

## SIMPLE METHOD FOR LOWPASS NARROWBAND FIR FILTER DESIGN

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### ABSTRACT

*This paper presents simple method for the narrowband low-pass FIR (Finite Impulse Response) filter design. The method is based on the IFIR (Interpolated FIR) structure where the model filter is the ST (Stepped Triangular) impulse response filter cascaded with the simple compensator.*

**Keywords:** FIR filter, ST filter, IFIR structure.

### 1. INTRODUCTION

Digital signal processing (DSP) is an area of science and engineering that has been rapidly developed over the past years [1]. This rapid development is a result of significant advances in digital computers technology and integrated circuits fabrication [1]. The typical operation in DSP is filtering. The function of the digital filter is to process a given input sequence  $x(n)$  to generate an output sequence  $y(n)$  with the desired characteristics. If the input signal  $x(n)$  is the unit sample sequence  $d(n)$ , the output signal would be the characteristic of the filter and is called unit sample response  $h(n)$ . The relation between the input sequence  $x(n)$  and the output sequence  $y(n)$  in the time domain is given by the well known convolution relation

$$y(n) = x(n) * h(n), \quad (1)$$

where  $*$  means the convolution. The corresponding relation in the frequency domain is

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}), \quad (2)$$

where  $Y(e^{j\omega})$ ,  $X(e^{j\omega})$ , and  $H(e^{j\omega})$ , are the corresponding Fourier transforms of the  $y(n)$ ,  $x(n)$ , and  $h(n)$ , respectively, and  $H(e^{j\omega})$  is the transfer function of the filter. Using z-transform we have the relation

$$Y(z) = X(z)H(z), \quad (3)$$

where  $Y(z)$ ,  $X(z)$  and  $H(z)$  are the corresponding z-transforms of  $y(n)$ ,  $x(n)$ , and  $h(n)$ , respectively. Depending of the length of  $h(n)$  the filters can be either FIR (Finite impulse response filters) or IIR (Infinite impulse response filters). FIR filters are often preferred to IIR because they have some very desirable properties such as linear phase, stability, and absence of limit cycle. The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response, [1]. FIR filters of length  $N$  require  $(N+1)/2$  multipliers if  $N$  is odd and  $N/2$  multipliers if  $N$  is even,  $N-1$  adders and  $N-1$  delays. The complexity of the implementation

increases with the increase in the number of multipliers. Over the past years there have been a number of attempts to reduce the number of multipliers like [2]-[6].

One of the most difficult problems in digital filtering is the design of narrowband filters, [1]. The difficulty lies in the fact that such filters require high-order designs in order to meet the desired specification. In turn, high order filters require a large amount of computation and so are difficult to implement.

We consider the design of lowpass narrowband FIR filters with cutoff frequencies considerably lower than the sampling rate. One efficient technique for the design of FIR filters is called the interpolation FIR (IFIR) technique [2]. We propose the ST (Stepped Triangular) [3] impulse response model filter cascaded with the simple compensator in an IFIR structure.

The rest of the paper is organized in the following way. Next section describes the ST impulse response filter, the compensator, and IFIR structure. The proposed method is described in Section 3 and illustrated with one example.

## 2. ST IMPULSE RESPONSE FILTER AND IFIR STRUCTURE

We review the ST (stepped-triangular) impulse response filter from [3]. The transfer function  $H_{ST}(z)$  for a given stepped triangular impulse response sequence  $h_{ST}(n)$  with  $N_2$  levels, and  $N_1$  samples at each level, can be presented as, [3]

$$H_{ST}(z) = \left[ \frac{1}{N_1 N_2} \sum_{i=0}^{N_1 N_2 - 1} z^{-i} \right]^K \left[ \frac{1}{N_2} \sum_{k=0}^{N_2 - 1} z^{-N_1 k} \right]^K. \quad (4)$$

Considering  $M=N_1 N_2$  from (3) we have

$$H_{ST_i}(z) = \left[ \frac{1}{M} \sum_{i=0}^{M-1} z^{-i} \right]^K \left[ \frac{1}{N_{2i}} \sum_{k=0}^{N_{2i}-1} z^{-N_{1i} k} \right]^K; i = 1, \dots, I. \quad (5)$$

where  $I$  is an integer.

