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PARTING STRAIN VECTOR IN HOMOGENEOUS AND ISOTROPIC SPACE

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ABSTRACT
In this paper, the intensity vector component of strain deviator is calculated. This is need full for the precise definition the shearing strain intensity (or strain intensity) which is by definition the vector. Analysis was performed in the different vector spaces. In the homogenous and isotropic vector space, the vector strain is decomposed on the spherical component and component of deviator. It is well known that the spherical component of the vector strain influences to the change of volume due to deformation, and the vector component of deviator strain influences to the change of form due to deformation.
The component vector of strain deviator is decomposed to the normal and the tangential component. The normal component vector of strain deviator is represented in the vector space of the normal strain, and the tangential path in the space vector of the tangential strain. Like this decomposition made it possible for to compute the intensity vector component of the strain deviator.
Key words: vector space, isotropic vector space, strain deviator, spherical vector.

1. INTRODUCTION
The strain vector $B\mathbf{n} = \boldsymbol{\varepsilon}$, in the general case, is a function the position vector of the observed point P and unit vector $\mathbf{n}$ of the vector $\overrightarrow{PQ}$ of the observed points P,Q for which strain is defined. For the selected point P, it is defined with acting the lineal operator B (tensor) to the unit vector $\mathbf{n}$. In the homogenous vector space, the strain vector $B\mathbf{n}$ is decomposed on the normal $\boldsymbol{\varepsilon}_n$ and the shearing $\boldsymbol{\varepsilon}_t$ component. In the homogenous and isotropic vector space, the normal component $\boldsymbol{\varepsilon}_n$ is the spherical S $\mathbf{n}$ component and the shearing component $\boldsymbol{\varepsilon}_t$ is the component of deviator D $\mathbf{n}$. The component of deviator D $\mathbf{n}$ is composed of the perpendicular D_1 $\mathbf{n}$ and tangential D_2 $\mathbf{n}$ component.
The perpendicular D_1 $\mathbf{n}$ component of the deviator D $\mathbf{n}$ has the geometrical representation in the vector space of the principal normal strain and he is equally deviator D $\mathbf{n}$ in this place. The tangential D_2 $\mathbf{n}$ component of the deviator D $\mathbf{n}$ has the geometrical representation in the vector space of the principal shearing strain and he is equally deviator D $\mathbf{n}$ in this place. With respect to the mutual perpendicularity of the perpendicular D_1 $\mathbf{n}$ and the tangential component D_1 $\mathbf{n}$, the magnitude of the deviator D $\mathbf{n}$ can is calculated.

2. PARTING DEFORMATION VECTOR IN HOMOGENEOUS SPACE
Similarly, as the decomposition of matrix $\mathbf{A}$ of linear operator A on the symmetric part and antisymmetric part and the deformation vector can be decomposed to two vectors [1]

$$A\, d\mathbf{r}_0 = \mathrm{du} = d\mathbf{u}^d + d\mathbf{u}^r, \qquad (1)$$

where the vector $d\mathbf{u}^d$ describes the pure deformation and the vector $d\mathbf{u}^r$ describes the rigid rotation of the uniform strained region. The vectors (1), through the medium of linear operators, can be represented by

$$A\, d\mathbf{r}_0 = B\, d\mathbf{r}_0 + C\, d\mathbf{r}_0, \qquad (2)$$

where by

$$B\,d\mathbf{r}_0 = \frac{1}{2}\,[A\,d\mathbf{r}_0 + (A\,d\mathbf{r}_0)^{\tau}] \qquad (3)$$

the vector of the pure deformation is defined, and by

$$B\,d\mathbf{r}_0 = \frac{1}{2}\,[A\,d\mathbf{r}_0 + (A\,d\mathbf{r}_0)^{\tau}] \qquad (4)$$

the vector of the rigid rotation is defined.

