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MATHEMATICAL MODEL OF DYNAMIC BEHAVIOR OF RIGID ROTOR INSIDE THE ROLLING ELEMENT BEARING

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ABSTRACT
Rolling bearings are one of the most important uproar and vibration generator into mechanical constructions. According to that, it is very important to know ascendancy of rolling bearing construction to main parameters which determine level of uproar and vibrations produced inside machines. This paper present new mathematical model of dynamic behavior of rigid rotor inside the rolling bearing. The new model is experimentally validated on a vibration test of rotor.
Keywords: Rolling Bearing, Dynamic behavior, Vibrations, Specific Frequency

1. INTRODUCTION
Requirements for uproar and vibration reduction at contemporary mechanical industry products are more and more uttered. Accomplishment of these requirements is triggered from one side by more and more claimed and refined market, and from the other side by string of positive lows and regulations, which lately are presetting increasing standards with respect of permitted level of uproar and vibrations generated by these products. One of main uproar and vibrations generator at mechanical constructions are built in rolling bearings. Partial attention is to be devoted to control and analysis of rolling bearings vibrations. Because of that, it is of particularly importance to know rolling bearing construction parameters ascendancy on uproar and vibration level these product generate. If our aim is to control vibrations or hand-over rotating moment with great concisness, we have to make effective mathematical model as a basis for anticipating of rotor dynamic behavior which is based on rolling bearings. Without effective mathematical model and without in detail analysis of bearings construction parameters impact to rotor dynamic behavior, it is very hard to resolve these problems. This paper present new mathematical model of dynamic behavior of rigid rotor inside the rolling bearing. The new model is experimentally validated on a vibration test of rotor.

2. ROLLING BEARING BALANCED RIGID ROTOR DYNAMIC MODEL
Very first researches of rigid rotor sleeve in rolling bearing moving was done by obavio je Krjuchkov in 1959. [4]. He deduced that centre of cross section trajectory of rolling bearing rotor has ideal geometry, with inner radial iradiance, so could be presented by absolute sinusoid equation, according to:

$$y = \Delta \cdot \left| \sin \frac{\pi \cdot s}{\lambda} \right| \tag{1}$$

Here is:

- Δ – top-top (pp) rotor oscilating amplitude [6];
- λ – line distance between bearing`s rolling bodies;
- s – curvilinear coordinate of bearing`s rolling bodies centres of gravity (Fig.1.).

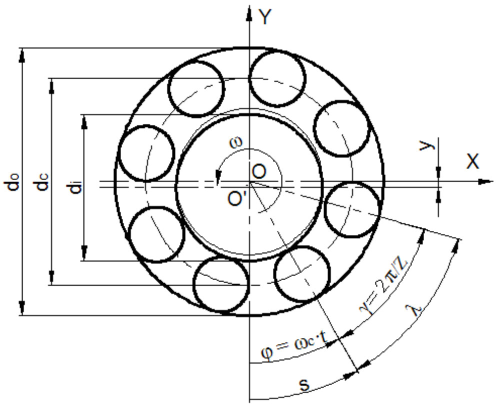

Figure 1. Schematic presentment of rolling bearing kinematics

The path of bearing`s rolling bodies centres of gravity is divisional axis, i.e., centres of rolling bodies are moving per divisional cirkumference of the cage. Divisional cirkumference diameter of the cage (dc) is equal to arithmetical middle between inner(di) and external rolling path diameter (do):

$$d_c = \frac{d_i + d_o}{2} \tag{2}$$

According to that, line distance between bearing's rolling bodies (λ) we could get if we divide length of divisional cirkumference of the cage with totally number of rolling bodies (z), according to equation:

$$\lambda = \frac{\pi \cdot d_c}{z} \tag{3}$$

Curvilinear coordinate of bearing`s rolling bodies centres of gravity (s) could be depicted as product of radius of divisional cirkumference of the cage and angular coordinate φ=φ(t) of rolling bodies centres of gravity (Fig.1.), according to:

$$s = r_c \varphi = \frac{1}{2} d_c \varphi \tag{4}$$

Angular coordinate (φ) of bearing rolling bodies centres of gravity could be depicted onto function of time (t) and raw speed of the cage (ωc), according to equation:

$$\varphi = \omega_c \cdot t = \frac{\omega}{2} \cdot \left(1 - \frac{d_b}{d_c} \cdot \cos\alpha\right) \cdot t \tag{5}$$

Here is:

- ω – rotor angular speed,
- db – diameter of rolling bodies
- α – working contact angle of bearing;
- t – time.

Expression for raw speed of the cage ωc is managed within literary work under ordinal number [7]. Now expression (4) for curvilinear coordinate (s) could be written as:

$$s = \frac{1}{2} \omega_c \cdot d_c \cdot t = \frac{\omega}{2} \left(1 - \frac{d_b}{d_c} \cdot \cos\alpha\right) \cdot \frac{d_c \cdot t}{2} \tag{6}$$

If we expression (6) for curvilinear coordinate (s) and expression (3) for line distance between bearing`s rolling bodies (λ) get in equation (1) we get:

$$y = \Delta \left| \sin\left(\frac{z}{2} \cdot \omega_c \cdot t\right) \right| = \Delta \left| \sin\left[\frac{z}{2} \cdot \frac{\omega}{2} \cdot \left(1 - \frac{d_b}{d_c} \cdot \cos\alpha\right) \cdot t\right] \right| \tag{7}$$

With relation to:

$$y = \Delta \left| \sin\left(\frac{f_o}{2} \cdot t \right) \right| \tag{8}$$

Where is:

$$f_o = \frac{z}{2} \cdot \omega \cdot \left(1 - \frac{d_b}{d_c} \cdot \cos\alpha \right) \tag{9}$$

Dimension fo in expression (9) is called characteriyed frequency of external bearing ring and it is speed which rolling bodies have while going through some fixed point on external bearing ring. This frequency we often call frequency of rotor rolling across bearing rolling bodies [7].

