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15th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2011, Prague, Czech Republic, 12-18 September 2011

DECOMPOSITION OF NONLINER-PHASE FIR FILTER INTO LINEAR-PHASE SECTIONS

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ABSTRACT
This paper presents the method to decompose an FIR (Finite impulse response) filter, with non-linear phase, into the linear-phase sections. In that way the original problem of the overall nonlinear phase filter design is decomposed into less complex designs. The main idea is to describe an FIR filter of the order N in terms of linear-phase FIR sub filters of the order m, m=1, 2, 3, 4. The method is illustrated with one example.
Keywords: FIR filter, non-linear phase, linear-phase section.

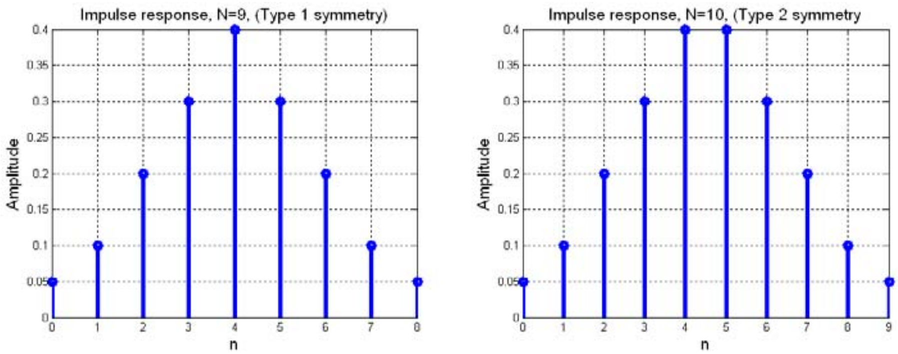
1. INTRODUCTION
It is well known that the decomposition of the original problem into less complex sub-problems, can reduce the complexity of the overall FIR (Finite impulse response) design. The main goal of this paper is to describe a nonlinear phase FIR filter of the order N, in the form of the linear-phase sub-filters of the desired order, and thus generalize the idea presented in [1].
The FIR filter of the length N, which is the order of the filter, has the following transfer function,

$$H(z) = \sum_{n=0}^{N-1} h_n z^{-n}, \tag{1}$$

where h_n are the filter coefficients. If the filter has a linear phase, then the coefficients of the filter h_n have one of the fourth type of symmetry, [2]. Type-1, and type-3 filters have an N odd, while the type-2, and 4 filters have an N even. As an example Fig.1 shows first and second type of symmetry filters. The symmetry of the filter coefficients reduces the number of the multipliers approximately by half, which depends on the type of the symmetry.
The rest of the paper is organized in the following way. Next section describes the decomposition of the FIR filter into parallel sub-filters. Section 3 presents the decomposition of sub-filters into linear-phase (LP) sub-filters. Finally, last section presents the decomposition of the overall filter and is illustrated with one example.

2. PARALLEL SUB-FILTERS
Here we consider the decomposition of the FIR filter of the order N into K parallels subfilters of the order 5. Two cases are possible depending if $N/5$ is integer or not an integer.

First case: *N/5 is integer*.
From (1) we write



a. *b.*

Figure 1. Impulse responses of linear-phase filters.

$$H(z) = \sum_{k=0}^{K-1} z^{-5k} H_{k,5}(z), \tag{2}$$

where $H_{k,5}(z)$ is the k-th subfilter of the order 5,

$$H_{k,5}(z) = \sum_{n=0}^{5-1} h_{5k+n} z^{-n}\,. \tag{3}$$

Example 1: Consider N=15, resulting in K=N/5=3.
From (1) we write

$$\begin{aligned} H(z) &= \sum_{n=0}^{14} h_n z^{-n} = h_0 z^0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + h_4 z^{-4} + \\ &+ z^{-5}\left[h_5 z^0 + h_6 z^{-1} + h_7 z^{-2} + h_8 z^{-3} + h_9 z^{-4}\right] + \\ &+ z^{-5\times 2}\left[h_{10} z^0 + h_{11} z^{-1} + h_{12} z^{-2} + h_{13} z^{-3} + h_{14} z^{-4}\right] = \\ &= H_{0,5}(z) + z^{-5} H_{1,5}(z) + z^{-10} H_{2,5}(z) = \sum_{k=0}^{2} z^{-5k} H_{k,5}(z) \end{aligned} \quad . \tag{4}$$

Second case: *N/5 is not integer*.
In this case

$$K = \left\lfloor \frac{N}{5} \right\rfloor, \tag{5}$$

where $\lfloor x \rfloor$ means the integer part of x.

$$H(z) = \sum_{k=0}^{K-1} z^{-5k} H_{k,5}(z) + z^{-5K} H_{K,m}(z)\,. \tag{6}$$

The only difference relating to the First case is that the last K-th sub-section has an order m, where m<5, as shown in the following example.

