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AUTOMATIZED DETERMINATION OF THE GEOMETRIC CHARACTERISTICS OF HELICOIDAL SHELL ON CYLINDRICAL SHELL

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ABSTRACT
This paper contains procedure for automatized determination of the geometric characteristics of helicoidal shell on cylindrical one that is applied at machines for special purposes. According to a differential equation in displacement for bending of a helicoidal shell subjected to uniform pressure and a differential equation for cylindrical shell loaded by uniformly distributed couple along helix, the boundary conditions are defined. The differential equation is solved numerically by using software MATLAB. The values of the shell thickness are determined for different parameters. The different models of helicoidal shell are analyzed.
Keywords: helicoidal shell, cylindrical shell, bending

1. INTRODUCTION

A helicoidal shell encountered at special construction machines, helicoidal transporters and similar are most often forged out of steel plate or steel strap at special machines, then welded on the cylindrical shell. At helicoidal pipe-like transporters, a helicoidal shell is welded from the inner part of the pipe, and whereas at special machines for snow cleaning and similar, it is welded on thin cylindrical shells of a bigger diameter. A number of scientific papers relating to bending of helicoidal shell are very small. An application of equations of shell theory and their solution in case of helicoidal shell is complicated. There have been published papers in this field by J. W. Cohen [1] and S. G. Mikhlin [2].

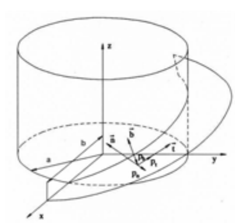

Figure 1. Helicoidal shell on cyllindrical shell

The helicoidal shell is a part of the surface of conoidal helix which is, along helicoidal line, placed on cyllinder and from the outer side, it is limitted by helicoidal line of the same stroke height and by a constant diametre like on Figure 1. The influence of a continnualy distributed load vertical to the shell surface is analised in this paper.

2. HELICOIDAL SHELL BENDING

In paper [4] there has been derived an equation along displacement for a helicoidal shell whose parameter equations of middle surface are: $x = \theta^1 \cos\theta^2$, $y = \theta^1 \sin\theta^2$, $z = k\theta^2$. The shell is seized at the shell of $\theta^1 = a$, and it is free at the shell of $\theta^1 = b$. Equations of classical shell theory can not be used due to the fact that a mixed coefficient of the second fundamental surface from differs from zero, i. e. coordinate lines and not curve lines.

Our suppositions were that only displacements in tangential plan $u_{<1>}$, $u_{<2>}$ can be ignored if compared do displacements in direction of vertical u_3 and the derived parameter are not the function of θ^2.

Differential equation of helicoidal shell bending of the order along displacement u_3, is derived in the form of:

$$\frac{d^4u_3}{dr^4} + \frac{2r}{r^2+k^2}\frac{d^3u_3}{dr^3} - \frac{r^2+k^2(\nu+1)}{\left(r^2+k^2\right)^2}\frac{d^2u_3}{dr^2} + \frac{r\left[r^2+k^2(3\nu+4)\right]}{\left(r^2+k^2\right)^3}\frac{du_3}{dr} + \left[\frac{2k^2(\nu+1)\left(3k^3-8r^2\right)}{\left(r^2+k^2\right)^4} + \frac{24k^2(1-\nu)}{h_h{}^2\left(r^2+k^2\right)^2}\right]u_3 + \frac{P}{B_h} = 0 \tag{1}$$

where: r - curving coordinate on helicoidal surface, h_h - bending shell tickness, k=H/2π - helicoidal surface, H - helicoidal surface stroke height, E - elasticity module, ν - Poisson's coefficient, P - uniformly distributed pressure along shell surface, B_h - stiffness at shell benting determined by a relation $B_h=Eh_h^3/12(1-\nu^2)$.

