13th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2009, Hammamet, Tunisia, 16-21 October 2009

STUDY OF GLASS AND CERAMIC MATERIALS BY ESPI

Juraj Slabeycius, Sona Rusnakova, Dana Bakosova
Faculty of Industrial technologies
Alexander Dubcek University in Trencin
I.Krasku 491/30, 02001 Puchov
Slovak republic

Stanislav Minarik
Faculty of Materials Science and Technology
Slovak University of Technology in Bratislava
Paulínska 16,917 24 Trnava
Slovak republic

ABSTRACT

Electronic speckle pattern interferometry (ESPI) is a very useful tool for investigation of natural vibration of oscillating objects. In contrast to classical measurement by accelerometer, ESPI allows to visualize whole vibrational field of object, and therefore not only to estimate the natural frequency, but also to identify the shape of vibrational mode. Frequencies of natural vibrational modes depend on elastical properties of material, especially on Young modulus E, Poisson ratio μ and density of material ρ. This fact gives us in principle the possibility to estimate the elastic constant of materials from the vibrational measurement.

Thin rectangular plates from glass and ceramics were investigated. By aid of electronic speckle pattern interferometry the natural frequencies of various vibrational modes were determined. The method of elastic constant of materials estimation was proposed.

Keywords: ESPI, natural frequencies, elastic properties, glass, ceramics

1. INTRODUCTION

Glass and ceramics belong to very old materials, which were known from ancient age. Nevertheless, there are modern and perspective materials, very frequently used in industry. There are huge amount of various types of glass and ceramic material with very different mechanical, electrical and thermal properties. The present paper is devoted to the investigation of mechanical parameters of these materials, such as Young modulus and Poisson ratio by electronic speckle pattern interferometry (ESPI). We have studied natural vibration of thin square plates from glass and ceramics. Frequencies of natural vibrational modes depend on elastical properties of material, especially on Young modulus E, Poisson ratio μ and density of material ρ. This fact gives us in principle the possibility to estimate the elastic constant of materials from the vibrational measurement. A vibration of rigid body is very complicated process, which can be mathematically described only under many simplified assumptions. Although the Kirchhoff's plate theory is more than 150 years old, vibrations of plates are studied till present time. Last twenty years appears huge number of publication developing theory of plate vibration. Review of actually stay of problematic we can find for example in [1,2,3]. The solutions for the various type of boundary conditions were sought. Nevertheless, analytical solution of plane vibration equation exist only for few very special cases, therefore the numerical methods are used. The exploitaition of computers becoms unavoidable. The most software tools are based on finite element method. Unfortunatelly, these tools in many cases don't give results agreeing with experiment. Our ESPI measurements can be used as a test of applicability and reliability such software tools.

2. VIBRATION OF THIN PLATES

Vibrations of plates have been studied for many years. According to classical Kirchhoff theory, which neglects the shear effects and rotatory inertia, the free transversal vibration of the thin homogenous plate we can write as follows [1]:

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \rho h \frac{\partial^2 w}{\partial t^2} = 0\,, \tag{1}$$

where M_{xx}, M_{xy}, M_{yy} are the stress couples, x,y are orthogonal plate coordinates, h is thickness and ρ density of plate. Plate deflection is denominates as w. Taking into account the momentum equilibrium equations, we can transform (1) into the form

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = -\frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2}\,, \tag{2}$$

where

$$D = \frac{E h^3}{12\left(1-\mu^2\right)} \tag{3}$$

is the plate stiffness. Although no analytical solution exists for the case of a rectangular plate with free boundaries, several approximate methods have been proposed [4, 5]. Hurlebaus [6] derived the solution in the form of exact series. Gaul [7] proposes the simplified formulas for natural frequencies of isotropic and orthotropic square plate, which allow determining elastic parameters from experimental data. The procedure based on equivalent wavelength was described in [8]. Other way is based on the numerical modeling of vibration by computer. Modern software complexes based on finite element method is of great importance. They allow imitating a behavior of investigated object on base of its geometry, material properties, loads and other input data. There are a plenty of such software complexes, among them: Diana, COSMOS, MARC, ABAQUS, ISPA, SAMSEF, ANSYS, MSC/NASTRAN, ACELAN and others. The idea of method is to compute natural frequencies of vibrational modes and then to use minimizing procedure to fit them with experimental data.

