13th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2009, Hammamet, Tunisia, 16-21 October 2009

THE THEORETICAL CONSIDERATION ABOUT THE MECHANICAL RESONANCES

Carmen Bal
Technical University of Cluj Napoca
15 Constantin Daicoviciu Street, Cluj Napoca
Romania

ABSTRACT
In this paper we present some theoretical notion about the mechanical resonances, and his characteristically size of it. Also we present relative dimension of the mechanical resonance mathematical demonstrated, when the periodical force that actions on the weight due to one series of the longitudinal waves, which circulate in the pipe and work with one piston attached to a mass.
Keywords: resonances**,** sonic pressure, sonic flow, sonic capacity, movement, energy.

1. GENERAL NOTION
If relatives dimensions of the spheres are chosen in the way that the natural periods of vibration of the weight (left free to oscillate, after the periodical force is no longer applied) coincides with the periods of the force applied, the formed system of table and springs is named resonator.

- Theoretically such a resonator is capable to increase the amplitude of the oscillator, when friction is not present, to the infinite.
- If the movement of the table is stopped by an obstacle by every impact, the amplitude of the movement is limited; the energy is dissipated in a number of hits, not being consumed thru friction. So, all percussion will produce an energy discharge. It can be seen that the energy of the hit is proportional with the weight and the square of the speed.
- Obviously the most power full hit is obtained when from its construction the resonator allows the maximum hit speed.
- Other condition necessary for the maximum pressure is that the springs should be designed in a way that the system will not be resonator for the private period of the line. These two conditions allow the calculation of the relative size of the springs from the resonator.
- If instead of realising the resonator for maximum pressure, we increase the amplitude of the resonator by removing the obstacle, the energy continuously decreases till its nullification, in case it is reached the amplitude on which every growth is continuously stopped by the internal friction of the resonator.
- If we reduce the amplitude under this value, the energy's pressure will decrease again until null, obviously in the moment the amplitude will reach zero, because the weight will then rest. If a periodical axial force is applied to a mass supported by two springs, it will start to oscillate under a certain law.

2. THEORETICAL CONSIDERATION ABOUT THE MECHANICAL RESONANCES
In the industrial applications of the resonators, when from a tool is demanded that the maximum of percussion or of percussive power with a minimum of weight, the condition above mentioned, of the optimum amplitude meets this request.
In practice there are cases when it is necessary to build a heavy hammer which will have high

amplitude or a long course, but will have light hits. The expected result can be obtained by putting the resonators mass closer to the maximum amplitude and not the optimum amplitude.
If the resonator discharges its energy trough consecutive hits, the curve represents the speed or the flow from the resonation's piston is not continuous, it's discontinuous, because in the moment of the hit the speed gets immediately from a finite value to zero. This discontinuity of the flow can be named as, *deformation* of the flow supplied to the resonator.
We can see that the deformation is bigger at the optimum amplitude and disappears when the maximum amplitude is reached, and in this case, the curve of the flow speed obtained is a perfect harmonic curve. We can see that the piston obtains a negligible deformation of the flow, in the resonator must be adapted an amplitude which near to the maximum amplitude
In case that from the main transmission line, small tools are operated, this deformations do not cause any difficulties, but in case of the heavy resonators (force hammers) the deformation of the flow causes disorders in the functionality of other tools, simultaneously auctioned trough the same transmission line.
Calculations regarding the discharge of energy in a pressure form can be applied also in the cases when the energy is discharged in any other matchlessly way, for example in case of a water pump with simple effect.
We consider a mass having a inertia coefficient L_s and a capacity C_s, placed in a liquid column which pulses under an instant sonotronic pressure p_s .In this case we can write:

$$p_{s_i} = L_s \cdot \frac{dQ_i}{dt} + \frac{1}{C_s} \cdot \int Q_i \cdot dt = L_s \cdot \frac{dQ_i}{dt} + \frac{S}{C_s} \cdot y \qquad (1)$$

Where: $Q_i dt = Sy$, and S is the cross-section of the resonator's piston and y represents the distance crossed by it (figure.1).
The instant sonotronic presure has an armonic evolution of the folowing form [4.19]:

$$p_{s_i} = p_m + p_{a_{max}} \cdot \sin(\omega t + \varphi) \qquad (2)$$

We consider $p_m = 0$ and the relation (2) becomes:

$$p_{s_i} = p_{a_{max}} \cdot \sin(\omega t + \varphi) \qquad (4.3)$$

We define (1) in relation with ωt, we obtain:

$$\left(p_{a_{max}} \cdot \sin(\omega t + \varphi)\right)' = \left(L_s \cdot \frac{dQ_i}{dt} + \frac{1}{C_s} \cdot \int Q_i \cdot dt\right)'$$

$$\omega \cdot p_{a_{max}} \cdot \cos(\omega t + \varphi) = \omega^2 \cdot L_s \cdot \frac{d^2 Q_i}{d(\omega t)^2} + \frac{Q_i}{C_s} \qquad \text{or}$$

$$\omega \cdot C_s \cdot p_{a_{max}} \cdot \cos(\omega t + \varphi) = \omega^2 \cdot L_s \cdot C_s \cdot \frac{d^2 Q_i}{d(\omega t)^2} + Q_i \qquad (4)$$