In special case, we obtain for k=0:

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{du_3}{dr}\right)\right]\right]\right\} = -\frac{P}{B}, \tag{2}$$

this representing a known differential equation of circular plate bending at symmetrical load. In another special case for k→∞, we obtain:

$$\frac{d^4u_3}{dr^4} = -\frac{P}{B}, \tag{3}$$

this representing a differential equation of bending of a infinitely long plate into cyllindrical surface. Boundary conditions are derived in the form of:

$$u_3 = 0,\ \frac{du_3}{dr} = 0 \ \text{(for r=a)}, \tag{4}$$

$$,\ \frac{d^3u_3}{dr^3} + \frac{r}{r^2+k^2}\frac{d^2u_3}{dr^2} - \frac{1}{r^2+k^2}\frac{du_3}{dr} + \frac{4k^2(\nu+1)}{\left(r^2+k^2\right)^3}u_3 = 0 \ \text{(for r=b)}, \tag{6}$$

3. CYLINDRICAL SHELL BENDING LOADED BY A CONTINUAL COUPLE ALONG HELICOIDAL LINE

A procedure of estimating cylindrical shell exposed to bending by a continual coupling along helicoidal line is given in the paper [5]. At construction elements of helicoidal shell shape on cylindrical shell, a load from helicoidal shell is transmitted onto cylindrical shell through the cross-section line of middle surface, i. e. through the helix. When the helicoidal shell is loaded with pressure, the resultant of that load on the cylinder constituted of a continual coupling in direction of tangent on the helicoidal shell on cylinder and a continually distributed force in tangential cylinder plane. The suppositions that displacements due to forces are negligible if compared to displacement due to a coupling and those displacements along the helicoidal line equal aero are introduced in the paper. As the loads and boundary conditions are given in the helix, curving coordinates of points on the middle surface of cylindrical shell are introduced, thus coordinate lines make two families of interpolated orthogonal helics. Relations of restrictive shell theory in tensor shape are used for cylindrical shell as the introduced coordinate lines are not the ones of cylindrical surfaces. Boundary conditions are defined in central points of cross-section of any helix from the family φ=const. with helics $\psi=0$ along which the coupling is distributed. Cross-section points are: $\psi=0$ and $\psi=\psi^*=2\pi k^2/\left(R^2+k^2\right)$. Based on the four boundary conditions, unknown constants in solving homogenous linear differential equations with constant coefficient along the displacement in direction of vertical on cylinder are determined. The solution of differential equations is derived in the form of:

$$u_3(\psi)=-\frac{\overline{M}}{2st}\left[\frac{\sin t\psi^*}{chs\psi^*-\cos t\psi^*}shs\psi\cos t\psi+\left(shs\psi-\frac{shs\psi^*}{chs\psi^*-\cos t\psi^*}chs\psi\right)\sin t\psi\right], \qquad (8)$$

where,

$$s=\sqrt{\frac{n+m}{2}},\ t=\sqrt{\frac{n-m}{2}},\ m=\frac{k^2+\nu R^2}{2k^2}, \qquad n^2=\frac{C}{B_c}\frac{R^2\left(R^2+k^2\right)^2}{k^4}, \qquad (9)$$

where $C=Eh_c/(1-\nu^2)$ - stiffnes to stetching. Value $\overline{M}$ is determined by an expression:

$$\overline{M}=\frac{M}{2B_c}\frac{R^2\left(R^2+k^2\right)}{k^2}, \qquad (11)$$

where M - is a continual coupling along helicoidal line, R - radius of cyllindrical shell, h_c - chyllindrical shell thickness. The function slope $u_3(\psi)$ at helicoidal line is given by an expression:

$$\left.\frac{du_3}{dS}\right|_{S=0}=\frac{k}{R\sqrt{R^2+k^2}}\frac{\overline{M}}{2st}\frac{tshs\psi^*-s\sin t\psi^*}{chs\psi^*-\cos t\psi^*}, \qquad (12)$$

where S - length of helicoidal line arc.

Value $\psi=\psi_m$ for which the function $u_3(\psi)$ reaches its extreme value is determinated by an expression:

$$tg(t\psi)=\frac{D_1+D_2ths\psi}{D_3+D_4ths\psi}, \qquad (13)$$

where,

$$\begin{aligned} D_1&=tshs\psi^*-s\sin t\psi^*, \quad & D_2&=t\left(\cos t\psi^*-chs\psi^*\right),\\ D_3&=s\left(chs\psi^*-\cos t\psi^*\right), \quad & D_4&=-\left(sshs\psi^*+t\sin t\psi^*\right). \end{aligned} \qquad (14)$$

The relation (13) represent a transcendental equation out of which, for concrete values s, t, ψ^* by some iterrative methods, the value ψ_m is determined, for which $u_3(\psi)$ has an extreme value. From the expression, it is evident that ψ_m does not depend on the value of concentrated moment M.

