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THE INFLUENCE OF THE DIFFERENT GEOMETRICAL FACTORS ON THE WORKING STRESSES OF TEETH SIDES OF GEARS

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ABSTRACT
In this paper is analyzed the influence of the different geometrical factors on the working stresses of teeth sides of cylindrical involute gears with external and internal meshing, and at the same time is done the comparison of the difference of this stresses of gears with external meshing toward stresses of gears with internal meshing. Also was analyzed contact (Hertzian) stresses for all the contact interval of meshing and mathematical model was creating for suiting of those changes. Achieved results was presented graphically and they was a good form to facilitate the suite of this changes for further researchers.
Keywords: Contact (Hertzian) stresses, external meshing, internal meshing.

1. INTRODUCTION
The contacting stresses are known as important subject on damage of teeth sides of the gears. This problem for a certain number of contacting surfaces was solved by H. Hertz, whose expression is used to evaluate influence of different factors in conveyance of teeth sides of cylindrical involute gears with external and internal meshing.
Damage of teeth sides of the gears is as a result of gears' material fatigue concerning the cyclic change of the working load. Such type of the damage is known as material abrasion (erode) or Pitting. The general expression for the working stresses is:

$$\sigma_H = \sqrt{\frac{F_n \cdot E}{b \cdot \rho} \frac{1}{2\pi\left(1-\nu^2\right)}} \qquad \text{... (1)}$$

Expression (1) presents the contact of two cylindrical bodies in general in which cannot be noticed needed elements for following the variation of the stresses at the meshing interval of teeth sides of the gears, neither for gears with external meshing nor for gears with internal meshing. Therefore, the mathematic model for calculating the normal stresses at the teeth sides along entire contact line for external and internal meshing is elaborated further.
If the teeth sides of gears with external meshing (figure 1.a) and those with internal meshing (figure 1.b) have a moment contact point *Y*, the curve radius of the teeth sides is ρ_{y1} for the pinion and ρ_{y2} for the gear and can be calculated by:

$$\rho_{y1} = r_{b1} \tan\alpha_{y1} \quad \text{and} \quad \rho_{y2} = r_{b2} \tan\alpha_{y2} \qquad \text{... (2)}$$

The reduced radius of curvature at the tooth side of the gear with external meshing and of the one with internal meshing at moment contact point *Y* of the meshing is:

$$\rho = \frac{\rho_{y1} \cdot \rho_{y2}}{\rho_{y2} \pm \rho_{y1}} \qquad \text{... (3)}$$

After needed transformations and substitutions in (1) the normal stresses at teeth side for the instant contact point *Y* are expressed by:

$$\sigma_{HY} = \sqrt{\frac{F_t \cdot E}{b \cdot d_1} \cdot \frac{u \pm 1}{u}} \cdot Z_E \cdot Z_{HY} \qquad \text{... (4)}$$

