ )]\\ \delta_i(A_i-J_i)[\ddot{\varphi}_i\sin(\varphi_i-\varepsilon_i)-\dot{\varphi}_i^2\cos(\varphi_i-\varepsilon\ )]\\ M_i\end{bmatrix} \quad (6)$$

Where for respective *i, j or l* element: ***m*** is disk masses; ***A*** is disk's equatorial moment of inertia; ***J*** is disk polar moment of inertia; ***e*** is linear eccentricity (static des-equilibration); ***δ*** is angular eccentricity (dynamic des-equilibration); ***α**** is precession angle; ***β**** is nutation angle; ***φ*** is rotational angle; ***c, b, k, p*** are stiffness, damping coefficients of shaft, bearings or foundation respectively.
Therefore, the general mathematical model is built with extension of the matrixes given above:

$$[M]\cdot[\ddot{q}]+([G]+[K])\cdot[\dot{q}]+[P]\cdot[q]=[Q] \quad (7)$$

Where: **[M]** is matrix for masses of the disks, masses of the bearings, polar and equatorial moments of inertia; **[G]** is matrix representing gyroscopic effect; **[K]** is matrix that presents damping coefficients for disks, shaft, lubricant and foundation; **[P]** is matrix for stiffness coefficients; **{q}** is vector for generalized coordinates with its first and second derivate; **{Q}** is vector of generalized load-external excitations.

3. GENERAL MODEL OF THE TURBOGENERATOR

Based on the dynamic model in *Figure 1* can be adopted the turbogenerator consisting three turbines coupled with an electric generator supported in five bearing-foundation systems, *Figure 4.*

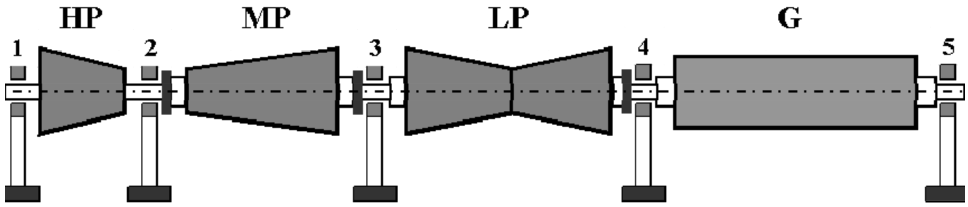
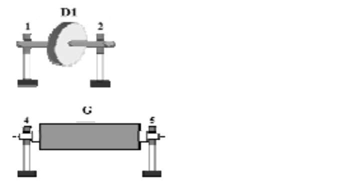
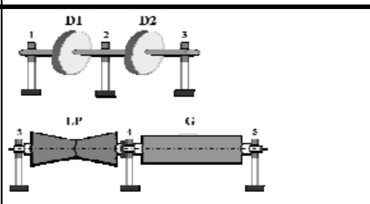
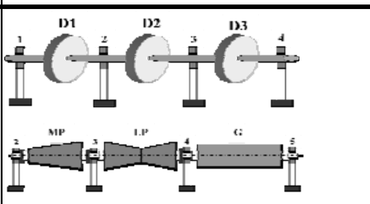
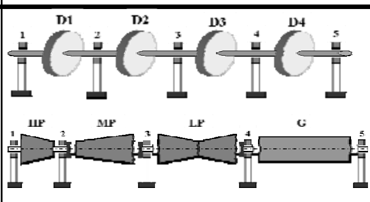


Figure 4. Adopted dynamic model for Turbogenerator
(HP,MP,LP-high, mid, low pressure turbine; G-electric generator; 1,2,3,4,5-bearing-foundation system)

Depending on number of turbines, number of bearings and the type of support the degree of freedom for the dynamic model is determined. DoF determines the number of differential equations of the system (7):

DoF = nq = (n x 2) + (n x 2) + (n x 1) + (nb x 2) + (nb x 2) (8)

Table 1. Types of dynamic models for turbogenerator

	Dynamic Model	#	RB	EB	BwLsRF	BwLsEF	DoF
Elastic shaft with one disk supported in two bearings		1	×				5
		2		×			5
		3			×		9
		4				×	13
Elastic shaft with two disks supported in three bearings		5	×				10
		6		×			10
		7			×		16
		8				×	22
Elastic shaft with three disks supported in four bearings		9	×				15
		10		×			15
		11			×		23
		12				×	31
Elastic shaft with four disks supported in five bearings		13	×				20
		14		×			20
		15			×		30
		16				×	40

RB - *Rigid Bearing*
EB - *Elatic Bearing*
BwLsRF - *Bearing with Lubricant supported in Rigid Foundation*
BwLsEF - *Bearing with Lubricant supported in Elastic Foundation*
DoF - *Degree of Freedom*

4. CONCLUSIONS

Referring to the *Table 1* it can be noticed that:

- The dynamic model of the turbogenerator is very complicated for design representing a system with big number of the DoF, while its mathematical model can solved with Matlab using Runge-Kutta procedure with ODE's;
- DoF highly increases when system is supported in bearings with lubricant in elastic foundation.

Therefore, the generalized types of the adopted dynamic model shown in *Table 1* with respective mathematical model present a good base for design of different types of turbogenerator in easy and quick way.

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