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EVALUATION OF GEOMETRY IN COMPLEX SHAPE DESIGN

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ABSTRACT
Evaluating complex geometric shapes in a CAD model can be difficult. Subjective evaluation is typically personal, and requires „an eye" for the type of work a designer is doing. Objective evaluation requires some sort of measurable criteria for determining a pass or fail.
One way to evaluate complex geometric shapes, in particular transitions between surfaces around common edges, is to use curvature combs and reflective techniques. Using a complex CAD model of a plastic bottle, this work explains mentioned techniques and appropriate workflow during geometry evaluation.
Keywords: complex shape, design, curvature combs, zebra stripes

1. INTRODUCTION
Growing demands exist for more complex yet aesthetically pleasing design in many industries and markets [1]. Complex shapes are found in molded consumer products, parts created as castings, and sheet metal products in automotive industry. These parts have specific requirements, such as draft surfaces, smooth surface continuity between part faces, and surfaces that are based on splines rather than line and arc profiles. Successful complex shape design requires good 3D modeling skills and clear understanding of all techniques for geometry evaluation. This paper explains the terms such as curvature, geometry continuity, as well as curvature combs and zebra stripes techniques for evaluation of curves and surfaces. In addition, a CAD model of a bottle was used in order to show how mentioned techniques are used in the design process.

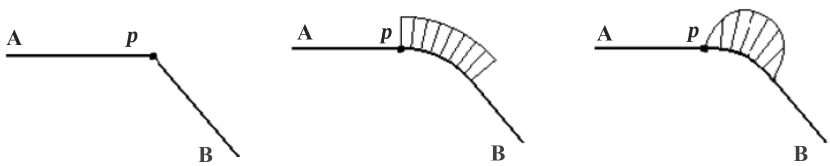
2. DEFINISIONS OF CURVATURE AND CONTINUITY
Curvature is used to describe both curves and surfaces. For complex curves and surfaces the value is constantly changing over the geometry. Its instantaneous value is simply the inverse of the radius ($C=1/R$) of the curve or surface at a contact point [2]. For a straight line, the radius at any point along the line is infinite, so the curvature at all points along the line is zero. For a circle or arc, the radius is constant at all points along the curve, therefore the curvature is also constant and nonzero. For a spline, the radius and, thus curvature, are constantly changing along the length of the curve. The curvature at any point on a surface requires more complex calculation but describes the same information as the curvature of a curve.
Continuity describes the curvature changes between connected curves or surfaces [3]. Although is typically judged between part surfaces, the underlying curves are the geometry that controls the quality of the surfaces and the continuity between surfaces. As with curvature, continuity applies equally to curves

and surfaces. Continuity conditions between both curves and surfaces can be specified using different tools in CAD software. For practical purposes, there are three types of geometry continuity: contact, or G0 continuity; tangent, or G1 continuity; and curvature, or G2 continuity. While there are orders of continuity higher than G2, they are used rarely in CAD and therefore will not be addresed here.

G0 continuity – curves are joined at their endpoints and surfaces contact at a common edge but that is all. Mathematical definition: Curve “A” is said to be “continuous” with curve “B” at a point “p” if the solution to function A at “p” is equal to the solution to B at “p”, such that A(p)=B(p). The connection forms a geometric discontinuity, a sharp change in direction between two curves or a sharp edge between surfaces (Figure 1a).[2]

G1 continuity – connected curves or surfaces are tangential. Mathematical definition: Curve “A” is said to be “tangent” to curve “B” at a point “p” if the derivative of A (A′) at “p” is equal to the derivative of B (B′) at “p”, such that A′(p)= B′(p). Figure 1b shows the curvature of two lines joined by an arc that is tangent to both lines. While G1 continuity technically provides a smooth transition between the two surfaces, it is not aesthetically very pleasing. This is because of the sudden change in the radius of the two adjoining faces. In a finished product, this abrupt change can be both seen and felt. [2]