**Example 1:** Consider  $M=8$  yielding in  $I=2$ . Therefore we have:  $i=1: N_1=2, N_2=4$ ;  $i=2: N_1=4, N_2=2$ . Figures 2a and 2b show the impulse responses for  $i=1$  and 2, respectively. Figure 2c shows the corresponding magnitude responses.

Note that all ST filters have the same zeros in the magnitude responses, and that all ST filters provide a high attenuation. However, there is an increase in the passband droop. Consequently ST filter needs the compensator filter.

We adopt the simple  $2M$ -order compensation filter introduced in [4]

$$G(z^M) = A[1 + Bz^{-M} + z^{-2M}], \quad (6)$$

where

$$A = -2^{-(b+2)}; B = -(2^{b+2} + 2), \quad (7)$$

and  $b$  is an integer. The compensator filter has only one coefficient  $B$  and the scaling factor  $A$ . Both can be realized using additions and shifts. Therefore, the compensation filter is a multiplierless filter.

The magnitude response is

$$\left| G(e^{jM\omega}) \right| = \left| 1 + 2^{-b} \sin^2(\omega M / 2) \right|. \quad (8)$$

The parameter  $b$  depends on the parameter  $K$  of the CIC filter. More details can be found in [4].

The basic idea of an IFIR structure is to implement a FIR filter as a cascade of two lower order FIR blocks. One section is the expanded model filter  $G(z^M)$  and another is the interpolator  $I(z)$ . The filter  $G(z^M)$ , where  $M$  is the interpolation factor, is obtained by introducing  $M-1$  zeros between each sample of the unit sample response of  $G(z)$ . The function of the interpolator filter  $I(z)$  is to eliminate images introduced by  $G(z^M)$ . More details can be found in [2].

## 3. PROPOSED METHOD

We propose that the compensated ST filter is the expanded model filter in and IFIR structure. In this design

$$\omega_p \leq \omega_s / 3, \quad (9)$$

where  $\omega_p$  and  $\omega_s$  are the passband and stopband frequencies. The design is described in the following steps:

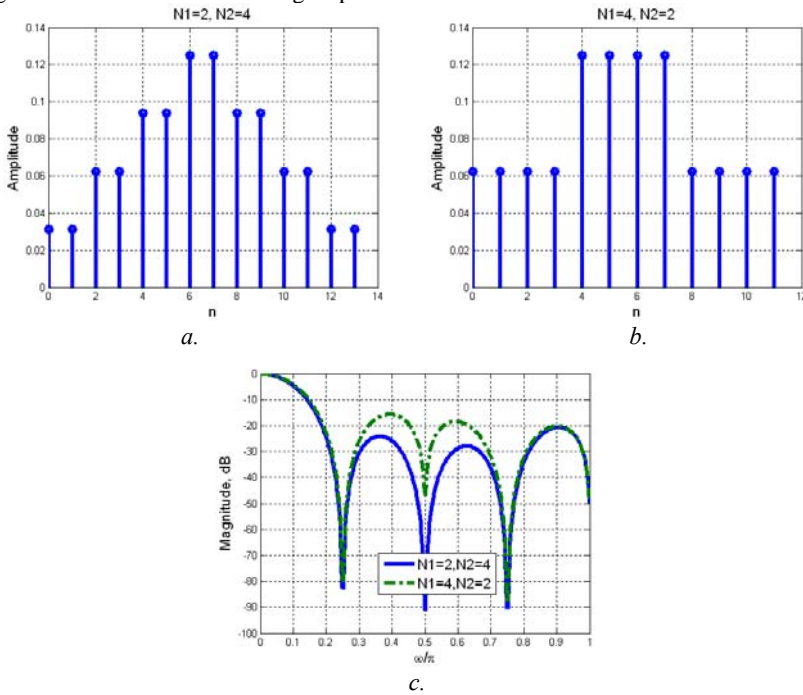


Figure 1. ST filters.

