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MATHEMATICAL MODEL FOR DETERMINATION OF ELASTIC SPRINGBACK OF INTERNAL WALL AFTER RIFLING USING METHOD OF COLD FORMING BASED ON FACTOR PLAN 2^3

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ABSTRACT
A mathematical model was developed for determining the elastic springback of internal wall of the sample after rifling, using method of cold forming by means of rifling tools, based on experimental measurements for the three variable influential parameter (the influence of groove relative depth, the geometry of the sample cross-section, and material type) with minimum and maximum values according to factor plan 2^3 and on the basis of regression analysis.
Key words: Elastic springback, experimental measurements, influential parameters, factor plan 2^3.

1. MATHEMATICAL MODEL FOR DETERMINATION OF ELASTIC SPRINGBACK OF INTERNAL WALL AFTER RIFLING USING METHOD OF COLD FORMING BASED ON FACTOR PLAN 2^3

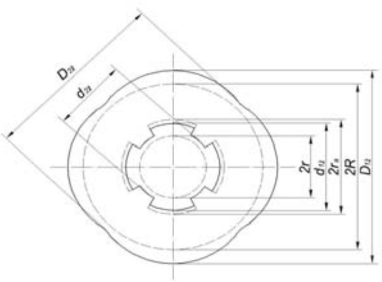

Figure 1. Sample after rifling by tool drawing

The three influential factors are investigated in factor experiment 2^3 on two levels, and for these cases the plan matrix was set up. The models for elastic springback are set in this paper, with the matrix of factors: the influence of groove relative depth, sample geometry, and material type. Experimenatal research are carried out with triple repetition of the experiment for the same combination of factor levels, which enables the evaluation of the experimental errors, and values of elastic springback are obtained, (Tables 1 and 2), and average values were used.

2. MATHEMATICAL MODEL FOR DETERMINATION OF ELASTIC SPRINGBACK $\Delta r_1 = \Delta r_1(r_a/r; R/r; k_{sr}/G)$

Model for determining the elastic springback is set in the following form:

$$\Delta r_1 = \Delta r_1(r_a/r;\ R/r;\ k_{sr}/G)$$

$$\Delta r_1 = \Delta r_1(X_1;X_2;X_3)$$

Where:

- r_a/r – Ratio between the tool radius and radius of the sample aperture, the influence of groove relative depth,
- R/r - Ratio between external and internal radius of the sample, the cross-sectional geometry of the sample,
- k_{sr} - Specific deformation resistance of the material and

G - Shear modulus of the material.
X_1 - Influence factor of the relative depth of groove
$X_{1min} = (r_a/r)_{min}$, $X_{1max} = (r_a/r)_{max}$
X_2 - Influence factor of the geometry of sample cross-section
$X_{2min} = (R/r)_{min}$, $X_{2max} = (R/r)_{max}$
X_3 - Influence factor of the type of material
$X_{3min} = (k_{sr}/G)_{min}$ *for C10E*, $X_{3max} = (k_{sr}/G)_{max}$ *for AlCu5*

As complete three-factor orthogonal plan provides, in addition to the main effects of factors, the assessment and effects of mutual effects of the first and second order, it was chosen for mathematical model describing the process:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{123}x_1x_2x_3 \tag{1}$$

Test results are listed in Table 1. Encoding of factors was derived through the transformation equations:

$$x_i = \frac{X_i - X_{oi}}{w_i}; \quad x_1 = \frac{X_1 - 1{,}080}{0{,}050}; \quad x_2 = \frac{X_2 - 2{,}600}{0{,}380}; \quad x_3 = \frac{X_3 - 245{,}0}{55{,}0}$$

Mathematical model describing the process is in the following form:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{123}x_1x_2x_3 \tag{2}$$

Table 1. Matrix plan with experimental results

Plan points	PLAN - MATRIX								EXPERIMENTAL RESULTS			
	x_0	x_1	x_2	x_3	x_1x_2	x_1x_3	x_2x_3	$x_1x_2x_3$	y_1	y_2	y_3	$\bar{y}_i$
1	1	-1	-1	-1	1	1	1	-1	8,0	8,0	10,0	8,667
2	1	1	-1	-1	-1	-1	1	1	16,0	17,0	16,0	16,333
3	1	-1	1	-1	-1	1	-1	1	10,0	9,0	10,0	9,667
4	1	1	1	-1	1	-1	-1	-1	22,0	20,0	22,0	21,333
5	1	-1	-1	1	1	-1	-1	1	12,0	10,0	13,0	11,667
6	1	1	-1	1	-1	1	-1	-1	26,0	27,0	26,0	26,333
7	1	-1	1	1	-1	-1	1	-1	14,0	13,0	15,0	14,000
8	1	1	1	1	1	1	1	1	34,0	37,0	34,0	35,000
Basic level		1,080	2,600	245,0								
Inter.variat.		0,050	0,380	55,0								
Upper level		1,130	2,980	300,0								
Lower level		1,030	2,220	190,0								

