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# STICK−SLIP FRICTION MODELING IN TUBE EXPANSION

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**ABSTRACT**
*A model to study the dynamics of a stick-slip phenomenon in expansion of tubes was developed. During the permanent deformation process, the system experiences large friction forces at mandrel/tubular interface. This result in an increase of drawing force required for expansion as well as variation in tube thickness along its length. Three different sets of equation; one each for slip phase, stick phase and the transition from stick to slip were derived using equilibrium equations, incompressibility conditions and Karnopp's model. A zero velocity interval was used to define slip, stick and transition phases. The model captured the effect of stick-slip phenomenon on varying thickness reduction along the tube length. The fluctuation in the displacement-time plot clearly indicates the time when the mandrel stuck to the tube. Subsequent slip phase resulted in higher thickness reduction in tube compared to the preceding section. This uneven thickness lowers the structural integrity of the tube during its service life. The results showed that the velocities of mandrel and friction coefficient are critical parameters in minimizing thickness variation after expansion as well as lowering the force required to expand the tube.*
**Keywords:** Tube expansion, Mandrel-tube dynamics, Stick-slip phenomenon, Thickness variation

## 1. INTRODUCTION

Tube expansion is a metal forming process involves pushing or pulling a conical mandrel to permanently enlarge the inner diameter of the tubular. This expansion process is a combination of three nonlinearities; (i) material nonlinearities due to elasto-plastic deformation of the tubular; (ii) geometric nonlinearity due to the large deformation; and/or (iii) contact conditions at the mandrel/tubular interface. The contact conditions at the interface can result in failures of the tubular during expansion or premature failure of the expanded tubular during its service life. It is believed that these failures may occur due to the poor understanding of the dynamics of the expansion process.
Previous studies used plasticity theories to develop analytical models for tubular expansion neglecting dynamic effects [1-2]. The models demonstrate the variation in the force required for expansion with respect to expansion ratio, friction coefficient, and mandrel geometry. Recently, a simplified dynamic model was developed to show the stick-slip phenomenon of mandrel [3]. No further literature was found on studying the dynamics of stick-slip phenomenon in tubular expansion. However, significant research work was done on general theoretical development of stick-slip phenomenon, friction modeling, induced vibration, etc. Experiments indicate a functional dependence upon a huge variety of parameters, including sliding speed, acceleration, sliding distance, temperature, normal load, surface preparation, and of course material combination [4]. Recently, [5] conducted a comparitive study between eight different friction models which were applied on a control valve as a case study. The study revealed that Karnopp, Lugre and Kano were the only three models which were able to reproduce the stick-slip phenomenon and passed all the tests. Thus, in the present paper, Karnopp's model [6] is used to study the stick-slip phenomenon in tubular expansion process. It is selected because of its simplicity and the advantages of generating ordinary non-stiff differential equations which can be integrated with any standard ODE-solver.

## 2. STICK−SLIP MODEL

Expansion process induces wall thickness imperfections due to local plastic deformation as a result of stick and slip of the mandrel. When the mandrel starts moving through the tubular, the chance of sticking to the expanded tubular increases due to lack of proper lubrication, tubular surface irregularities, and the presence of welded and/or threaded connections. This means that higher force is required to overcome the excess of friction and push the mandrel forward. When the applied force reaches the limit to overcome maximum static friction, the mandrel tends to slip relative to the expanded tubular. This phenomenon is known as a "stick-slip" phenomenon. The study of stick-slip vibrations is faced with difficulties, as during the stick-slip motion two different mechanisms are taking place; the motion is governed by a static friction force in the stick phase and a velocity dependent kinetic friction force in the slip phase. Modelling of these two mechanisms yields a set of differential equations with discontinuous region as shown in Figure 1(a). A standard method to solve this discontinuity problem is by applying a smoothing method. The classical switch model using a switching method between slip, stick and transition phases poses the discontinuity problem. Hence Karnopp [6] model that is using a zero velocity interval to define slip, stick and transition phases is used as shown in Figure 1(b).