3. THE STRAIN VECTOR IN HOMOGENEOUS SPACE

The strain vectors $\boldsymbol{\varepsilon} = B\,\mathbf{n}$ of the one dimensional element (the vector of the unit elongation) in the orthonormal base ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) can be represented by [2]

$$\boldsymbol{\varepsilon} = B(\sum_{i=1}^{3} \vec{e}_i a_{in}) = \sum_{i=1}^{3} a_{in} B\vec{e}_i \qquad (5)$$

where $a_{in} = cos\ \alpha_{in} = (\mathbf{e}_i \cdot \mathbf{n})$ (i = 1, 2,3).

For the symmetrical linear operate B exists the right orthonormal base ($\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$) and real numbers (principal normal strain) ε_1, ε_2, ε_3 such that is [3]

$$B\,\mathbf{e}_1' = \varepsilon_1\,\mathbf{e}_1', \quad B\,\mathbf{e}_2' = \varepsilon_2\,\mathbf{e}_2', \quad B\,\mathbf{e}_3' = \varepsilon_1\,\mathbf{e}_3'. \qquad (6)$$

The coordinate system S = (O; $\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$) rises with a rotation from the coordinate system S = (O; $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$). If (a_{n1}, a_{n2}, a_{n3}) are the components of a vector $\mathbf{n} = \frac{d\vec{r}_0}{|d\vec{r}_0|}$ in the system S and (a_{n1}, a_{n2}, a_{n3}) the components of same the vector in system S', then for

$$\mathbf{n} = a_{n1}\,\mathbf{e}_1 + a_{n2}\,\mathbf{e}_2 + a_{n3}\,\mathbf{e}_3 = a_{n1}'\,\mathbf{e}_1' + a_{n2}'\,\mathbf{e}_2' + a_{n3}'\,\mathbf{e}_3'$$

is obtained

$$a_{nj}' = \mathbf{n} \cdot \mathbf{e}_j' = a_{n1}\,(\mathbf{e}_1 \cdot \mathbf{e}_j') + a_{n2}\,(\mathbf{e}_2 \cdot \mathbf{e}_j') + a_{n3}\,(\mathbf{e}_3 \cdot \mathbf{e}_j') = \sum_{i=1}^{3} a_{ni} a_{ij} \quad (j = 1, 2, 3). \qquad (7)$$

The strain vector $\boldsymbol{\varepsilon}$ in the coordinate system S' = (O; $\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$) is given by

$$\boldsymbol{\varepsilon} = B\mathbf{n} = \sum_{i=1}^{3} a_{ni}' B\vec{e}_i' = \sum_{i=1}^{3} \varepsilon_i\,\mathbf{e}_i'\ a_{in}' = (\sum_{i=1}^{3} \varepsilon_i \vec{e}_i' \vec{e}_i')\cdot \mathbf{n}. \qquad (8)$$

4. STRAIN VECTOR IN HOMOGENEOUS AND ISOTROPIC SPACE

In the homogeneous and isotropic [4] vector space the strain vector $B\,\mathbf{n}$ is given by

$$B\mathbf{n} = \sum_{i=1}^{3} B\vec{e}_i a_{in} = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} B\vec{e}_i = \frac{1}{\sqrt{3}} \sum_{i,j=1}^{3} \varepsilon_{ji} \vec{e}_j. \qquad (9)$$

In the coordinate system S' = (O; $\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$) the strain vector $B\,\mathbf{n}$ has the introduction in the vision

$$B\,\mathbf{n} = \sum_{i=1}^{3} a_{ni}' B\vec{e}_i' = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} \varepsilon_i\,\mathbf{e}_i'. \qquad (10)$$

5. PARTING STRAIN VECTOR IN HOMOGENEOUS AND ISOTROPIC SPACE

In the coordinate system S' = (O; $\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$) by

$$\varepsilon = \frac{1}{3}\,(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) \qquad (11)$$

the magnitude of the normal component $\boldsymbol{\varepsilon}_n$ of strain vector $B\,\mathbf{n}$ is given and this is the mean normal strain ε [5]. In the vector space, of the principal normal strain ε_1, ε_2, ε_3, by

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 - 3\varepsilon = 0 \qquad (12)$$

a plane, in a general form, is given. The plane (12) is oriented with unit normal **n** witch has the cosines of a direction mutually equal. The plane (10) is called the deviatoric plane π and for the positive principal normal strain $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ is presented in the figure 1.