2.1. Calculating model parameters

Method of multi-factor orthogonal plans simplifies the model equations [1,2,3,4,5], of diffuse systems. The simple form of the equation parameters of response functions (multiple regression coefficients) is obtained using the properties of orthogonal plans, according to which the correlation matrix transforms into the diagonal matrix.
The values of the model parameters are:

$$b_i = \frac{1}{N}\sum_{u=1}^{N} x_{iu}\overline{y_u}, \qquad i = 0,1,2,\ldots,N; \quad N = 2^k \tag{3}$$

therefore, calculating with mean values of equation:

b 0 = 17,875; *b1 = 6,8725;* *b2 = 2,125;* *b3 = 3,875;*
b12 = 1,2925; *b13 = 2,0425;* *b23 = 0,625;* *b123 = 0,2925*

and model in coded coordinates is:

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{12} x_1x_2 + b_{13} x_1x_3 + b_{23} x_2x_3 + b_{123} x_1x_2x_3$$

$$\hat{y} = 17{,}875 + 6{,}8725\, x_1 + 2{,}125\, x_2 + 3{,}875\, x_3 + 1{,}2925\, x_1x_2 + 2{,}0425\, x_1x_3 + 0{,}625\, x_2x_3 + 0{,}2925\, x_1x_2x_3 \tag{4}$$

Transformation into natural coordinates $y = f(X_1, X_2, X_3)$ is performed using abovementioned transformation equations.

2.2. Checking model significance

To calculate the error of the experiment, according to the calculations from the table (1), for the output values obtained, the sum of squares equation at the same repetition is:

$$s^2(y) = s_E^2 = 10/16 = 0,624875; \quad f_E = N(m-1) = 8(3-1) = 16, \tag{5}$$

System of repetition of experiments in this example is the orthogonal plan and dispersion of model parameters is done according to the equation [1,2,3,4,5]:

$$s^2(b_i) = \frac{s^2(y)}{Nn} = \frac{s_E^{\,2}}{Nn} = \frac{0,624875}{8 \cdot 3} = 0,02603 \text{ or } s(b)_i = 0,161358 \tag{6}$$

Table 2. Experimental data

Plan	EXPERIMENTAL REZULTS								
points	y1	y2	y3	$\bar{y}$	si2	ŷi	(yi-ŷ)2	$\Sigma y_i^2/(n-1)$	$(\Sigma y_i)^2/(n*(n-1))$
1	8,0	8,0	10,0	8,667	1,333	8,701	0,001	114,0	112,667
2	16,0	17,0	16,0	16,333	0,333	16,233	0,010	400,5	400,167
3	10,0	9,0	10,0	9.667	0,333	9,787	0,014	140,5	140,167
4	22,0	20,0	22,0	21,333	1,333	21,455	0,015	684,0	682,667
5	12,0	10,0	13,0	11,667	2,333	11,785	0,014	206,5	204,167
6	26,0	27,0	26,0	26,333	0,333	26,213	0,014	1040,5	1040,167
7	14,0	13,0	15,0	14,000	1,000	14,115	0,013	295,0	294,000
8	34,0	37,0	34,0	35,000	3,000	34,875	0,016	1840,5	1837,5
				17,875	10,000		0,098		

Rating significance of any first order model parameter is performed independently of the others. The aim of evaluation is to test the null hypothesis $b_i=0$. This means that the group of non-signifficant parameters can be excluded from the model, without correcting values of other signifficant parameters that remain in the model.

