12th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2008, Istanbul, Turkey, 26-30 August, 2008

REDUCED PARAMETERS FOR THE DYNAMIC STUDY OF A SYSTEM OF ONE DEGREE OF FREEDOM

Lluïsa Jordi, Enrique Zayas, Salvador Cardona
Department of Mechanical Engineering
Technical University of Catalonia
Av. Diagonal, 647. Barcelona
Spain

ABSTRACT
In the machine field, it is usual to use mechanisms of one degree of freedom in which their configuration, and therefore any geometrical condition, is defined by a unique variable or independent coordinate. Usually, this variable corresponds with the movement of the input member of the mechanism that, often, coincides with a crank that is driven by a rotary motor. In order to obtain, both the movement equation of the mechanism and the constrain actions, Lagrange's equations are used. This implies to define a set of dynamic parameters, reduced parameters of the system, which are function of the independent coordinate. In this paper, a procedure to determine these parameters using PAM –Program of Analysis of Mechanisms– is exposed. This program makes static, kinematic and kinetostatic analysis of planar mechanism of one or more degrees of freedom that are controlled by the same number of actuators, angular or linear. This program obtains and allows exporting all the kinematic and dynamic variables of the mechanism, for all the time steps, from which it is possible to obtain the reduced parameters. The exposed procedure increases the possibilities of a program easy to use as PAM, which does not have the capacity for making a direct dynamic analysis. As example, the study of the direct dynamic of a single-dwell bar mechanism is exposed.
Keywords: reduced parameters, Lagrange's equations, dynamic, single-dwell bar mechanism

1. INTRODUCTION
The dynamic analysis of mechanisms by means of manual procedures is, usually, hard and tough, either by the number of equations to propose if Newton's laws are used, or by the difficulty to determine inertial forces if virtual work method is used, or by the difficulty to find the kinetic energy if the chosen method is the Lagrange's equations [1, 2, 3]. In these two last cases the situation becomes worth if some constraint action is needed.
The use of specific software to study multibody systems may be not justified or, even, may be unfeasible in some cases, as in the case that the study of the mechanism must be done in real time as part of a simulation and control of a productive process. In these cases, a model of low cost, both in implementation and computational point of view, is necessary.
In this paper, the dynamic study of the mechanisms of one degree of freedom using a set of reduced parameters, function of the independent coordinate used in the kinematic analysis of the mechanism, is exposed. These reduced parameters are determined by means of a kinetostatic analysis and using them the dynamic of the system is written as a second order differential equation easy to integrate. If some constraint action is needed, it is obtained by means of an algebric expression that includes the reduced parameters as well as the velocity and the acceleration obtained through the integration process.

2. DYNAMIC OF THE SISTEM

The use of ordinary Lagrange's equations for holonomic systems is proposed [4] for the study of the dynamic of the system:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\frac{\partial E_{\mathrm{c}}}{\partial \dot{q}_{\mathrm{i}}}-\frac{\partial E_{\mathrm{c}}}{\partial q_{\mathrm{i}}}=F_{\mathrm{i}}^{*} \qquad \dots (1)$$

Where E_{c} is the kinetic energy of the system, q_{i} the generalized coordinates and F_{i} the generalized force associated to the coordinate q_{i}.

If the aim is to find some constraint action (bearing force or constraint moment), this constraint can be substituted, conceptually, by an actuator that guarantees the kinematic condition imposed by the constraint –constraint actuator– (figure 1). Thus, the bearing force or the constraint moment can be determined applying the Lagrange's equations to a system with one additional generalized coordinate and applying, later, the kinematic condition imposed by the constrain to the obtained equations.