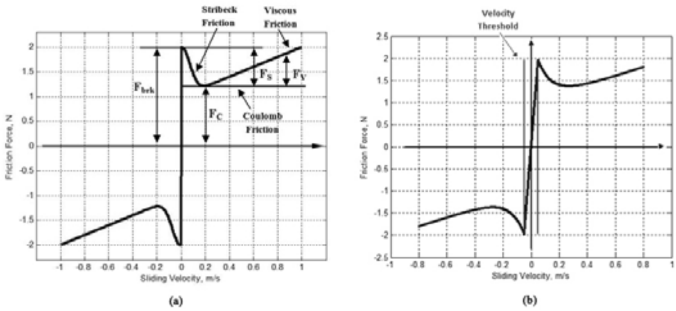


*Figure 1. Friction Models: (a) Classical Model, and (b) Karnopp's Model*

## 3. GOVERNING EQUATION

Assuming thin wall tubular, conical shape mandrel to expand the tubular, constant rate of deformation and uniform contact pressure at tubular/mandrel interface, the governing equations can be derived using equilibrium and incompressibility conditions. An infinitesimal element of the expanded tubular wall subjected to contact pressure and friction as well as the induced hoop and longitudinal stresses [1]. As the mandrel moves through the tube, each part of the tube passes through the same process, and if the tube is sufficiently long, the expansion takes smoothly. Considering a small element of the tubular wall expressions for the principal strain increments are given by:

$$d\varepsilon_r = \frac{dt}{t}, \qquad d\varepsilon_\theta = \frac{dr}{r}, \qquad d\varepsilon_z = \frac{dz}{z} \qquad \text{... (1)}$$

where, $r$ and $t$ represents the tube radius and thickness at the expansion zone. The subscripts $r$, $\theta$, and $z$ denote the radial, circumferential, and longitudinal directions. The incompressibility condition in terms of the principal strains is given by:

$$d\varepsilon_r + d\varepsilon_\theta + d\varepsilon_z = 0 \qquad \text{... (2)}$$

Using Tresca's yielding criterion and simplification results in following equation for required expansion force:

$$F_e = \frac{2\pi r_1 t_1 m \sigma_y \left(r_2^2 - r_1^2\right)\left(1 + \mu \cot\alpha\right)\cos\alpha - 2 r_1^2 M \ddot{x}}{\left(1 + \mu \cot\alpha\right)\left(r_2^2 - r_1^2\right)\cos\alpha + 2 r_1^2} \qquad \text{... (3)}$$

where, $\sigma_y$ is the material tensile yield strength, $m$ is a correction factor equal to 1.15 to satisfy von-Mises yield criterion, $M$ is the mandrel mass, $\mu$ is the friction coefficient with values ranging between the static ($\mu_s$) and the kinetic ($\mu_k$) friction coefficients, $r_1$ and $t_1$ represents the tubular inner radius and wall thickness before expansion, $r_2$ and $t_2$ are the tubular inner radius and wall thickness after expansion, $\alpha$ is the mandrel angle, and $\ddot{x}$ is the mandrel acceleration. Using longitudinal and hoop stresses relations from equilibrium and Levy-Mises flow rule, a relation for the tubular thickness variation due to expansion process is obtained as given below:

$$\frac{t_2}{t_1} = \left(\frac{r_2}{r_1}\right)\left[\frac{\pi h_2 P_c r_1^2 + M\ddot{x}\cos\alpha + \pi h_1 P_c\left(r_1^2 h_3 + r_2^2\cos\alpha\right)}{\pi h_2 P_c r_2^2 + M\ddot{x}\cos\alpha + \pi h_1 P_c\left(r_1^2 h_3 + r_2^2\cos\alpha\right)}\right]^{3/h_2} \quad \text{... (4)}$$

where, $h_1 = 1 + \mu\cot\alpha$, $h_2 = 4 - h_1$ and $h_3 = 1 - \cos\alpha$. $P_c$ is the contact pressure at the mandrel/tubular interface and it is given by: $P_c = \left((F_e/2\pi r_1 t_1) - m\sigma_y\right)(t_1/r_1)\cos\alpha$