3. ELECTRONIC SPECKLE PATTERN INTERFEROMETRY

Holographic interferometry is an optical method based on the interference of two optical wave fields – first one is scattered by object in primary state, the second one by object in load state [9,10]. Naturally, these fields cannot really exist in the same time. Hence, at least one of them is reconstructed by holographic way. It means that the optical wave scattered by object in primary state is recorded by classical holographic way (result of interference both object and reference beam is recorded on a hologram, than developed). Thereafter, hologram is placed on the same place, illuminated and the optical wave from primary state of object is reconstructed. This wave interferes with really exist wave scattered by object in load state. All this procedure is very complicated, time consumpting and requires high accuracy of hologram placing.

Because of that, the more convenient method based on the properties of laser speckles and digital processing of image was developed. Experimental setup is similar as for classical holography (laser source, beam splitter) but object and reference beam are directed not to the photo sensible plate, but are ducted to the CCD camera (Fig.1). Both object and reference beam, are recombined within the coherence length of the employed laser. The resultant is a speckle pattern formed from the coherent addition of the speckle pattern of the scattered object illumination and the reference beam [11,12].

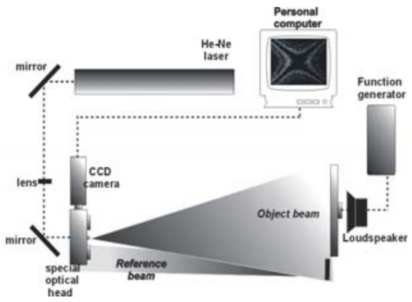
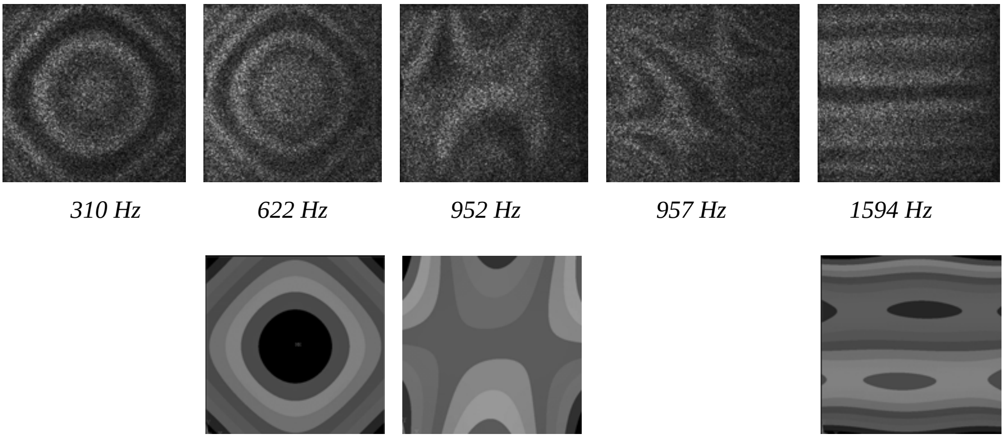


Figure 1. Experimental setup for ESPI measurements.

The intensity distribution of the speckle pattern is recorded by CCD camera. The properties of the interference pattern change with rigid body motions relative to the interferometric arrangement as well as with surface deformations. This is the variable optical path of the interferometer. Each recorded speckle image encodes in its speckle intensity distribution a single state of the object. The motions or displacements, respectively, between initial and displaced object state result in speckle intensity variations due to an interferometric phase change. Comparative analysis of the speckle intensities in subsequent interferograms yields information about the interferometric phase change of each speckle undergone from the first, initial object state to the second, i.e. displaced object state.

4. RESULTS AND DISCUSSION

The two types of material were investigated. The first was common ceramic plate, used as a tile. The second one was glass, used as plate glass. Both samples were the square plates with the size of the 200 x 200 mm. The type of modes and their natural frequencies strongly depend on boundary conditions. There are two extreme cases of boundary condition – free edges or clamped edges. We investigated the samples with free edges. This condition was assured with massive holder in which tested sample was pointed on the soft wires in all edges. A He-Ne laser with wavelength λ = 632,8 nm was used as the coherent light source for the ESPI equipment. Vibrations were excited by the loudspeaker feeded by sinusoidal signal of proper frequency.