In the studied case here, one degree of freedom mechanism, the determination of one constraint action leads to a system of two independent coordinates q_1 and q_2; the first one is associated to the real movement of the mechanism and the second one to the movement disabled by the constraint. The kinetic energy for a holonomic mechanism of two independent coordinates is [4]

$$E_{\mathrm{c}}=\frac{1}{2}m_1\left(q_1,q_2\right)\dot{q}_1^2+\frac{1}{2}m_2\left(q_1,q_2\right)\dot{q}_2^2+m_{12}\left(q_1,q_2\right)\dot{q}_1\dot{q}_2 \qquad \dots (2)$$

The following equations are obtained using Lagrange's equations and particularizing to the imposed movement by the constraint's actuator: $\ddot{q}_2=0$, $\dot{q}_2=0$ and $q_2=0$

$$\begin{cases} m_1\left(q_1\right)\ddot{q}_1+\dfrac{1}{2}m_{1q_1}\left(q_1\right)\dot{q}_1^2=T_{\mathrm{act}} \\ m_{12}\left(q_1\right)\ddot{q}_1+\left(m_{12q_1}\left(q_1\right)-\dfrac{1}{2}m_{1q2}\left(q_1\right)\right)\dot{q}_1^2=F_{\mathrm{E}} \end{cases} \qquad \dots (3)$$

Where $m_{1q_1}=\dfrac{\partial m_1}{\partial q_1}$; $\quad m_{12q_1}=\dfrac{\partial m_{12}}{\partial q_1}$; $\quad m_{1q2}=\dfrac{\partial m_1}{\partial q_2}$

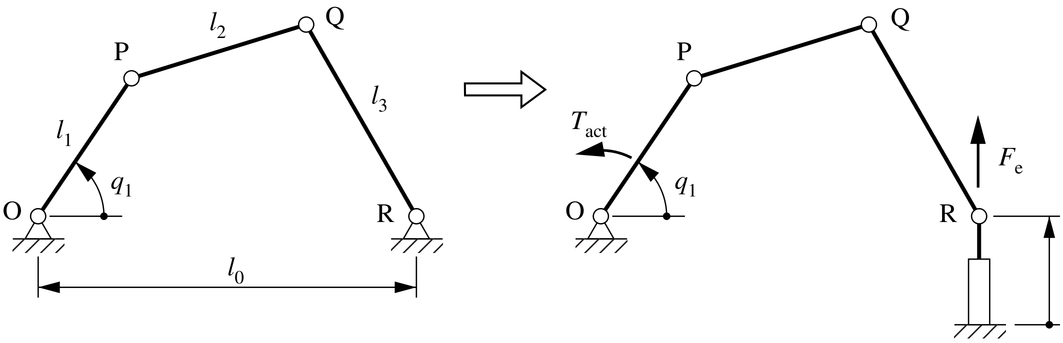


Figure 1. One degree of freedom mechanism and the constraint substituted by a constraint actuator.

The coefficients of the equation 3 can be obtained, for example, doing the following kinetostatic analysis [5]:

i) With $\ddot{q}_1=0\,\mathrm{rad/s^2}$ and $\dot{q}_1=1\,\mathrm{rad/s}$ it is obtained $m_2=\frac{1}{2}m_{1q_1}\left(q_1\right)=T_{\mathrm{act}}/\dot{q}_1^2$, from the first equation, and, from the second equation, it is obtained $m_{e_2}=\left(m_{12q_1}\left(q_1\right)-\frac{1}{2}m_{1q2}\left(q_1\right)\right)=F_{\mathrm{E}}/\dot{q}_1^2$.

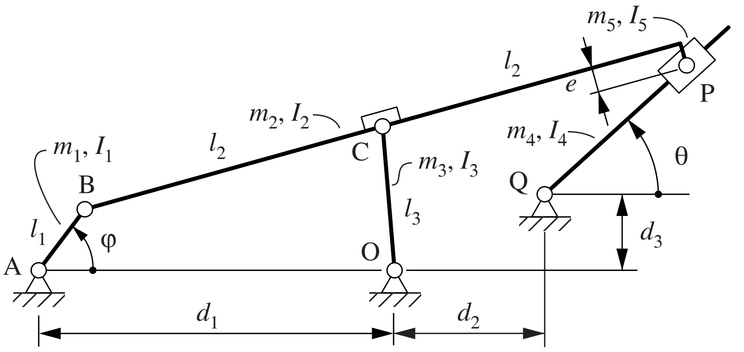
ii) With $\ddot{q}_1=1\,\mathrm{rad/s^2}$ and $\dot{q}_{1_0}=0\,\mathrm{rad/s}$ it is obtained $m_1=m_1\left(q_1\right)=\left(T_{\mathrm{act}}-\frac{1}{2}m_{1q_1}\left(q_1\right)\dot{q}_1^2\right)\Big/\ddot{q}_1$, from the first equation, and, from the second equation, it is obtained

$$m_{e_1} = m_{12}(q_1) = \left(F_{\mathrm{E}} - \left(m_{12q_1}(q_1) - \frac{1}{2} m_{1q2}(q_1)\dot{q}_1^2 \right) \right) \Big/ \ddot{q}_1 \ .$$

