

## THE APPLICATION OF HILBERT-HUANG TRANSFORM TO NON-STATIONARY ENVIRONMENTAL DATA SETS

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### ABSTRACT

*The paper devotes analysis of environmental data sets by using Hilbert-Huang transform (HHT). The environmental data were measured by meteorological stations which are deployed in the southern part of Czech Republic. The analysis is performed in both the time and frequency domains. The aim of the work is to investigate selected measured non-stationary time series by using the HHT method and also provide more physically meaningful interpretation of underlying non-linear dynamic systems. The results can help better describe dynamic characteristic of ecosystems.*

**Keywords:** environmental series, HHT, non-stationarity

### 1. INTRODUCTION

The status of each natural ecosystem in terms of biodiversity and stability is directly dependent on two factors. One is the energy balance, including incoming and outgoing flows of energy, the other is the water balance (hydrological). Monitoring and examination of ecosystems allows us to describe the link between directly and indirectly measured values as well as landscape elements. Observed ecosystems are examples of complex dynamical systems with distributed parameters which have a number of variables that are in interaction [1]. One can find out the links among the parameters by data analysis. Unfortunately, environmental time series represent mostly non-stationary non-linear stochastic processes. The available data analysis methods are for linear but non-stationary processes or for nonlinear but stationary and statistically deterministic processes [2, 3]. The Hilbert-Huang transform (HHT) is a new method, which was developed by N.E.Huang et al. [3] for the analysis of non-stationary and non-linear signals. This approach was tested on environmental data which were measured by meteorological stations (see Figure 1) which are deployed in the southern part of Czech Republic around the city of Třeboň. Various sensors read meteorological variables such as temperature, humidity, wind speed /direction, radiation, etc. Matlab programming environment was used for data processing.



*Figure 1. Meteorological station*

### 2. HILBERT- HUANG TRANSFORM

The HHT consists of two parts, the empirical mode decomposition (EMD) followed by Hilbert spectral analysis. The EMD algorithm decomposes the signal into components, called intrinsic mode functions (IMF) to which the Hilbert analysis can be applied. An IMF is defined as any function

having the same (or differing at most by one) numbers of zero-crossing and extrema, and also having symmetric envelopes defined by the local maxima and minima, respectively. The decomposition method and the Hilbert analysis are described e.g. in [2, 3]. Essentially, the EMD method is a sifting process. The sifting process can be stopped, when the residue  $r$  becomes a monotonic function from which no more IMF can be extracted. It holds

$$x(t) = \sum_{i=1}^n c_i(t) + r(t), \quad (1)$$

where  $c_i, i=1, 2, \dots, n$  are IMFs,  $r(t)$  is the residue at the time  $t$  and  $x(t)$  is the original signal at the time  $t$ .

For an arbitrary IMF  $c_i(t)$  the Hilbert transform is defined by the principal value (PV) integral

$$H\{c_i(t)\} = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{c_i(\tau)}{t-\tau} d\tau \quad (2)$$

This yields an analytical signal

$$z_i(t) = c_i(t) + jH\{c_i(t)\} = a_i(t)e^{j\phi_i(t)} \quad (3)$$

in which

$$a_i(t) = \sqrt{c_i^2(t) + (H\{c_i(t)\})^2}, \quad (4)$$

$$\phi_i(t) = \arctan\left(\frac{H\{c_i(t)\}}{c_i(t)}\right). \quad (5)$$

From (5) the instantaneous frequency  $\omega_i$  can be computed by

$$\omega_i(t) = \frac{d\phi_i(t)}{dt}. \quad (6)$$

After performing the Hilbert transform to each IMF component, the original signal  $x(t)$  can be expressed as the real part (RP) in the following form

$$x(t) = RP \sum_{i=1}^n a_i(t)e^{j\phi_i(t)} + r(t). \quad (7)$$

### 3. APPLICATION TO NON-STATIONARY ENVIRONMENTAL DATA SETS

The main objective of the research solved in Faculty of Mechanical Engineering, Czech Technical University in Prague under Project TOKENELEK consists in quantification of the main energy fluxes in the landscape (IR balance, sensible heat flux and evapotranspiration) on the basis of environmental data which were measured by meteorological stations. The research also contributes to the assessment of the role of ecosystems in the countryside and of the impact of human action on the landscape.