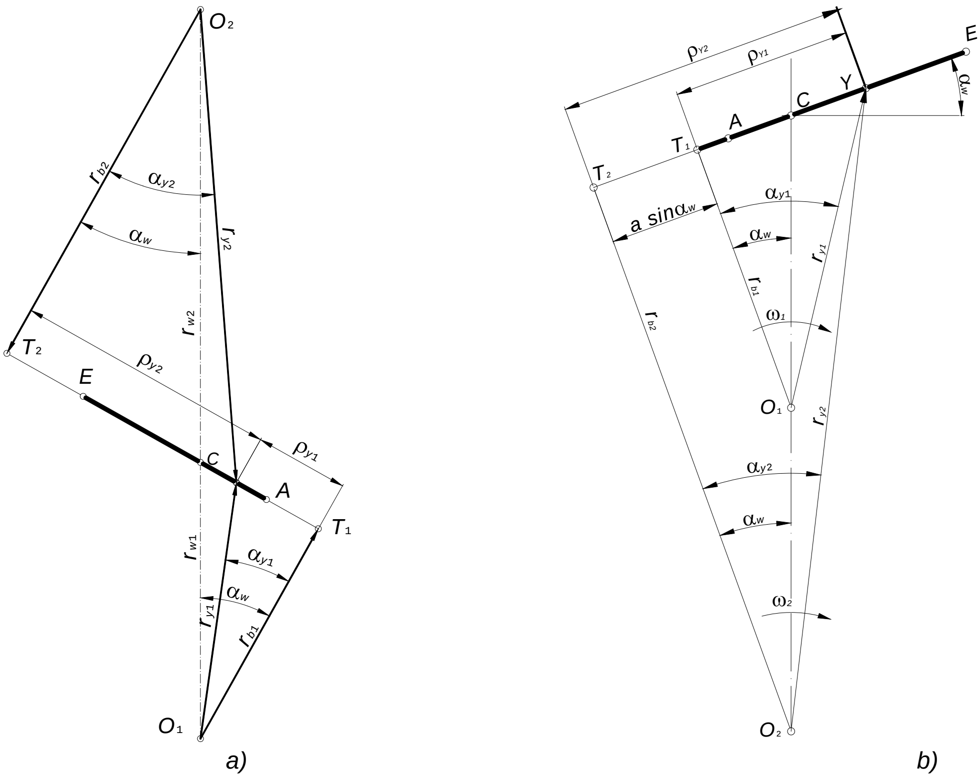

Figure 1. Radii of curvature of the teeth sides of the gears with cylindrical involute gears with: a) external meshing and b) internal meshing

2. ANALYSIS OF THE WORKING STRESSES AT TEETH SIDES OF THE GEARS

In figure 2.a) and figure 2.b) are shown the variation of radii of curvature at teeth sides ρ_{y1} and ρ_{y2}, according to expression (2) and reduced radius of curvature ρ from expression (3) for gear set with external and internal meshing.

Reduced radius of curvature ρ along the line of action has parabolic form with zero values at the beginning and at the end of the meshing interval (at points T_1 and T_2, ρ=0). Maximal value of reduced radius of curvature ρ is achieved for Pitch point (kinematics' pole) C, (for θ=20°, figure 2.a), and that only for gear ratio u=1. From expression (1) and figure 2.a) can be concluded that the beginning and the end of the interval is not allowed to be used for meshing of the gears teeth, because these parts are with fatal effect concerning the working stresses at teeth sides of the gears, especially when ρ=0 than working stresses at teeth sides of the gears σ_H have infinite high value ($\sigma_H=\infty$).

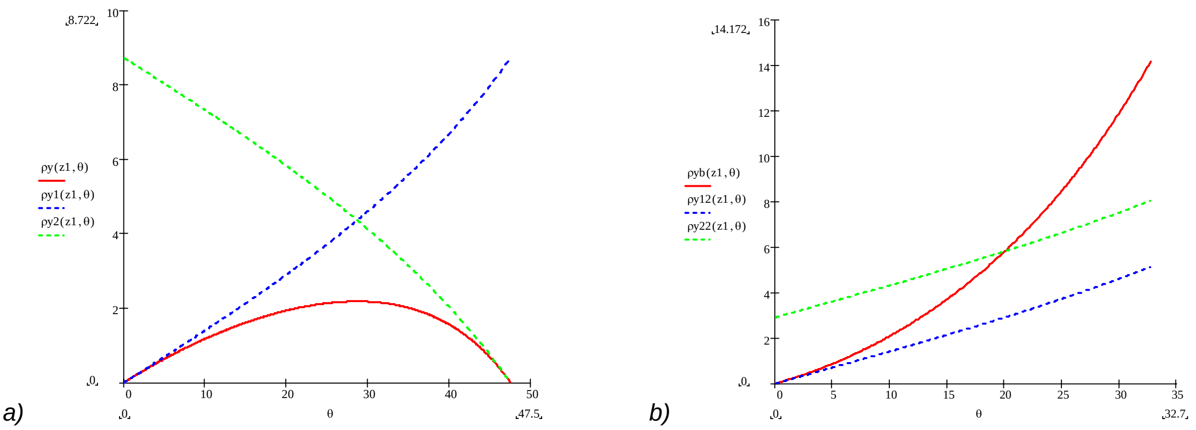

Figure 2. Variation of radii of curvature at teeth sides of the gears: a) for external, and b) for internal meshing.