3. STUDY OF A SINGLE-DWELL MECHANISM

The single-dwell mechanism showed in the figure 2 is studied, as an example of the procedure exposed in the previous section. In this mechanism the rotation angle θ of the rocker QP is keeping almost constant during an interval of movement of the crank AB. Geometric parameters and the mass and moments of inertia are showed in figure 2. It is assumed that the crank AB is balanced; thus its centre of mass coincides with the fixed revolute joint A. The centre of mass of the coupler BP is located in C and the centres of mass of the rockers OC and QP are in their middle point. The centre of mass of the slider coincides with the revolution joint P.

To determine the reduced parameters the software PAM –Program of Analysis of Mechanisms– is used. This program makes static, kinematic and kinetostatic analysis of planar mechanisms of one or more degrees of freedom that are controlled by the same number of actuators, angular or linear [6, 7]. This program obtains and allows exporting all the kinematic and dynamic variables of the mechanism, for all the time steps.



l_1 = 50 mm	m_1= 75·10^{-3} kg
l_2 = 200 mm	m_2= 500·10^{-3} kg
l_3 = 95 mm	m_3= 120·10^{-3} kg
e = 15 mm	m_4= 230·10^{-3} kg
d_1 = 327 mm	m_5= 150·10^{-3} kg
d_2 = 97 mm	I_1 = 22·10^{-6} kg m^2
d_3 = 50 mm	I_2 = 6,5·10^{-3} kg m^2
	I_3 = 100·10^{-6} kg m^2
	I_4 = 700·10^{-6} kg m^2
	I_5 = 50·10^{-6} kg m^2

Figure 2. Single-dwell mechanism used as example.

4. RESULTS

Figure 3 shows the reduced parameters corresponding to one revolution of the crank AB obtained by means of the procedure exposed in the section 2. The studied constraint action is the normal force in the prismatic joint. The calculations have been made using the software MATLAB®; for this reason the results of the program PAM have been imported into MATLAB.

Figure 4 shows the results obtained for the evolution of the mechanism when the torque of the actuator is null and the initials conditions are $\varphi = 0$ rad and $\dot{\varphi} = 1$ rad/s. It can be observed that the movement and the constraint action are periodic, with period 5,42 s, such as corresponding to the free movement system with cyclical kinematics without friction.

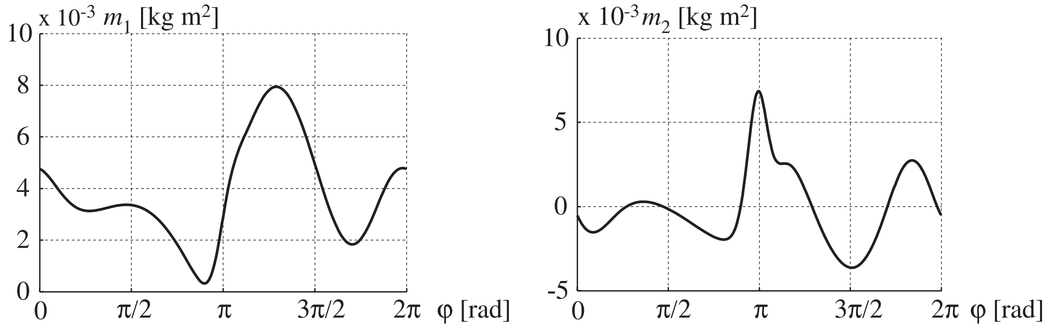


Figure 3a. Reduced parameters of the single-dwell mechanism studied.