But the ecosystems are both nonlinear and nonstationary. Therefore traditional analysis techniques can sometimes lead to misinterpretations of the characteristics of the data. As the HHT is applicable for nonlinear and nonstationary analysis it has been tested for observed environmental data.

The following quantities were monitored and recorded in 10-minute intervals: air temperature and humidity, incident and reflected global solar, soil humidity, precipitation, the direction and speed of wind, 10 values for soil temperature, atmospheric pressure and incident and emitted radiation in the IR region. The example of the decomposition of the air surface temperature  $T_z$  [°C] by the EMD algorithm is shown in Figure 2. The variances of the IMFs  $c1, c2, \dots, c11$  are compared in Figure 3. The IMFs  $c3, c4, c5, c6, c7, c8$  are more important components of the original signal  $T_z$  than the IMFs  $c1, c2, c9, c10, c11$ . The signal  $T_f$  composed according to

$$T_f = c3 + c4 + c5 + c6 + c7 + r, \quad (8)$$

includes only the major components. Figure 4 shows the original signal  $T_z$  and the filtered signal  $T_f$ . The instantaneous frequency for the IMF  $c4$  is depicted in Figure 5.

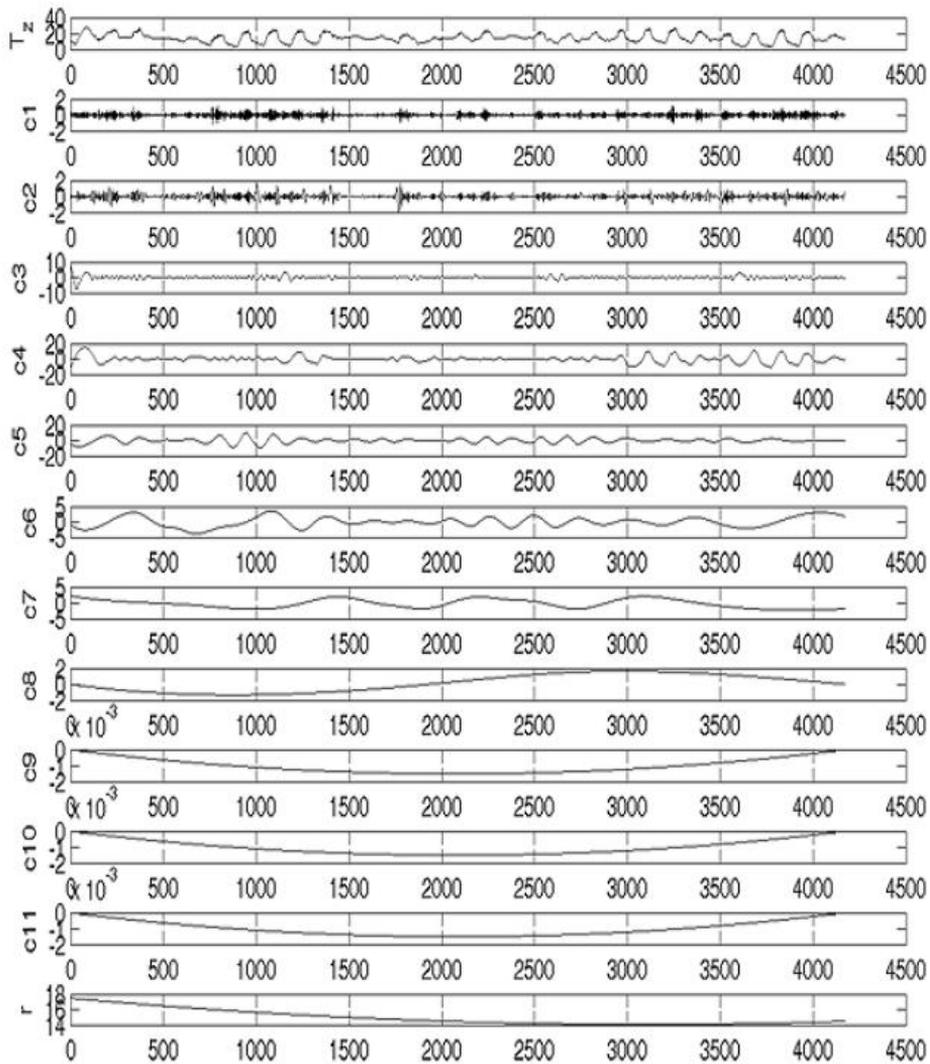


Figure 2. The empirical mode decomposition of the air surface temperature  $T_z$  [ $^{\circ}\text{C}$ ] into the IMFs  $c_1, c_2, \dots, c_{11}$  [ $^{\circ}\text{C}$ ] and the residua  $r$  [ $^{\circ}\text{C}$ ]. 4171 samples (10 min sampling interval).

