

## MULTISTAGE DECIMATION FILTER

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### ABSTRACT

*This paper presents one method for a multiplierless FIR (finite impulse response) decimation filter design. The method is based on the IFIR (interpolated finite impulse response) structure and the sharpening techniques. It is supposed that the decimation factor can be presented as the product of two factors. In that way the expanded model filter can be moved to the lower rate. The coefficients of the model filter are rounded to the nearest integers and the sharpening technique is applied to the model filter in order to improve the magnitude characteristics of the overall filter. The interpolator is the cascaded RRS (recursive running sum) filter and the simple compensator. As result the overall structure is multiplierless and exhibits a high attenuation in the stopband and a low passband droop.*

**Keywords:** decimation filter, IFIR filter, RRS filter.

### 1. INTRODUCTION

Sampling rate conversion (SRC) is a necessary task in many applications such is software radio, sigma-delta conversion, subband coding, among others [1].

The reduction of a sampling rate is called decimation, as shown in Fig.1, because the original sample set is reduced (decimated). Decimation consists of two stages: filtering and downsampling.

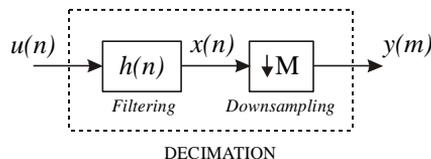


Figure 1. Decimation.

Downsampling reduces the input sampling rate by an integer factor  $M$ , which is known as a downsampling factor. If the original signal is not bandlimited to  $\pi/M$ , the replicas of the signal spectra introduced by downsampling will overlap. This overlapping effect known as aliasing, sometimes changes the signal irreversibly. Consequently the design of decimation systems is mainly a filter design problem where is a great need to get their best implementations in terms of power consumption while keeping area and performance in acceptable ranges [1].

There have been a lot of efforts for design of efficient decimator filters [1]-[8]. However, so far there is no universal solution that would be suitable for all applications [2].

The objective of the work presented in this paper is the design of the multiplierless decimation filter with no filtering at the high input rate, with a low passband droop and a high stopband attenuation, which are requirements usually used in the multistandard radio concept. We improve our previous result [7] replacing the interpolator with the cascaded RRS filter and the simple compensator.

The rest of the paper is organised in the following form. In the next section the overview of the IFIR structure, and sharpening techniques is presented. The proposed method is described in Section 3 and illustrated with one example.

## 2. IFIR and SHARPENING TECHNIQUES

The basic idea of an IFIR structure is to implement a FIR filter as a cascade of two lower order FIR blocks, as shown in Figure 2. One section is the expanded model filter  $G(z^M)$  and another is the interpolator  $I(z)$ .

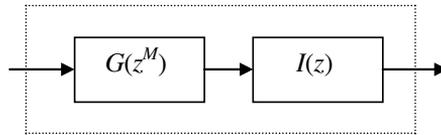


Figure 2. IFIR structure.

The filter  $G(z^M)$ , where  $M$  is the interpolation factor, is obtained by introducing  $M-1$  zeros between each sample of the unit sample response of  $G(z)$ . The function of the interpolator filter  $I(z)$  is to eliminate images introduced by  $G(z^M)$ . More details can be found in [1].

We round the impulse response as

$$h_r(n) = r \cdot h_I(n) = r \cdot \text{round}(h(n) / r), \quad (1)$$

where  $h(n)$  is an equiripple type FIR filter which satisfies given specification,  $h_I(n)$  is the new impulse response derived by rounding all coefficients of  $h(n)$  to the nearest integer, and  $\text{round}(\cdot)$  means the round operation. The rounding constant  $r$  determines the precision of the approximation of  $h_r(n)$  to  $h(n)$  and is in the form  $2^{-k}$ , where  $k$  is an integer. Considering that the integer coefficient multiplications can be accomplished with only shift-and-add operations, the rounded impulse response filter is multiplier-free.

To improve the gain response characteristics of the rounded filter, we propose to use the sharpening technique which can be used for simultaneous improvements of both the passband and stopband characteristics of a linear-phase FIR digital filter [8]. The technique uses the amplitude change function (ACF) which is a polynomial relationship of the form  $Hsh = f(H)$  between the amplitudes of the overall and the prototype filters,  $Hsh$  and  $H$ , respectively. We propose to use simple sharpening polynomial in the form [9]

$$Sh\{H\} = 3H^2 - 2H^3. \quad (2)$$

Additionally we use the RRS filter as an interpolator in IFIR structure.

$$I(z) = \left[ \frac{1}{M} \frac{1 - z^{-M}}{1 - z^{-1}} \right]^N. \quad (3)$$

The magnitude characteristic of the filter is given as

$$\left| I(e^{j\omega}) \right| = \left| \frac{\sin(\omega M / 2)}{M \sin(\omega / 2)} \right|^N. \quad (4)$$

We adopt the following RRS compensator from [9]

$$C(z^M) = B[1 + Az^{-M} + z^{-2M}], \quad (5)$$

where

$$B = -2^{-(b+2)}, \quad A = -[2^{(b+2)} + 2], \quad (6)$$

and

$b$  is integer which depends on  $N$  and the values of  $b$  are in the interval  $[-1, 2]$ . More details can be found in [9].

### 3. DESCRIPTION OF THE METHOD

The filter must eliminate all spectra replicas. Therefore its normalized stopband frequency  $\omega_s$  is equal to  $1/M$ . The passband is at least  $0.5 \omega_s$ .