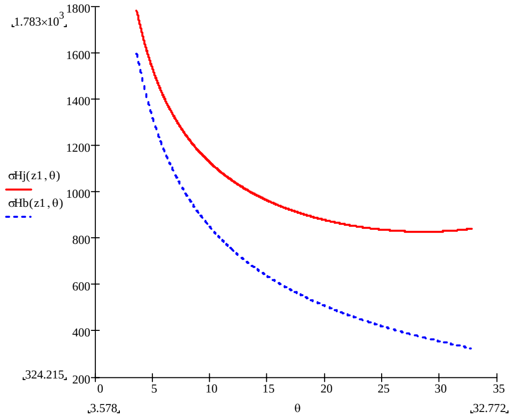

Figure3. Variation of working stresses at teeth sides along active line of action $\overline{AE}$.

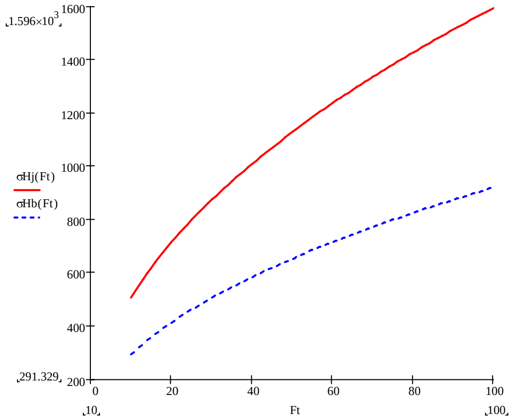

Figure 4. Influence of external load $\boldsymbol{F_t}$ *at working stresses at teeth sides of the gears.*

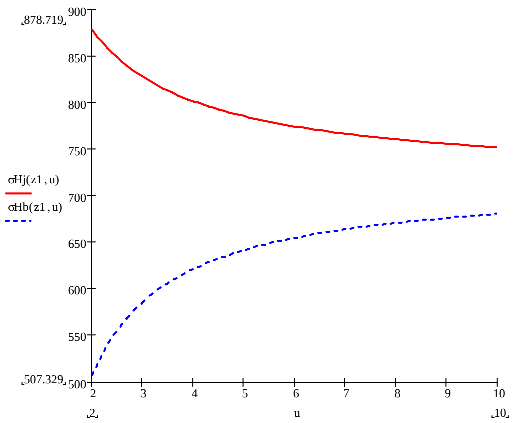

Figure 5. Influence of gear ratio ***u*** *at working stresses at teeth sides of the gears.*

For the gearset with internal meshing, reduced radius of curvature at teeth sides ρ_y, at the beginning of the meshing (point T_1) has value zero, while along contact line has increasing tendency. Therefore, comparing to the gearset of cylindrical involute gears with external meshing, these pairs have advantage concerning working stresses at teeth sides. Critical point is at the beginning of meshing (at point T_1) in which is $\rho=0$ ($\sigma_H=\infty$).
In figure 3. is presented variability of working stresses at teeth sides of the gears with external and internal meshing along the active length of line of action $\overline{AE}$.
From figure 3. can be notified that working stresses at teeth sides of the gears decreases from entering to the exit of meshing. So, concerning the loading of the teeth sides, the entering points of meshing are more in danger (liable) than those at the exit of the meshing.
In figure 4. is represented influence of the external load F_t at normal stresses of teeth sides with external and internal meshing.
From figure 4. is clear that for the similar parameters of the gears, with external load increase the working stresses at teeth sides of the gears with external meshing increases much faster than the working stresses at teeth sides of the gears with internal meshing.
In figure 5. the influence of gear ratio u to the working stresses at teeth sides of the gears with external and internal meshing is presented.
At figure 5. could be seen that gear ratio influences in different ways at the working stresses at teeth sides of the gears with external and internal meshing. For the teeth of the gears with external meshing the increase of gear ratio has positive impact, because it decreases the working stresses, while for the teeth of the gears with internal meshing, increase of the gear ratio has negative influence, increasing the working stresses at teeth sides of the gears.