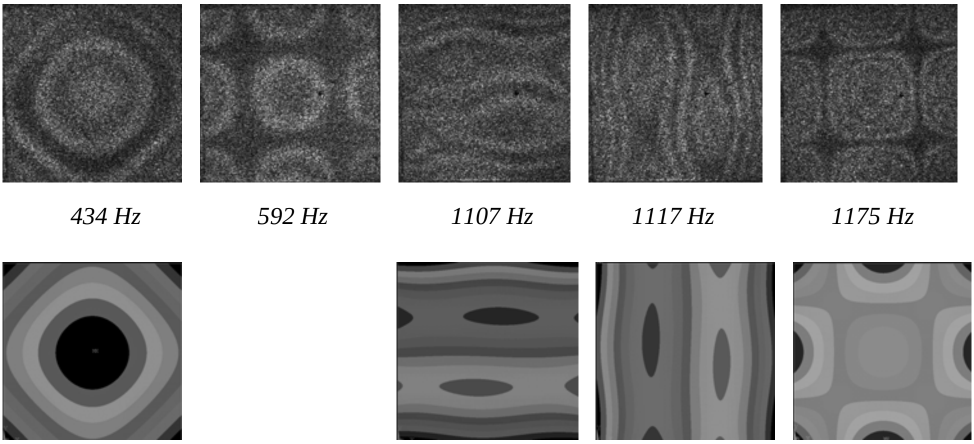
310 Hz *622 Hz* *952 Hz* *957 Hz* *1594 Hz*

-- *618 Hz* *948 Hz* -- *1604 Hz*

Figure 2. Experimentally obtained vibrational modes of the ceramics sample and its comparing with computer simulation by FEM for $E = 22$ GPa; $\mu = 0{,}2$; $\rho = 1760$ kgm^{-3}.

Thickness of ceramic sample was 6.3 mm, density 1760 kg/m^3. Experimental result are shown in the Fig. 2. Glass sample was made from table glass with density 2100 kg, plate thickness was 3,33 mm.

The sample was covered by thin layer of diffusive white paint for the ESPI measurements. Results are shown in the Fig. 3.



434 Hz *592 Hz* *1107 Hz* *1117 Hz* *1175 Hz*

424 Hz -- *1069 Hz* *1069 Hz* *1115 Hz*

Figure 3. Experimentally obtained vibrational modes of the glass sample and its comparing with computer simulation by FEM for $E = 40$ GPa; $\mu = 0{,}3$; $\rho = 2100$ kgm^{-3}.

The shape of modes obtained from ESPI measurements agree very good with computed ones Almost all measured modes was successfully identified with those obtained by computing. The coincidence is very good. Obtained values of natural frequencies can serve as the input data for optimising procedure, which will result to seeking material parameter – Young modulus E and Poisson ratio μ.

5. ACKNOWLEDGMENT

This work was partially supported by Slovak Science Grant Agency in framework of grant No. 1/0898/09. Authors would like to express his gratitude to VEGA for the support.

6. REFERENCES

[1] Krauthammer,T., Ventsel, E.: Thin plates and shells, Marcel Dekker Inc, New York, Basel, 2001.

[2] Mansfield, E.H.: The bending and stretching of plates, Cambridge Univ. Press, 1989.

[3] Hagedorn, P., DasGupta, A.: Vibrations and waves in continuous mechanical systems, John Wiley, 2007.

[4] Warburton, G.B.: The vibration of rectangular plates, Proc. Inst. Mechanical Engineers, 168,371-384, 1954

[5] Hearmon R.F.: The frequency of flexural vibration of rectangular orthotropic plates clambed or supported edges, J. Of. Appl. Mechanics, 26, 537/540, 1959.

[6] Hurlebaus, S., Gaul, L., Wang, J.T.: An exact series solution of for calculating the eigenfrequencies of orthotropic plates with completely free boundary. J. Sound and Vibration,244, 747-759, 2001.

[7] Gaul, L., Willner, K.,Hurlebaus, S.: Determination of material properties of plates from modal ESPI measurements, 17th international modal analysis conference , SPIE proceedings series, 1756-1762, 1999.

[8] Caldersmith, G.W.: Vibrations of orthotropic rectangular plates. Acoustica 56 ,144-152, 1984.

[9] Rastogi,P.K.(ed.):Digital Speckle Pattern Interferometry and Related Techniques, Wiley & Sons, 2001.

[10] Jones,R., Wykes,C.:'Holographic and Speckle Interferometry', Cambridge Studies in Modern Optics, Cambridge University Press, 1989.

[11] Slabeycius,J. et al: Testing of the Tyre Vibrational States bySpeckle Interferometry, 17th International Congress on Acoustics, Roma, 2001.

[12] Rusnáková, S. et al: ESPI - the tool for identity dynamic stiffness parameters of various composite materials,4th Youth symposium on Experimental solid mechanics, Bologna, 2005