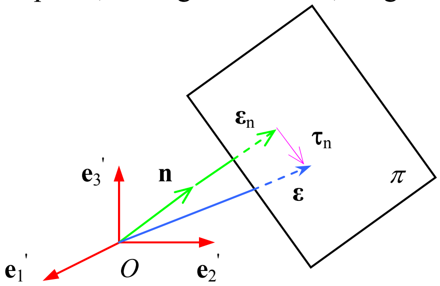

Figure 1. The deviatoric plane

In the vector space of the principal normal strain, the projections of normal strain vector $\boldsymbol{\varepsilon}_n$ on axes ε_1, ε_2, ε_3 are mutually equal and have the values equal to the mean ε normal strain. The components of the shearing strain (strain deviator) $\boldsymbol{\varepsilon}_t$ in the vector space of the principal normal strain ε_1, ε_2, ε_3 has the representation in the vision

$$\sum_{i=1}^{3}\varepsilon_i\ \mathbf{e}_i' - \sum_{i=1}^{3}\varepsilon\ \mathbf{e}_i' = \sum_{i=1}^{3}(\varepsilon_i-\varepsilon)\ \mathbf{e}_i'. \tag{13}$$

In the vector space of the strain vector $B\,\mathbf{n}$, the strain vector is given by

$$\boldsymbol{\varepsilon} = B\mathbf{n} = \sum_{i=1}^{3} a_{ni}' B\vec{e}_i' = \frac{1}{\sqrt{3}}\sum_{i=1}^{3}\varepsilon_i\ \mathbf{e}_i'. \tag{14}$$

The normal component $\boldsymbol{\varepsilon}_n$ of the strain vector $B\,\mathbf{n}$ has the representation in the vision

$$\boldsymbol{\varepsilon}_n = \frac{1}{\sqrt{3}}\sum_{i=1}^{3}\varepsilon\,\mathbf{e}_i', \tag{15}$$

the shearing component (strain deviator) $\boldsymbol{\varepsilon}_n$ of the strain vector $B\,\mathbf{n}$ can be charged in the frame

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_n = \frac{1}{\sqrt{3}}\sum_{i=1}^{3}\varepsilon_i\ \mathbf{e}_i' - \frac{1}{\sqrt{3}}\sum_{i=1}^{3}\varepsilon\,\mathbf{e}_i' = \frac{1}{\sqrt{3}}\sum_{i=1}^{3}(\varepsilon_i-\varepsilon)\ \mathbf{e}_i'. \tag{16}$$

In the coordinate system $S = (O;\ \mathbf{e}_1,\ \mathbf{e}_2,\ \mathbf{e}_3)$ the strain vector can be noted as

$$\boldsymbol{\varepsilon} = B\mathbf{n} = \sum_{i=1}^{3} a_{ni} B\vec{e}_i = \frac{1}{\sqrt{3}}\sum_{i=1}^{3} B\vec{e}_i = \frac{1}{\sqrt{3}}\sum_{j=1}^{3}\varepsilon_{ji}\vec{e}_j \quad (i = 1, 2, 3). \tag{17}$$

The intensity of the normal component $\varepsilon_\mathbf{n}$ here has the representation in the version

$$\boldsymbol{\varepsilon}_n = \boldsymbol{\varepsilon}\cdot\mathbf{n} = \frac{1}{\sqrt{3}}\Big(\sum_{i=1}^{3}B\vec{e}_i\Big)\cdot\mathbf{n} = \frac{1}{3}\Big(\sum_{i,j=1}^{3}\varepsilon_{ji}\vec{e}_j\Big)\cdot\Big(\sum_{j=1}^{3}\vec{e}_j\Big) = \frac{1}{3}\Big(\sum_{i,j=1}^{3}\varepsilon_{ij}\Big) \quad (\varepsilon_{ij} = \varepsilon_{ji}).$$

(18)

The shearing component (strain deviator) $\boldsymbol{\varepsilon}_t$ of the strain vector $\boldsymbol{\varepsilon}$ can charged in the frame