Example 2: We consider N=14, resulting in

$$K=\left\lfloor \frac{14}{5} \right\rfloor = 2\,. \tag{7}$$

From (1) we have

$$\begin{aligned} H(z) &= \sum_{n=0}^{13} h_n z^{-n} = h_0 z^0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + h_4 z^{-4} + \\ &+ z^{-5}\left[h_5 z^0 + h_6 z^{-1} + h_7 z^{-2} + h_8 z^{-3} + h_9 z^{-4}\right] + \\ &+ z^{-5\times 2}\left[h_{10} z^0 + h_{11} z^{-1} + h_{12} z^{-2} + h_{13} z^{-3}\right] = \\ &= H_{0,5}(z) + z^{-5} H_{1,5}(z) + z^{-10} H_{2,4}(z) = \sum_{k=0}^{1} z^{-5k} H_{k,5}(z) + z^{-5K} H_{k,4}(z) \end{aligned} \,. \tag{8}$$

3. DESCRIPTION OF FIR SUB-FILTERS IN THE FORM OF LP SUB-FILTERS

From (4) and (6) the general form of the sub-filters is given as $H_{k,5}$. In this section we investigate how describe the subfilter $H_{k,5}$ in terms of linear-phase (LP) subfilters. To this end we rewrite Eq.(3),

$$H_{k,5}(z) = \sum_{n=0}^{5-1} h_{5k+n} z^{-n} = h_{5k} z^0 + h_{5k+1} z^{-1} + h_{5k+2} z^{-2} + h_{5k+3} z^{-3} + h_{5k+4} z^{-4} \tag{9}$$

The equation (9) is rewritten in the following form.

$$\begin{aligned} &H_{k,5}(z) = \\ &= h_{5k} z^0 + h_{5k+1} z^{-1} + h_{5k+2} z^{-2} + h_{5k+1} z^{-3} - h_{5k+1} z^{-3} + h_{5k+3} z^{-3} + h_{5k} z^{-4} - h_{5k} z^{-4} + h_{5k+4} z^{-4} = \\ &= z^0\left[h_{5k} z^0 + h_{5k+1} z^{-1} + h_{5k+2} z^{-2} + h_{5k+1} z^{-3} + h_{5k} z^{-4}\right] + \\ &+ z^{-3}\left[(h_{5k+3} - h_{5k+1}) z^0 + (h_{5k+3} - h_{5k+1}) z^{-1}\right] + \\ &+ z^{-4}\left[(h_{5k+4} - h_{5k}) - (h_{5k+3} - h_{5k+1})\right]. \end{aligned} \tag{10}$$

Note that the sub-filters in the first and second rectangular brackets are LP filters of the order 5, and 2, respectively. We denote the LP sub-filters as

$$H_{k,5}^{LP}(z) = h_{5k} z^0 + h_{5k+1} z^{-1} + h_{5k+2} z^{-2} + h_{5k+1} z^{-3} + h_{5k} z^{-4}\,. \tag{11}$$

$$H_{k,2}^{LP}(z) = (h_{5k+3} - h_{5k+1}) z^0 + (h_{5k+3} - h_{5k+1}) z^{-1}\,. \tag{12}$$

From (10)-(12) we arrive at

$$H_{k,5}(z) = H_{k,5}^{LP}(z) + z^{-3} H_{k,2}^{LP}(z) + A_k z^{-4}\,, \tag{13}$$

where A_k is the constant

$$A_k = (h_{5k+4} - h_{5k}) - (h_{5k+3} - h_{5k+1})\,. \tag{14}$$

4. OVERALL FILTER DESCRIPTION

From (2), (3) and (13) we get the following expression for the overall filter presentation in terms of LP sub-filters.

$$H(z)=\sum_{k=0}^{K-1} z^{-5k} H_{k,5}(z)=\sum_{k=0}^{K-1} z^{-5k}\left(H_{k,5}^{LP}(z)+z^{-3}H_{k,2}^{LP}(z)+A_k z^{-4}\right). \qquad (15)$$

The method is illustrated in the following example.

Example 3: The transfer function of the filter of order N=10, and K=2, is given as

$$H(z)=$$
$$=0.1+0.12z^{-1}+0.95z^{-2}+0.8z^{-3}+0.94z^{-4}+0.9z^{-5}+0.82z^{-6}+0.7z^{-7}+0.83z^{-8}+0.82z^{-9}$$

From (15) we have

$$H(z)=\sum_{k=0}^{1} z^{-5k} H_{k,5}(z)=\left[H_{0,5}^{LP}(z)+z^{-3}H_{0,2}^{LP}(z)+A_0 z^{-4}\right]+z^{-5}\left[H_{1,5}^{LP}(z)+z^{-3}H_{1,2}^{LP}(z)+A_1 z^{-4}\right]=$$
$$=\left[0.1(1+z^{-4})+0.12(z^{-1}+z^{-3})+0.95z^{-2}+0.7(z^{-3}+z^{-4})+0.16z^{-4}\right]+$$
$$+z^{-5}\left[0.9(1+z^{-4})+0.82(z^{-1}+z^{-3})+0.7z^{-2}+0.01(z^{-3}+z^{-4})-0.09z^{-4}\right]$$

Note that the sub-filters in brackets have the same structure.

5. CONCLUSION

This paper presented the generalization of the non-linear phase filter description in the terms of the linear-phase sub-filters. We considered here the fifth-order LP sections. However using this procedure one can choose any desired order of the LP sections. The method can be useful for the implementation of non-linear phase filters like minimum-phase filters [3] and the polyphase components of LP FIR filters[4].

6. ACKNOWLEDGEMENT

Thos work is supported by the CONACYT, Mexico.

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