Using the free-body diagram of the mandrel, Tresca's yield criterion, and the hoop and the longitudinal stresses equations obtained from equilibrium, the governing equation of motion is obtained. Then, using equation of motion along with the switch model the state-equations of the stick-slip model were obtained. The switching criterions with state equations for acceleration are given by:

$$\begin{aligned}
&if\left|v_{rel}\right| > \eta \ and \ \left|-\frac{\pi r_2^2 B}{M}\ln\left(\frac{x+L}{L}\right)\beta_1\right| > F_{f_{stick}} \quad \ddot{x} = -\frac{\pi r_2^2 B}{M}\ln\left(\frac{x+L}{L}\right)\beta_1 + \frac{\beta_3}{M}\Delta F_f(v) \quad (Slipping)\\
&if\left|v_{rel}\right| \le \eta \ and \ \left|-\frac{\pi r_2^2 B}{M}\ln\left(\frac{x+L}{L}\right)\beta_1\right| > F_{f_{stick}} \quad \ddot{x} = -\frac{\pi r_2^2 B}{M}\ln\left(\frac{x+L}{L}\right)\beta_1 + \frac{\beta_3}{M}F_{f_{stick}}\operatorname{sgn}(\Delta F_e) \quad (Transition) \quad \text{...(5)}\\
&f\left|v_{rel}\right| < \eta \ and \ \left|-\frac{\pi r_2^2 B}{M}\ln\left(\frac{x+L}{L}\right)\beta_1\right| \le F_{f_{stick}} \quad \ddot{x} = -v_{rel}\sqrt{\frac{\pi r_2^2 B}{ML}} \quad (Sticking)
\end{aligned}$$

where, $\Delta F_f(v) = -F_{e\,stick}\operatorname{sgn}(v_{rel})/\left(1+\delta\left|v_{rel}\right|\right)$, the constant $\delta$ is taken to be 0.003 [s/m], $F_{f\,stick} = F_{e\,stick}\left(1+(\beta-1)\left(1-e^{-at_{stick}}\right)\right)$, $\Delta F_e = -\pi r_2^2 B\ln\left((x+L)/L\right)$ and the constant $\beta$ and $a$ are taken to be 1.16 and 4.0 [1/s], respectively. The terms $L$ and $x$ denote the length of the expanded section and the mandrel relative displacement with respect to the reference coordinates. Since the mandrel is pushed by the fluid pressure build up, a relationship of the incremental drawing force is obtained by using the bulk modulus $B$ of the water. $\beta_1$ and $\beta_3$ are given by $1+\left(\left(r_2^2/r_1^2\right)-1\right)(\cos\alpha/2)$ and $-\pi\left(r_2^2-r_1^2\right)\cot\alpha$, respectively. The stick and slip time states are given by ($b$ = 5×10$^5$ 1/s and $c$ = 1×10$^4$ 1/s):

$$t_{stick} = \begin{cases} -bt_{stick} & (Slipping)\\ -bt_{stick} & (Transition)\\ 1 & (Sticking)\end{cases} \qquad t_{slip} = \begin{cases} 1 & (Slipping)\\ -ct_{slip} & (Transition)\\ -ct_{slip} & (Sticking)\end{cases} \quad \text{... (6)}$$