According to that, rotor cross section centre, with relation to bearing housing orifice centre, continously oscilate during bearing works. These oscilations could be depicted with equation of abolute sinusoid (8). According to equation (8), balanced rigid rotor oscilating frequency in rolling bearing id equal to bearing external ring specific frequency. Dimension pp-amplitude of these vibrations is directly dependent on dimension of bearing`s rings relative moving. Relative moving of bearing`s rings is direct indices of dimension of radial working iradinace of rolling bearing. According to that, vibration on specific frequency of external bearing`s ring and its i njenim harmonics (k·fo), are the best indicator of radial working iradinace dimension and could be very important diagnostic parameter of rolling bearing working capacity and working correctness. In vibrations analysis technic, specific frequencies of rolling bearing vibrations are used for analysis of vibrations spectra for detection of damages locations and various irregularity on bearing.

3. RIGID ROTOR IN ROLLING BEARING VIBRATION EXPERIMENTALLY RESEARCH

In order to verify developed mathematical model, experimentally recording of rotor relative vibrations relatively to outer bearing ring is done, as well as frequency analisyis of vibrations spectra, for rolling bearing 6008. Vibrations measurement is done using the device for rolling bearings dynamic research, which is developed at the Faculty of Mechanical Engineering in Podgorica. The layout of device is showed at the picture *Figure 2.*

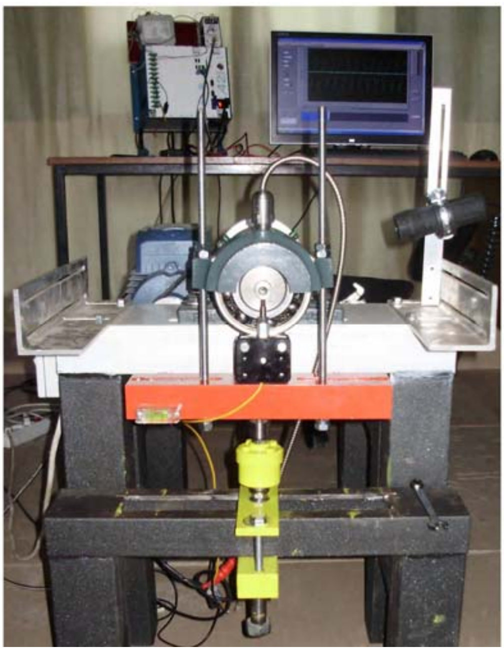

Figure 2. Device for rolling bearings dynamic research

Frequency layout of vibratory response is showed at *Figure 3*. Frequency analysis is done using MATLAB software. In the table at picture *Figure 3*. characteristic parameters of bearing construction which are needed for estimating of specific vibrations frequencies are given, and after that, values of them. Values of specific vibrations frequencies are got according to equation (9).

On the frequency diagram high values of vibratins amplitude on cyclical frequency of rotor rotating and its harmonics could be recognized, as well as on specific frequency of outer bearing ring. High level of vibrations of rotor rotating is consequence of residuary imbalance in system, while vibrations on the outer bearing ring are consequence of specific construction and rolling bearing working manner. These vibrations are called structural vibrations of rolling bearing. It couldn`t be evaded, and are present at every rolling bearing, even at the ideal construction rolling bearings. Structural vibrations could be major reduced only by radial iradiance reducing or by combination of outer radial load and bearing radial iradiance.

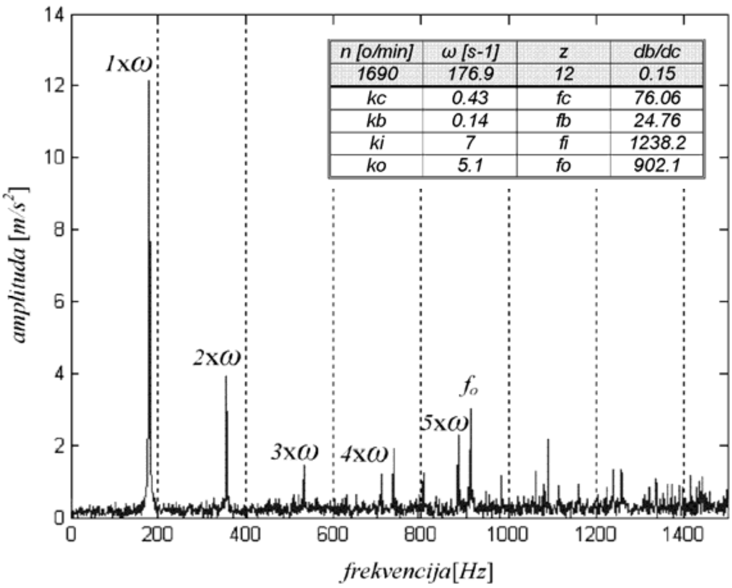

Figure 3. Frequently diagram of vibration acceleration of rolling bearing 6008

6. CONCLUSION

Balanced rigid rotor based on rolling bearings cross section centre coninuosly oscilate during working of bearing. These oscillations is possible to show using equation of absolute sinusoid, which is managed in this work. Frequency of these oscilations is equal to specific vibrations frequency of external bearing ring, i.e. to frequency of transistion of rolling bodies across some fixed point at the external bearing ring. Vibration on specific frequency of external bearing`s ring and its i njenim harmonics (k·fo), are the best indicator of radial working iradinace dimension and could be very important diagnostic parameter of rolling bearing working capacity and working correctness.

7. REFERENCES

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