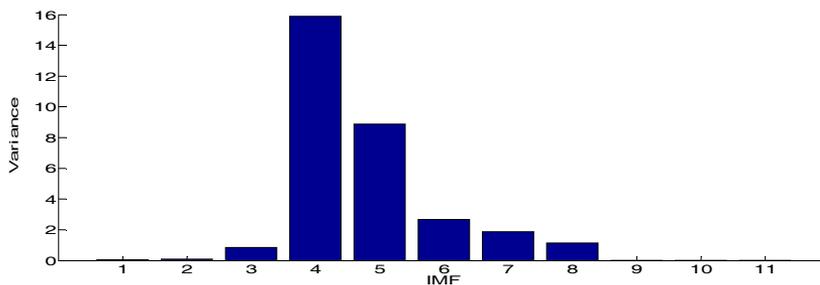


Figure 3. Variances of the IMFs  $c_1, c_2, \dots, c_{11}$

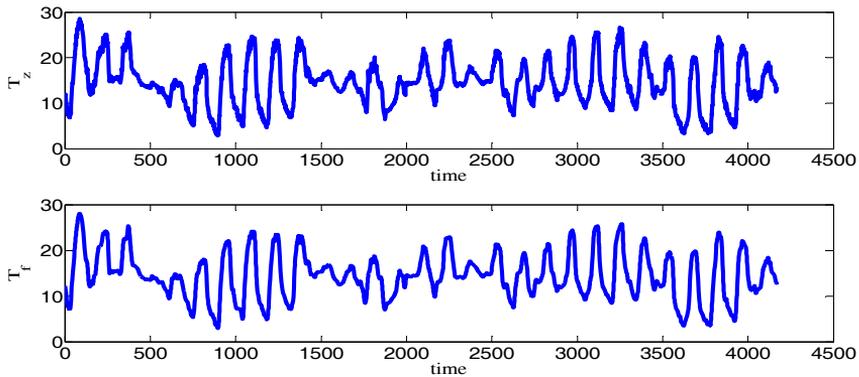


Figure 4. The courses of the signals  $T_z, T_f$

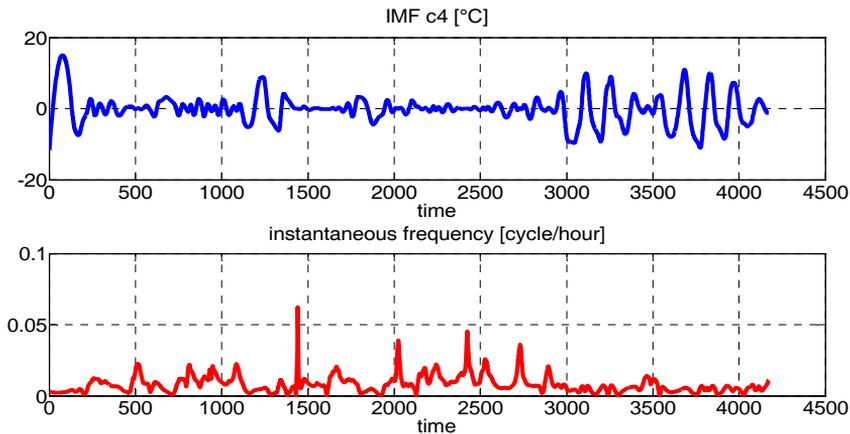


Figure 5. The courses of the IMF c4 and its instantaneous frequency

#### 4. CONCLUSION

The preliminary results show that the HHT has good performance and potential for analyzing environmental data sets without removing trend or make additional assumptions. The EMD algorithm can be treated as a time-frequency filtering method. The low-pass, high-pass and band-pass filters can be designed from IMF components. The next study will be focused on identification, prediction, simulation and fault detection using this technique for solution of environmental problems.

#### ACKNOWLEDGEMENTS

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