## 4. RESULTS AND DISCUSSION

This model has been solved numerically using MATLAB tool for a specific tubular expansion where an experimental data has been used to calibrate the model in order to obtain the values of the different constants that have been used in the model. The effect of the dynamics caused by the stick-slip phenomenon on the structural response of an expanded tubular has been simulated. Variation of the mandrel displacement, the drawing force required to push the mandrel forward, as well as the thickness variations were studied. Figure 2(a) shows the effect of the stick-slip phenomenon on the expansion force, and as can be seen from the figure that the developed model is capable of producing reasonable results that are in good agreements with the experimental data. Figure 2(b) shows the effect of the stick-slip phenomenon on the tubular thickness variation, it is important to note that a minor fluctuation in the tubular thickness is occurring as the mandrel changes its state from stick to slip. This increase in thickness reduction should be taken into account before undergoing any field trials because any small change in one of the fundamental parameters may cause a sharp increase in the thickness reduction which means unexpected failure may occur. Figure 2(c) shows that the absolute displacement of the mandrel increases with time indicating that the mandrel advances through the tubular as it is expanding. But, it also shows that the displacement exhibits little oscillations and that the mandrel gets stuck to the tubular for a short period of time but the level of oscillations is minimal and not affecting the mandrel movement. Figures 2(d-e) show a parametric study of the expansion force variation as a result of changing the friction coefficient and the mandrel speed while holding all other parameters constant and as can be noticed from the figures that the expansion force is increased as the friction coefficient increases. On the other hand, it was found that increasing the mandrel speed causes a reduction on the effect of the stick-slip phenomenon where the fluctuations in thickness and expansion force have been reduced as the mandrel speed gets increased.

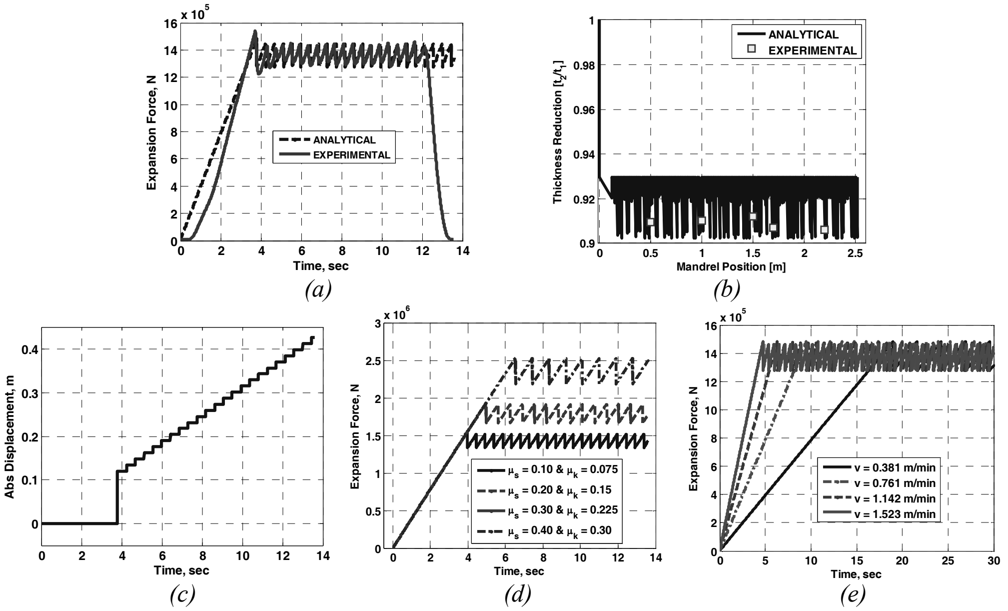


*Figure 2. (a) Force required for expansion, (b) Tubular thickness variation, (c) Mandrel displacement versus Time, (d) Expansion force for different friction coefficients at mandrel speed of 1.903m/min, (e) Expansion force for different mandrel speeds at fixed friction coefficients of $\mu_s$=0.084 and $\mu_k$=0.063*

## 5. CONCLUSION

The analytical model describing the dynamics of the stick-slip phenomenon of a tubular expansion process has been developed. It is found that the tube thickness reduction and expansion force, from the analytical model showing a good agreement with the experimental results. It is also clear from the comparison that expansion of the tubular by 20% results in tubular thickness reduction of approximately 9%. It is also found that the required force to propagate the mandrel forward in the 20% expansion process is fluctuating between a maximum value of 1517 kN in the sticking phase and a minimum valie of 1336 kN in the slipping phase.

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