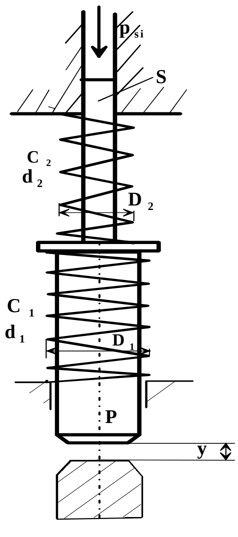

Fig. 1 Mechanical resonator with two arch

Because L_s şi C_s are in resonance with the relation, $\omega^2 \cdot L_s \cdot C_s = 1$, so the formula (4) becomes:

$$\frac{d^2 Q_i}{d(\omega t)^2} + Q_i = \omega \cdot C_s \cdot p_{a_{max}} \cdot \cos(\omega t + \varphi) \qquad (5)$$

We consider $Q_{i0} = 0$, when $\omega t = 0$, the solution of the differential equation is:

$$Q_i = A \cdot \sin \omega t + B \cdot \omega t \cdot \sin(\omega t + \varphi) \qquad (6)$$

Considering the time's origin (t = 0), the moment when $Q_i = 0$ şi y = 0, the relation (1) becomes:

$$p_{si_0} = L_s \cdot \left(\frac{dQ_i}{dt}\right)_0 = L_s \cdot \omega \cdot \left(\frac{dQ_i}{d\omega t}\right)_0 \qquad (7)$$

By differential (7) and replacing we obtain:

$$A = \frac{p_{a_{max}}}{2} \cdot C_s \cdot \omega \cdot \sin\varphi$$

$$B = p_{a_{max}} \cdot C_s \cdot \frac{\omega}{2} \tag{8}$$

Replacing the relations (8) in the relation (6) we obtain:

$$Q_i = \frac{\omega \cdot C_s \cdot p_{a_{max}}}{2} \cdot \left[\sin\omega t \cdot \sin\varphi + \omega t \cdot \sin(\omega t + \varphi)\right] \tag{9}$$

because $$S \cdot y = \frac{1}{\omega} \cdot \int Q_i \cdot dx \tag{10}$$

$$y = \frac{C_s \cdot p_{a_{max}}}{z \cdot S} \cdot \left[\sin\omega t \cdot \sin\varphi + \omega t \cdot \sin(\omega t + \varphi)\right] \tag{11}$$

From the relation (1), we can determine the energy stored in every moment, given by:

$$W = L_s \cdot \frac{Q_i^2}{2} + \frac{S}{C_s} \cdot \int Q_i \cdot y \cdot dt \tag{12}$$

but $\int Q_i \cdot dt = S \cdot y$, so the relation (12) becomes after replacement:

$$W = L_s \cdot \frac{Q_i^2}{2} + \frac{S^2 \cdot y^2}{2 \cdot C_s} + k \tag{13}$$

where *k* is a constant of integration.
Considering W = 0 when $Q_i = 0$ and y = 0, then:

$$W = L_s \cdot \frac{Q_i^2}{2} + \frac{S^2 \cdot y^2}{2 \cdot C_s} = \frac{1}{2} \cdot \left(L_s \cdot Q_i^2 + \frac{S^2 \cdot y^2}{C_s} \right) \tag{14}$$

replacing in the relation (4.14) pe Q_i and y, we obtain:

$$W = \frac{1}{2} \cdot \left\{ \frac{L_s \cdot \omega^2 \cdot C_s^2 \cdot p_{a_{max}}^2}{} \cdot \left[\sin\omega t \cdot \sin\varphi + \omega t \cdot \sin(\omega t + \varphi)\right] + \frac{S}{C_s} \cdot \frac{C_s^2 \cdot p_{a_{max}}^2}{4 \cdot S^2} \cdot \right.$$

$$\left. \cdot \left[\sin\omega t \cdot \cos\varphi - \omega t \cdot \cos(\omega t + \varphi)\right]^2 \right\}$$