Step 1:

The value of  $M$  is chosen as,

$$M = \left\lceil \frac{3}{2\omega_s} \right\rceil, \quad (10)$$

and the maximum value of  $N$  is chosen for given  $M$ .

Step 2:

The ST filter is cascaded with the compensator filter using  $b=1$ .

Step 3:

The model filter is

$$H_m(z) = [H_{ST}(z)G(z^M)]^K, \quad (11)$$

where  $K$  is chosen to obtain the desired attenuation in the side lobes except in the last one.

Step 4:

The interpolator filter is designed with the passband frequency  $\omega_p$ , and the stopband frequency  $\omega_{sl}=\omega_1-\omega_p$ , where  $\omega_1$  is the frequency where the maximum value of the last sidelobe occur.

The method is illustrated in the following example.

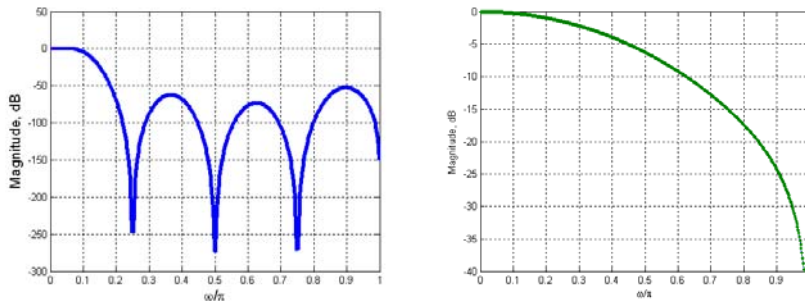
### Example 2.

We design the lowpass filter with the  $\omega_s=0.25$ . The passband frequency is  $\omega_p=\omega_s/3$ . The passband ripple is 1dB and the attenuation is 60 dB. The corresponding filter designed with ParksMcIclan algorithm has an order 31 and requires 16 multipliers.

We have  $M=8$ ,  $N_1=2$ , and  $K=3$ . Figure 2a shows the magnitude response of the cascaded ST filter and the compensator. Note that the last sidelobe has a low attenuation.

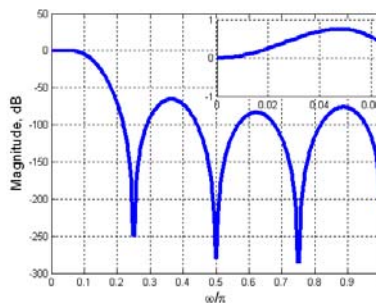
We design interpolation filter with  $\omega_p=.25/4$  and  $\omega_s=.9-\omega_p=0.85$ ,  $R_p=.01$ dB and  $A=40$ dB. The corresponding filter has an order  $N=3$ . The impulse response is rounded with the  $r=2^{-6}$ . Figure 2b shows the magnitude response of the rounded interpolator. Coefficients of the filter are [5, 27, 27, 5].

Finally the model filter is cascaded with the rounded interpolator. The corresponding resulting magnitude response is shown in Fig. 2c, along with the passband zoom.



a. ST filter and compensator.

b. Rounded Interpolator.



c. Resulting filter: Overall magnitude response and the passband zoom.

Figure 2 . Illustrating Example 2.

#### 4. CONCLUDING REMARKS

We present novel method for multiplierless design of a lowpass FIR filters based on ST filters and the IFIR structure. The expanded model filter is the cascade of the ST filter and the simple compensator, while the interpolator is a low order filter which attenuates a highest side lobe of the ST filter in order to satisfy the stopband requirements. The designed filters must satisfy the requirement that the passband frequency  $\omega_p$ , and the stopband frequency  $\omega_s$  are in the following relation,  $\omega_p = \omega_s/3$ .

#### ACKNOWLEDGEMENT

This work is supported by CONACYT Mexico.

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