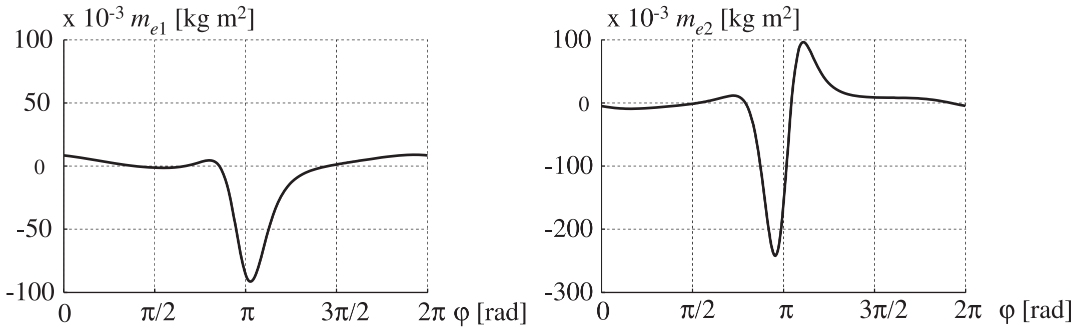


Figure 3b. Reduced parameters of the single-dwell mechanism studied.

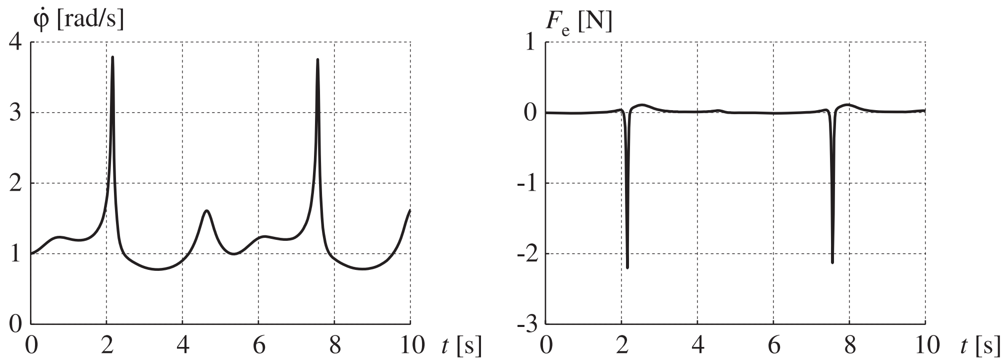


Figure 4. Angular velocity of the crank and bearing force in the slider when the torque of the actuator is null.

5. CONCLUSIONS

The approach proposed for the study of the dynamics of mechanisms of one degree of freedom, not necessarily with planar movement, based in the use of the reduced parameters gives a simple and easy procedure to analize them.

The determination of the reduced parameters, based in the kinetostatic analysis, increases the possibilities of programs of easy use, as PAM, that do not have the capacity for making direct dynamic analysis.

The procedure exposed is easily extensible to the constant forces or those that are function of the position, such as the weight. These forces can be reduced to the independent coordinate and be determined by means of a kinetostatic analysis. Obviously, the same does not occur with forces that are function of the velocity and therefore these ones must be considered in the movement differential equation.

6. REFERENCES

[1] S. Cardona y D. Clos, Teoria de Màquines, Ed. UPC, Barcelona, 2000.

[2] J. García de Jalón y E. Bayo, Kinematic and dynamic simulation of multibody systems, Springer-Verlag, New York, 1994.

[3] E. J. Haug, Computer-Aided Kinematics and Dynamics of Mechanical Systems, Allyn and Bacon, Massachusetts, 1989.

[4] J. Agulló, Mecànica de la partícula i del sòlid rígid Barcelona, Publicacions OK Punt, Barcelona, 2002.

[5] S. Cardona y L. Jordi, Aplicación de engranajes no circulares para adaptar las características de un mecanismo a las prestaciones del motor y cumplir las especificaciones de movimiento, 8º Congreso Iberoamericano de Ingeniería Mecánica, 2007.

[6] D. Clos y J. Puig, PAM, un programa de análisis de mecanismos planos de n grados de libertad enfocado a la docencia universitaria, Anales de Ingeniería Mecánica, vol 15 (1), pp. 757-765, 2004

[7] S. Cardona et al. Curs d'autoaprenentatge de simulació de mecanismes, Universitat Politècnica de Catalunya, Barcelona, 2006.