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_n = \frac{1}{\sqrt{3}}\sum_{i=1}^{3}(B\vec{e}_i - \varepsilon\vec{e}_i) = \frac{1}{\sqrt{3}}\sum_{i,j=1}^{3}(\varepsilon_{ji}\vec{e}_j - \varepsilon\delta_{ij}\vec{e}_j) = \frac{1}{\sqrt{3}}\sum_{i,j=1}^{3}(\varepsilon_{ji} - \varepsilon\delta_{ij})\vec{e}_j \tag{19}$$

or

$$\boldsymbol{\varepsilon}_t = D\,\mathbf{n} = \frac{1}{\sqrt{3}}\ (D\,\mathbf{e}_1 + D\,\mathbf{e}_2 + D\,\mathbf{e}_3). \tag{20}$$

6. PARTING VECTOR OF STRAIN DEVIATOR IN HOMOGENEOUS AND ISOTROPIC SPACE

The deviatoric $D\,\mathbf{n}$ component of the strain vector $B\,\mathbf{n}$ can be decomposed on the perpendicular $D_I\,\mathbf{n}$ and the tangential $D_2\,\mathbf{n}$ component of the deviator $D\,\mathbf{n}$ (Figure 2).

The perpendicular $D_I\,\mathbf{n}$ component has the version in the vision

$$D_I\,\mathbf{n} = \frac{1}{\sqrt{3}}\ (D_I\,\mathbf{e}_1 + D_I\,\mathbf{e}_2 + D_I\,\mathbf{e}_3) = \frac{1}{\sqrt{3}}[(\varepsilon_{11} - \varepsilon)\,\mathbf{e}_1 + (\varepsilon_{22} - \varepsilon)\,\mathbf{e}_2 + (\varepsilon_{33} - \varepsilon)\,\mathbf{e}_3]. \tag{21}$$

The magnitude of the vector (21) is

$$|D_I\,\mathbf{n}| = \frac{1}{\sqrt{3}}[(\varepsilon_{11} - \varepsilon)^2 + (\varepsilon_{22} - \varepsilon)^2 + (\varepsilon_{33} - \varepsilon)^2]^{1/2} = \frac{1}{3}[(\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2]^{1/2}. \tag{22}$$

The tangential $D_2\,\mathbf{n}$ component has the version in the vision

$$D_2\,\mathbf{n} = \frac{1}{\sqrt{3}}\,(D_2\,\mathbf{e}_1 + D_2\,\mathbf{e}_2 + D_2\,\mathbf{e}_3) \;=\; \frac{1}{\sqrt{3}}(\,\varepsilon_{21}\,\mathbf{e}_2 + \varepsilon_{31}\,\mathbf{e}_3 \;+ \varepsilon_{12}\,\mathbf{e}_1 + \varepsilon_{32}\,\mathbf{e}_3 + \varepsilon_{13}\,\mathbf{e}_1 + \varepsilon_{23}\,\mathbf{e}_2).$$

(23)

After (23), the linear operator D_2 transports the vectors of the normal $\mathbf{e}_i$ (i = 1, 2, 3) in the here plane, i.e. $D_2\,\mathbf{e}_1 = \varepsilon_{21}\,\mathbf{e}_2 + \varepsilon_{31}\,\mathbf{e}_3$, $D_2\,\mathbf{e}_2 = \varepsilon_{12}\,\mathbf{e}_1 + \varepsilon_{32}\,\mathbf{e}_3$ and $D_2\,\mathbf{e}_3 = \varepsilon_{13}\,\mathbf{e}_1 + \varepsilon_{23}\,\mathbf{e}_2$. This is the proof that the vector $D_2\,\mathbf{n}$ is perpendicular to the vector $\mathbf{n}$ and that lies on the deviatoric planeπ. If consider symmetric of the shear components of strain vector $B\,\mathbf{n}$, i.e. $\varepsilon_{12} = \varepsilon_{21}$, $\varepsilon_{13} = \varepsilon_{31}$ and $\varepsilon_{13} = \varepsilon_{31}$, then from (23) is obtained