4. MODELS FOR AUTOMATIZED PROJECTING OF HELICOIDAL SHELL ON CYLINDRICAL SHELL

Using the programme for numerical solution of differential equations, with the function of differential equations, with the function of automatized determination of the helicoidal shell thickness, based on the criterion of the allowed value of maximum stress in radial direction (σ_d=150 N/mm^2) there Table 1. The analysis has been made for the models with the same thickness of helicoidal and cylindrical shells. Other parameters are: p=1.5 bars, H=140 mm, E=200000 N/mm^2, ν=0.3. The value of maximum stress in radial direction is estimated on the base of relation: $\sigma_r=6\hat{M}_{<11>}/h_h^2$ in the points with coordinate: $r=h_c/2+h_h$.

Table 1. Values of calculated thickness of helicoidal shell [mm] for different models

		Model of Helicoidal Shell on Cylindrical Shell											
		1	2	3	4	5	6	7	8	9	10	11	12
a[mm]		127	280	30	80	130	230	30	30	30	5	100	10
b[mm]		200	350	100	200	300	550	200	250	300	80	350	60
Thickness h_h	Estimated	3.932	3.804	4.227	7.101	9.988	18.910	10.683	13.904	17.099	4.845	15.136	3.064
	Accepted	4	4	4.5	7.5	10	19	11	14	17.5	5	15.5	3.5

A dependence diagram of maximum stress in radial direction on the height of helicoidally shell height for pressures 1, 1.5 i 2.5 bars is shown in Figure 2. A dependences of maximum stresses in radial direction on helicoidal shell stroke height for models from 1 to 9 as indicated in the diagrams are given in Figure 3-5.

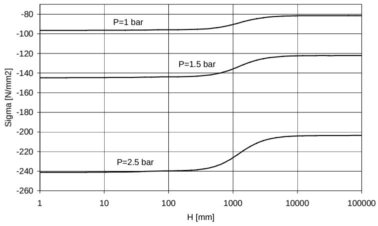

Figure 2. Maximum stress in radial direction for pressures 1, 1.5 i 2.5 bars

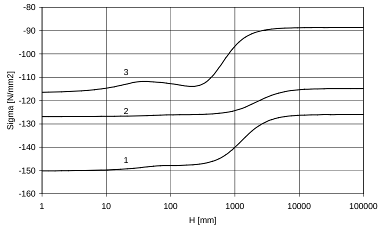

Figure 3. Maximum stress in radial direction for models 1,2,3

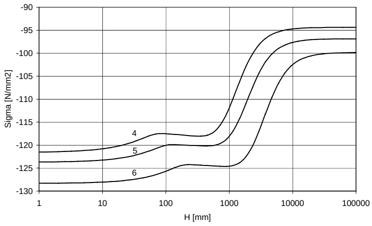

Figure 4. Maximum stress in radial direction for models 4,5,6

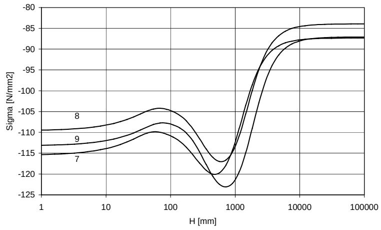

Figure 5. Maximum stress in radial direction for models 7,8,9

5. CONCLUSION

The derived differential equations of helicoidal shell bending and cylindrical shell bending loaded by a continual coupling along a helicoidal line have enabled an efficient solution of the problem of helicoidal shell bending on cylindrical shell.

Based on the derived stress dependence diagrams, it has been shown, for most shell models, that stress in radial direction in dependence of helicoidal bending is the greatest when $k \to 0$. With an increase of steps these stresses decreas inconsiderably at the beginning and then with an increase of k they decrease. When $k \to \infty$, stress in radial direction for most models are the smallest ones. The maximum stress dependence in radial direction for most shell models have the shape of logistic curve, as shown in Figure 2-4. This conclusion has a practical significance in estimating construction shapes of helicoidal shell as it enables control estimations by using simpler equations for bent circulat plates. However, this conclusion may not relate to the models of helicoidal shells at which the relation of the shell width and inner diameter is great. At such models, the stress values in radial directions are not maximu for shells with bending of k=0, this being evident from the derived diagrams in Figure 5. The established mathematical model makes it possible to analyze the influence of relation of helicoidal shell width and inner diameter on the values of maximum stress in radial direction, for different values of helicoidal shell bending.

6. REFERENCE

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