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THERMAL ANALYSIS OF ELECTRICAL MACHINES

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ABSTRACT
This paper presents the energy losses, heating or cooling of the homogeneous body of electrical machine during its operation. Overheating and cooling of electrical machine is analyzed in analytical and graphical way depending on the temperature of the homogeneous body and accumulation thermal time constant. During analysis it is seem that heating constant and cooling constant are not the same, and the temperature of the homogeneous body, in function of time and thermal time constant, is non-stationary and achieves to be stationary.
Keywords: heating, cooling, electrical machine

1. INTRODUCE
Heating of electrical machines caused as a result of energy losses appear during their work. All of energy that loses converted into heat energy. Mechanical losses and they in the iron do not depend on the loads, so they are taken constant. Losses in the contacts are in the right proportion with the load (2*I*), while those in copper increases with the load quadratic (RI^2). From above can be issued the general expression for the lost power in the machine [1]:

$$P_h = A + BI + CI^2$$

It is clear that the heating of machine depends on the loads, so it will be greater for larger losses.
Though, electrical machines contains of several parts with different thermal features, the heat process study based on the heating theory of ideal rigid body. With rigid body implied the homogeneous ideal body, where all the body points have the same temperature and the whole its surface has the same ability to spread the heat.

2. THERMAL - PHYSICAL PARAMETERS
Let be assume that the body in unit time released heat power P_h, so within the time $d\tau$ will be released in the body the heat energy $dQ = P_h d\tau$. In the general case a part of this heat accumulated into the body increasing its temperature, while the remained rest, through surface body, given to the surrounding environment.
If within time $d\tau$, temperature increasing from t to $t+dt$, i.e. for dt, then the amount of accumulated heat in the body will be [2]:

$$dQ_1 = m \cdot c \cdot dt \qquad \ldots (1)$$

Where: m, kg – body mass; c, J/(kg°C) – body specific heat (specific thermal capacity). The amount of thermal energy that the body gives up surround depends by cooling surface S of the body, the ability of this body surface to release heating (heat transmission coefficient k) and of body overheating against the environment, which is equal to the difference of body temperature t_t and environment temperature t_a:

$$dQ_2 = S \cdot k\, (t_t - t_a)\, d\tau = S \cdot k \cdot t_{ov} \cdot d\tau \qquad \text{… (2)}$$

In this way, the equation of body heat is:

$$dQ = dQ_1 + dQ_2 = m \cdot c \cdot dt + S \cdot k \cdot t_{ov} \cdot d\tau \qquad \text{… (3)}$$

When the ambient temperature is constant t_a = konst., changing of body overheating is equal to the its temperature change *dt*, so expression (3) takes this form [3]:

$$P_h d\tau = dQ = m \cdot c \cdot dt + S \cdot k \cdot t \cdot d\tau \qquad \text{… (4)}$$

During heating, overheat of body and the amount of heat that is given to environment reflected increasing, while accumulated heat in the body and the change by its overheating reflected decreasing. In this way after a very long time (theoretically infinite) all of the released heat in the body is given to the exterior environment, so the body overheat achieves stationary value ($t=t_m=const$ and $dt=0$). Under these conditions the equation (4) is written:

$$P_h d\tau = S \cdot k \cdot t_m \cdot d\tau \qquad \text{… (5)}$$

Where:

$$t_m = P_h/(S \cdot k) \qquad \text{… (6)}$$

Stationary overheat t_m, is so high, when greater is the amount of released heat in the body P_h and lesser the intensity of heat provision, i.e. so as more less the production $S \cdot k$. If it is accepted that the all released heat accumulated into the body, then body overheat will increase against environment. In this case, the body will achieve stationary overheat during the time β, which is determined by thermal balance:

$$P_h\, \beta = m \cdot c \cdot t_m \qquad \text{… (7)}$$

Where:

$$\beta = (m \cdot c \cdot t_m) / P_h \qquad \text{… (8)}$$

In view of the expression (6), the equation (7) is:

$$\beta = (m \cdot c)/(S \cdot k) \qquad \text{… (9)}$$

Time β (ek. 9) is called the body heating constant. Heating constant does not depend on the amount of released heat in the body; it is in the right proportion with body total thermal coefficient ($m \cdot c$) and in indirect proportion with the intensity of the heat provision ($S \cdot k$).

By division both sides of the equation (4) with $S \cdot k$ and in view of the expressions (6) and (9) obtained:

$$t_m d\tau = \beta\, dt + t\, d\tau \qquad \text{… (10)}$$

Or by separating variables:

$$d\tau / \beta = dt / (t_m - t) \qquad \text{… (11)}$$

After integration of both sides of expression (11) obtained:

$$\tau / \beta = - \ln (t_m - t) + C \qquad \text{… (12)}$$

Constant of integration C is determined by conditions on the border where $\tau = 0$ is t = 0, so:

$$C = \ln t_m \qquad \text{… (13)}$$

After replacing the expression (13) in expression (12) obtained:

$$\tau / \beta = - \ln (t_m - t) + \ln t_m \qquad \dots (14)$$

Whereof:

$$t_n = t = t_m (1 - \exp(-\tau / \beta)) \qquad \dots (15)$$

Expression (15) is given graphically in fig. 1. From figure it is seem that overheat temperature of electrical machine, looked as homogeneous body, done under curved line exponential and depends on parameters t_m and β.

After time $\tau = (4 \div 5)\beta$ overheat practically reaches stationary value t_m. Thus, for example the machine which has heating constant β = lh, for the time (4÷5) h will achieves its stationary temperature.