Knowing that by resonance, $\omega^2 \cdot L_s \cdot C_s = 1$, we obtain:

$$W = \frac{1}{8} \cdot C_s \cdot p_{a_{max}}^2 \left[\omega^2 t^2 + \sin^2 \omega t - 2 \cdot \omega t \cdot \sin\omega t \cdot \cos(\omega t + 2 \cdot \varphi)\right] \tag{15}$$

Of which value is independent of φ in every moment given by ωt = kπ. As a result the energy absorbed by a resonator is proportional with the square of the time.The medium energy at the given moment will be:

$$W = \frac{1}{8} \cdot C_s \cdot \omega^2 \cdot p_{a_{max}}^2 \cdot t = \frac{p_{a_{max}}^2 \cdot t}{8 \cdot L_s} \tag{16}$$

We observe that in the resonator lies a battery with a very high efficiency.
Mechanical power in every moment is given by the relation:

$$P = \frac{dW}{dt} = \frac{p_{a_{max}}^2 \cdot t}{4 \cdot L_s} \tag{17}$$

If the considered moment is in a long time interval from the beginning of the resonator oscillations, the relations (10) and (11) can be simplified by omitting the first terms from the brackets.

$$Q_i = \frac{\omega \cdot C_s \cdot p_{a_{max}}}{2} \cdot \omega t \cdot \sin(\omega t + \varphi) \tag{18}$$

$$y = -\frac{C_s \cdot p_{a_{max}}}{z \cdot S} \cdot \omega t \cdot \cos(\omega t + \varphi) \tag{19}$$

The stored energy is given by (from the relation 16):

$$W = \frac{C_s \cdot p_{a_{max}}^2}{8} \cdot (\omega t)^2 \qquad (20)$$

If we write:

$$Q_i = Q_{a_{max}} \cdot \sin(\omega t + \varphi), \qquad (21)$$

From (21) we can express p_{amax}:

$$p_{a_{max}} = \frac{2 \cdot Q_{a_{max}}}{\omega \cdot C_s \cdot \omega t} \qquad (22)$$

By replacing (22) in the relation (20), after simplification we obtain:

$$W = \frac{C_s \cdot 4 \cdot Q_{a_{max}}^2 \cdot (\omega t)^2}{8 \cdot (\omega t)^2 \cdot \omega \cdot C_s^2} = \frac{Q_{a_{max}}^2}{2 \cdot \omega^2 \cdot C_s} \qquad (23)$$

From the resonance according to $\omega^2 \cdot L_s \cdot C_s = 1$, so $\omega^2 \cdot C_s = \frac{1}{L_s}$ witch replaced in the relations (23) we obtain:

$$W = \frac{Q_{a_{max}}^2 \cdot L_s}{2 \cdot} \qquad (24)$$

In conclusion the relation (24) shows that the stored energy from a resonator is equal with the maximum kinetic energy of the mass that forms the resonator. In the above analyse friction has been omitted from the circuit. If the resonator is used in relation with a machine that produces work or if friction is present in the circuit, the conditions are different. If the friction coefficient is C_f the sonotronic pressure is given by the relation:

$$p_i = C_f \cdot Q_i + L_s \cdot \frac{dQ_i}{dt} + \frac{1}{C_s} \cdot \int Q_i \cdot dt \qquad (25)$$

The solution of the equation will be:

$$Q_i = A \cdot e^{-\beta \cdot \omega t} \cdot \cos(\omega t + \psi) + \frac{p_{a_{max}}}{C_f} \cdot \sin \omega t$$

Conclusion*: The maximum utile efforts to one hammer who have one source, we can calculated the power*, $P \cong \frac{1}{2} \cdot p_{a_{max}} \cdot Q_{a_{max}}$, were $Q_{a_{max}} = r \cdot \omega \cdot S$, as good as the flow is the sinusoidal simple form and in phases with $p_{a_{max}}$. *This maximum efforts are obtained if the resonator is so construct, as together witch him condenser to be in resonance with the sonomotric pressure and to be in equilibrium in the course were are produce knocking, under action of the condenser resorts and to the medium of the pressure of line, consideration to static work.*

In this condition, were one resonator work with the optimal amplitude, produce a big deformation to the instantaneous flow. So the hammer must have a small dimension, so that to absorbed to the principal transmission line, a relative little flow.

2. REFERENCES

[1] Constantinescu, G., (1985), *Theory of the sonicity*, Bucureşti, Ed.Academiei.

[2] Carmen Bal, (2007)**,** *Caloric effect in the circuits by harmonic flow***,** Cluj Napoca, Ed. ALMA MATER.

[3] Carmen Bal, (2006), *Research and contributions about the drive systems with the harmonic flow*, The doctoral thesis Technical University of Cluj Napoca.

[4] Pop I. Ioan**,** Carmen Bal, (2007), *The sonicity applications.* Iaşi**,** Performantica.