We suppose that the decimation factor can be presented as the product of two factors

$$M = M_1 M_2 \quad (7)$$

We design the decimation filter with the given specification using the IFIR structure with the factor  $M_1$ , as shown in Figure 3.a. In that way the original decimation filter is designed using two less order filters, the model filter  $G(z)$  and the interpolation filter  $I(z)$ . The interpolator filter is RRS filter (3) cascaded with the compensator (5)  $C(z^{M_1})$

Using well known multirate identity [1] we move the expanded model filter  $G(z^{M_1})$  and the compensator  $C(z^{M_1})$  after downsampling with the factor of  $M_1$ , as shown in Figure 3. b.

Next issue is to eliminate the multiplications. To this end, we apply the rounding technique to the model filter. The resulting filter is a multiplierless filter because the integer multiplications can be realized by only additions and shifts. Unfortunately, this filter has a distorted magnitude response. In order to improve its magnitude response and finally satisfy the desired specification we apply the sharpening technique to the model filter. We propose to use simple sharpening polynomial (2)

Finally, using the polyphase representation of the RRS interpolation filter we get the structure shown in Figure 3.c, where  $I_{pr}(z)$  represents the polyphase components of the RRS filter. Note that there is no filtering at the high input rate.

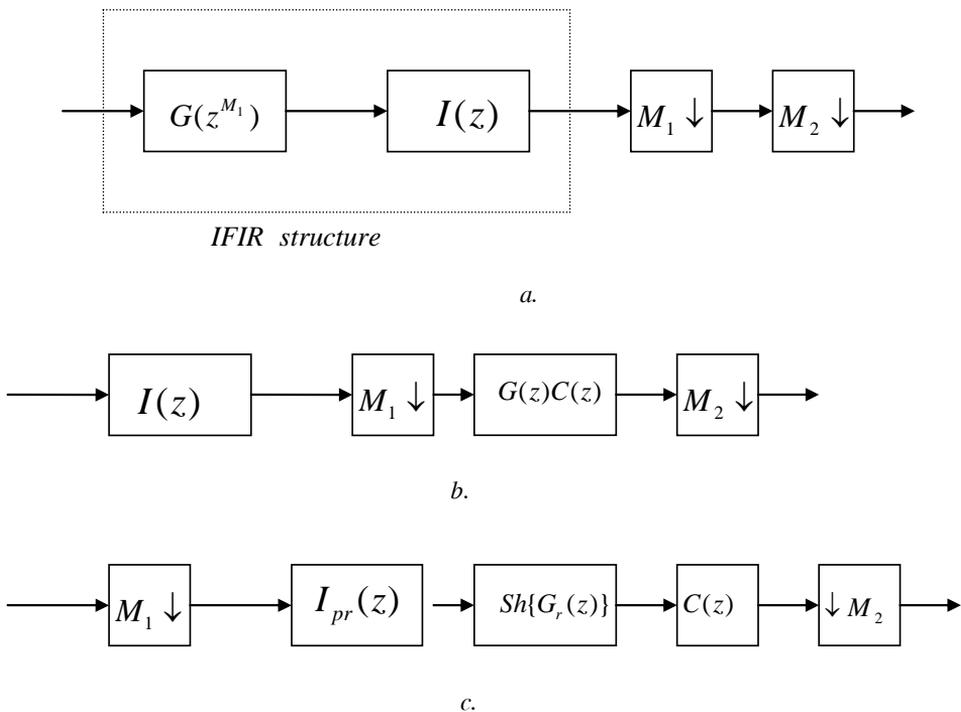


Figure 3. Proposed structures.

Method is illustrated in the following example.

### Example 1

We design the decimation filter with the decimation factor  $M=8$ . The normalized stopband frequency is  $1/8$  and the corresponding passband frequency is  $0.064$ . The maximum passband ripple is  $0.1$  dB and the minimum stopband attenuation is  $40$  dB. The corresponding equiripple filter has an order of  $69$  and requires  $35$  multipliers. Using  $M_1=4$  we get the model filter of the order  $18$ . Applying the rounding technique with the factor  $r=0.0039$  in the model we get the rounded model filter with  $8$  integer multiplications. The sharpening polynomial (2) is applied to the model filter. RRS filter has  $N=5$  and the compensator parameter  $b=0$ . The resulting magnitude response is shown in Fig. 4. a. The passband (Fig. 4.b) and the stopband show that the desired specification is satisfied.

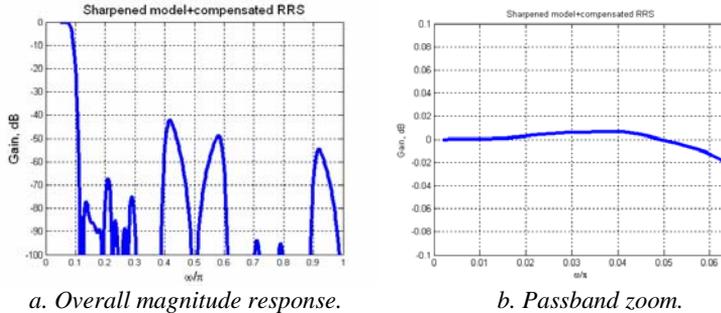


Figure 4. Example 1.

## 4. CONCLUSION

We present simple method for an efficient decimation filter design. The method is based on the IFIR structure where the interpolator is the cascade of  $N$  RRS filters and the simple compensator. Using the multirate identity the model filter and the compensator are moved to lower rate. Using the polyphase decomposition the RRS filter can also be moved to the lower rate. As a consequence there is no filtering at high input rate. Additionally the expanded model filter is expressed in terms of integer coefficients using rounding technique. The sharpening technique is used to improve the magnitude characteristic and to satisfy the desired specification.

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