When the body does not release heat, stationary overheat becomes zero. Replacing $t_m = 0$ in the expression (11) obtained cooling equation of body [4]:

$$d\tau / \beta = dt / t \qquad \dots (16)$$

After integration of both sides by expression (16) obtained:

$$\tau / \beta = - \ln t + C \qquad \dots (17)$$

Integration constant C is determined by conditions: where $\tau = 0$ and temperature $t = t_m$, so:

$$C = \ln t_m \qquad \dots (18)$$

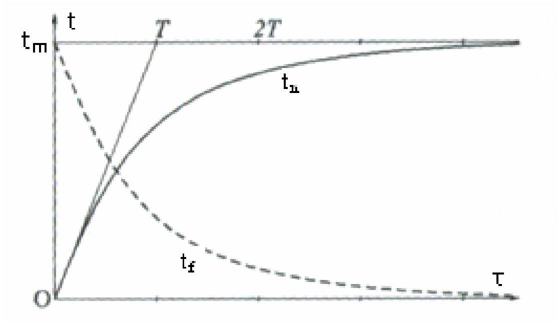

Figure 1. Heating and cooling curved of the homogeneous body

After replacing the expression (18) in expression (17) obtained:

$$\tau / \beta = - \ln t + \ln t_m \qquad \dots (19)$$

Whereof:

$$t_f = t = t_m \exp(-\tau / \beta) \qquad \dots (20)$$

Expression (20) is given graphically by broken lines in fig. 1. From figure it is seem that the body practically go cooling for $\tau = (4 \div 5)\beta$.

In general case the heating and cooling constant is not meant to be the same. Derived expressions here can be used for practical research of overheat rotating machines and transformers.

3. ANALYSIS OF HEATING AND COOLING TEMPERATURE AND THERMAL TIME CONSTANT

In view of the upper expressions, by means of the simulations respectively the diagrams which are presented in continuity, it is analyzed the heating and cooling temperature for a body. This body of a thermal machine has these characteristics: thermal power P_h=10; 50; 100 W; conduction coefficient λ=0.7W/(mK); overall heat transmission coefficient k=1.47W/(m^2K); density ρ=1000; 1500; 2000kg/m^3; specific heat c=920J/(kgK); total area surfaces S=0.54m^2; volume 0.162m^3; mass m=15÷300kg; the time where the parameters are analyzed is τ=0,100,...,1000000s.

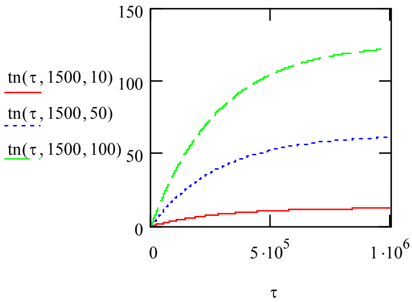

Figure 1. Change of heating temperature of a body in function of time τ; ρ=1500 kg/m^3; P_h=10; 50;100W;

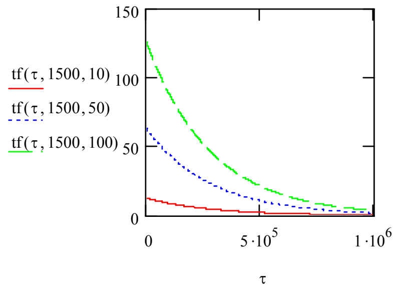

Figure 2. Change of cooling temperature of a body in function of time τ; ρ=1500 kg/m^3; P_h=10; 50;100W;

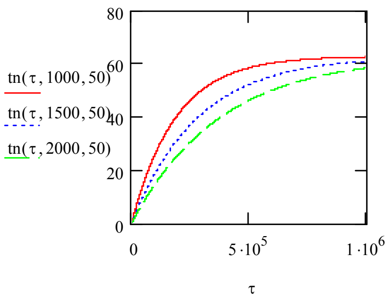

Figure 3. Change of heating temperature of a body in function of time τ; ρ=1000; ρ=1500; ρ=2000kg/m^3; P_h=50W

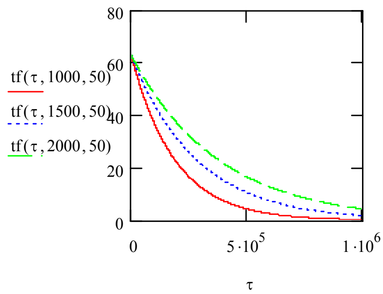

Figure 4. Change of cooling temperature of a body in function of time τ; ρ=1000; ρ=1500; ρ=2000kg/m^3; P_h=50W

4. CONCLUSION

Mathematical model, that above is presented, describes the dynamic of body temperature respectively heating and cooling temperature. Depending of thermal power of a body are achieves these overheat temperatures: t_m (P_h=10W) =12.56^{0}C; t_m (P_h=50W) =62.83^{0}C; t_m (P_h=100W) =125.66^{0}C; Depending of density of a body achieves these accumulation heat constants: β(ρ=1000)=1.873·10^5 s β(ρ=1500)=2.809·10^5 s; β(ρ=2000)=3.746·10^5 s; From formulas and diagrams it is shown that after a determined time achieved to a constant heat exchange, respectively heating and cooling constant temperature, expressed through the time constant accumulation. For differently thermal power appeared evidently differently heating and cooling temperature (figures 1 and 2), and also for differently body density appeared for determinate time the differently heating and cooling temperature , while after infinite time appeared to same and constant value (figures 3 and 4). Achieved results consist in exactness and verity of practical side and also to secure the control of body heating and cooling thermal machines.

5. REFERENCES

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