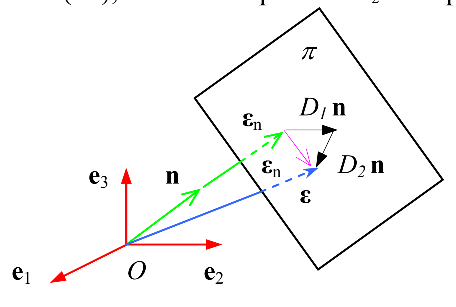

Figure 2. The deviatoric components of the strain vector

$$D_2\,\mathbf{n} = \frac{1}{\sqrt{3}}[\varepsilon_{12}\,(\mathbf{e}_1 + \mathbf{e}_2) + \varepsilon_{23}\,(\mathbf{e}_2 + \mathbf{e}_3) + \varepsilon_{31}\,(\mathbf{e}_3 + \mathbf{e}_1)$$

$$= \sqrt{\frac{2}{3}}\,(\varepsilon_{12}\,\mathbf{f}_1 + \varepsilon_{23}\,\mathbf{f}_2 + \varepsilon_{31}\,\mathbf{f}_3), \qquad (24)$$

where $\mathbf{f}_1 = \dfrac{\vec{e}_1 + \vec{e}_2}{|\vec{e}_1 + \vec{e}_2|} = \dfrac{\sqrt{2}}{2}\left(\vec{e}_1 + \vec{e}_2\right)$, analogous is $\mathbf{f}_2 = \dfrac{\sqrt{2}}{2}\left(\vec{e}_2 + \vec{e}_3\right)$ and $\mathbf{f}_3 = \dfrac{\sqrt{2}}{2}\left(\vec{e}_3 + \vec{e}_1\right)$. The magnitude of the vector (24) is

$$|D_2\,\mathbf{n}| = \sqrt{\frac{2}{3}}\,(\varepsilon_{12}{}^2 + \varepsilon_{23}{}^2 + \varepsilon_{31}{}^2)^{1/2}. \qquad (25)$$

In respect of the perpendicular of the vectors $D_1\,\mathbf{n}$ and $D_2\,\mathbf{n}$ from (22) and (25) follows

$$|D\,\mathbf{n}| = (|D_1\,\mathbf{n}|^2 + |D_2\,\mathbf{n}|^2)^{1/2}$$

$$|D\,\mathbf{n}| = \frac{1}{3}\,\{[(\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2] + 6\,(\varepsilon_{12}{}^2 + \varepsilon_{23}{}^2 + \varepsilon_{31}{}^2)\}^{1/2} \qquad (26)$$

the magnitude of the deviator D $\mathbf{n}$ [6].

The strain deviator $D\,\mathbf{n}$ is equal his the perpendicular component $D_1\,\mathbf{n}$ in the coordinate system $S' = (\,O;\ \mathbf{e}_1{}',\ \mathbf{e}_2{}',\ \mathbf{e}_3{}')$ where the unit vectors $\mathbf{e}_i$ (i = 1,2,3) are parallel to the principal normal strain vector ε_i (i = 1,2,3). The strain deviator $D\,\mathbf{n}$ is equal his the tangential component $D_2\,\mathbf{n}$ in the coordinate system $S'' = (\,O;\ \mathbf{f}_1{}',\ \mathbf{f}_2{}',\ \mathbf{f}_3{}')$ where the unit vectors $\mathbf{f}_I{}'$ (i = 1,2,3) are parallel to the principal shear strain vector η_i (i = 1,2,3).

7. CONCLUSION

In the homogenous vector space, customary the parting for the strain vector is on the normal and the tangential component. For further a parting, the strain vector needed is installing in a meditation vector spaces, which are beside homogenous and isotropic. In this way constructed the vector space, the normal component of the strain vector transforms in a spherical and shearing in deviatory component. Again, the deviatoric component consists of the perpendicular and the tangential component. The perpendicular component has a geometrical version in the vector space of the normal strain, and the tangential in the vector space of the shearing stress. In respect of the perpendicular of the perpendicular and the tangential component of the strain vector, the intensity of the deviator component in the arbitrary select system of a coordinates it is possibly calculated.

8. REFERENCES

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