3. FACTORS THAT INFLUENCE ON TRANSMISSION OF THE GEARS TEETH

Influence of different factors in the working stresses at teeth sides of the gears with external and internal meshing is included in the following:

$$Z = \sqrt{K_v\, K_{H\alpha}\, K_{H\beta}}\, Z_\varepsilon\, Z_\beta\, Z_H \qquad \dots (5)$$

Through expression (5) is analyzed influence of the parameters such are: displacement coefficient of the profile x and gear ratio u in working stresses at teeth sides of the gears with external and internal meshing.

In figure 6 the influence of the displacement coefficient of the profile *x* in factor *Z* for pair with external and internal meshing is graphically presented.
From figure 6 can be noticed that with increase of the displacement coefficient of profile *x*, the factor Z (calculated by expression (5)) decreases for the external and internal meshing. Decrease of this factor influences directly in decreasing of working stresses at teeth sides that presents a good advantage for meshing gearset.
In figure 7 is presented graphically influence of the gear ratio *u* to the factor *Z* for both, external and internal meshing.
Graphs in figure 7 show that will increase of the gear ratio *u* to the factor *Z* for both, external and internal meshing which is negative concerning working stresses.

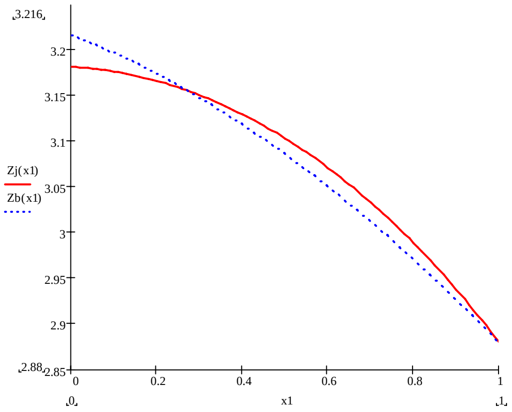

*Figure 6. Influence of the coefficient of displacement of the profile **x** at factor Z for external and internal meshing.*

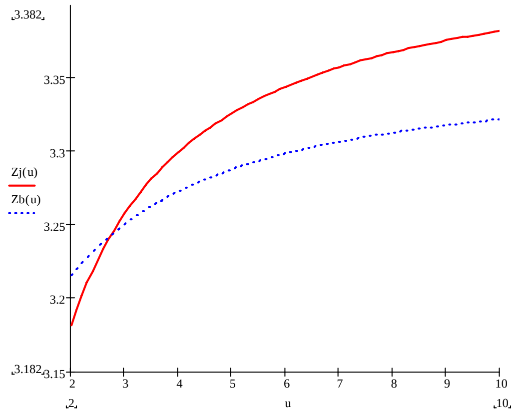

*Figure 7. Influence of gear ratio **u** at factor Z for external and internal meshing.*

4. CONCLUSIONS

In this paper is analyzed influence of the different factors in working stresses at teeth sides of the cylindrical involute gears with external and internal meshing and based in the achieved results can be concluded that:

- *The gear ratio **u** and external (nominal) load F_t increase working stresses that is not convenient for transmission (lifting power) of the gears teeth;*
- *The increase of the values for the displacement coefficient of profile **x**, influences positively directly on the transmission (lifting power) of the gears teeth;*
- *The radii of curvature have decisive influence on transmission of the gearset, because if the edges of the meshing interval (line of action) are used than stresses at teeth sides increases to infinite, therefore these parts are not allowed to be used for meshing presenting the destructive character for gears teeth.*

Nevertheless, in all cases the gearset with internal meshing are presented more favorable for use concerning the aspect of working stresses comparing to the gearset with external meshing under the same working conditions.
Therefore, wherever design and constructive conditions allow, it is preferred that pairs of the gears with internal meshing be exploited, furthermore knowing their many other advantages.

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