According to t-criterion, with degrees of freedom $f_E = N(m-1) = 8(3-1) = 16$, we obtain the calculated values t_r for t-criterion from equation: $t_{ri} = \frac{|b_i|}{s(b_i)}$

$$t_{r0} = \frac{|b_i|}{s(b_0)} = \frac{17,875}{0,1623} = 110,778; \quad t_{r12} = \frac{|b_{12}|}{s(b_{12})} = \frac{1,2925}{0,1623} = 8,01; \quad t_{123} = \frac{|b_{123}|}{s(b_{123})} = \frac{0,295}{0,1625} = 1,8127$$

and similarly:

$t_{r1} = 42,5916; \; t_{r2} = 13,1694; \; t_{r3} = 24,0149; \; t_{r13} = 12,6581; \; t_{r23} = 3,87337.$

For degrees of freedom $f_E = N(n-1) = 16$ (using t-criterion for assesment of signifficance of model parameters we take the number of degrees of freedom (f_E) with which dispersion of experiment $s^2(y) = (s^2_E)$, is determined and assumed signifficance level $\alpha = 5\%$ will be equal to the nominal value $t_t = 1,75$, and according to the signifficance condition $t_{ri} > t_t$ parameters: $b_0, b_1, b_2, b_3, b_{12}, b_{13},$ *and* b_{23} *and* b_{123} are signifficant, and model finally becomes:

$$\hat{y} = 17,875 + 6,8725\,x1 + 2,125\,x2 + 3,875\,x3 + 1,2925\,x1x2 + 2,0425\,x1x3 + 0,625\,x2x3 + 0,2925\,x1x2x3 \tag{7}$$

or, in natural coordinates (using abovementioned transformation equations):

$$\hat{y} = 58,18 - 49,087\,X_1 - 1,156\,X_2 - 0,045\,X_3 - 0,55\,X_1 X_2 + 0,01495\,X_1X_3 - 0,2639\,X_2X_3 + 0,2799\,X_1X_2X_3 \tag{8}$$

The model for determination of elastic springback in function of covered influential factors has the folowing form:

$\Delta r_1 = 58,18 - 49,087(r_a/r) - 1,156(R/r) - 0,045\,(k_{sr}/G) - 0,55(r_a/r)(R/r) +$
$0,01495(r_a/r)(k_{sr}/G) - 0,2639(R/r)\,(k_{sr}/G) + 0,2799(r_a/r)(R/r)\,(k_{sr}/G)$ (9)

2.3. Checking model adequacy

Checking the adequacy of the model, in the general case, consists of comparing the dispersion of experimental results with regression line (s^2_R) and dispersion of the experimental results in points of multi-factorial space (s^2_E) over Fisher's criterion. The Fisher 's criterion uses s^2_E instead of s^2_R, representing dispersion of mean values of experimental results with respect to the regression line.

The next step of dispersion analysis in the mathematical process modeling includes checking the adequacy of the model. Since it is a system of repeating the experiment n - times at each point of hypercube, the dispersion will be related to the adequacy of models:

$$S^2{}_{LF} = \frac{m(\bar{y}_1 - \hat{y}_1)}{Nm - (m+1) - N(m-1)}; \qquad f_{LF} = Nm - (m+1) - (m-1) \tag{10}$$

$$s^2_{LF} = \frac{3 \cdot 0,098786}{24 - (3+1) - 8(3-1)} = 0,073665; \quad f_{LF} = 24 - (3+1) - 8(3-1) = 4 \tag{11}$$

The next step of the dispersion analysis in the mathematical modeling process includes checking the adequacy of the model. Since it is a system of n - times repeated experiments at each point of hypercube, the dispersion will be related to the adequacy of models:

Then from equation: $F_{rLF} = \dfrac{s^2_{LF}}{s^2_E}$ (12)

the calculated value is obtained: $F_r = 0,073665/0,624875 = 0,117887$ (13)

Since the obtained computational value is less than table value $F_t = 3$ *for* $f_{LF} = 4$, $f_E = 16$ *i* $\alpha = 5\%$, *i.e.* $F_r = 0,0589435 < F_t = 3$, and the obtained model (multiple regression equation) adequately describes the relevant three-factor process.

2.4. Determination of the limits of model reliability

Limits of reliability of model parameters are defined by equations:

$b_i \pm \Delta b_i = b_i \pm ts(b_i) = b_i \pm t\sqrt{c_{ii}}\,\overline{s(y)}$ (14)

therefore:

$\Delta b_i = \pm ts(b_i) = \pm 1,75 \cdot 0,161358 = \pm 0,2823763$ (15)

where: $t = t_t = 1,75$ *for* $f_E = 16$ *and* $\alpha = 5\%$.

3. CONCLUSION

Using factorial plan 2^3 and experimental research, performed with the triple repetition of the experiment, with the same combination of factor levels, the drwaing force values are obtained, in a mathematical model that encompasses three influential factors. In the experimental part, the influential factors are varied from minimum to maximum values. Targeted, small scale experimental tests, with the selection of influential factors and applying the abovementioned factorial plan, we can get the reliable mathematical models for calculating the required values.

4. REFERENCES

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