G2 continuity – the curvature varies smoothly between connected curves or surfaces. Mathematical definition: Curve “A” is said to be “G2” continuous with curve “B” at a point “p” if the A′ and A′′ at “p” are equal to B′ and B′′, respectively, at “p,” such that A′(p)= B′(p), and A′′(p)= B′′(p). Figure 1c shows two lines joined by a spline. Because the curvature of a spline is constantly changing, it can match the zero curvature of the adjacent lines. When two adjacent faces have G2 continuity, it is often said they are *curvature continuous*.[2]



a) b) c)

Figure 1. Geometry continuity; a) G0 continuity, b) G1 continuity, c) G2 continuity

3. TECHIQUES FOR GEOMETRY EVALUATION

One of the most powerful ways of assessing curvature on a curve in space is to use C*urvature combs* (also known as *Porcupine analysis* and *Curvature graph*). This technique gives a graphic representation of the curvature in the form of a series of lines called a *comb*. The length of the lines represents the curvature. The longer the line, the greater the curvature (and smaller the radius), as can be seen in Figures 1b,1c,2. When the comb crosses the curve, it indicates an inflection point. An inflection point is where the curve changes convexity. In the illustration below there are two very similarly looking curves, both of which appear to have two inflection points. When C*urvature combs* is shown on the right illustration it becomes obvious that the below curve is not smooth and has many inflection points. Although the *Curvature combs* purely are used for curvilinear elements in space it can be used to evaluate surfaces by using an intersection plane.[3]

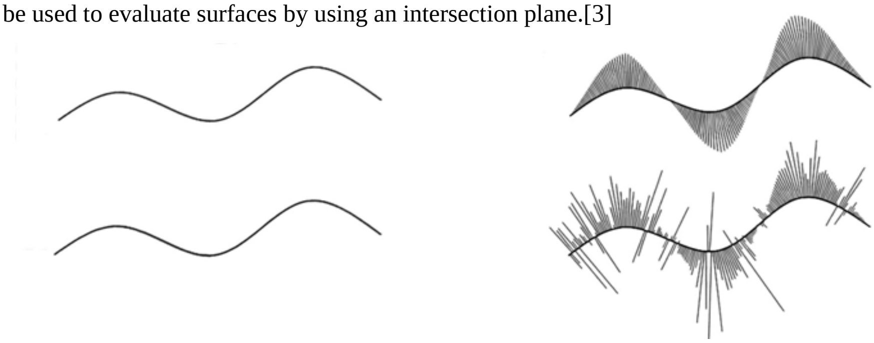


Figure 2. Geometry evaluation with Curvature combs

When the problem is evaluating complex surfaces, in particular the transitions between surfaces around common edges, reflective techniques are used. The most widely used reflective technique is *Zebra stripes.* It simulates putting a perfectly reflective part in a room that is either cubic or spherical and where walls are painted with black-and-white stripes. In high-end shape design, surface quality is measured qualitatively by using light reflections from the surfaces. Reflective stripes make it easier to visualize when an edge is not smooth. The three cases that *Zebra stripes* can help to indentify are as follows:

Contact. Surfaces intersect at an edge, but are not tangent across the edge. This condition exists when stripes do not line up on either side of the edge. [2]

Tangency. Surfaces are tangent across an edge, but have different radius of curvature on either side of the edge (non-curvature continuous). This condition exists when stripes line up across an edge but the stripe is not tangent across the edge. [2]

Curvature Continuity. Surfaces on either side of an edge are tangential and match in radius of curvature. *Zebra stripes* are smooth and tangential across the edge. [2]

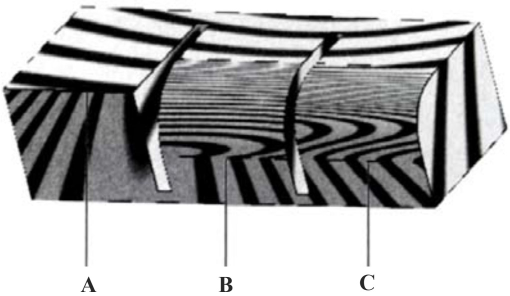


Figure 3. Geometry evaluation with Zebra Stripes

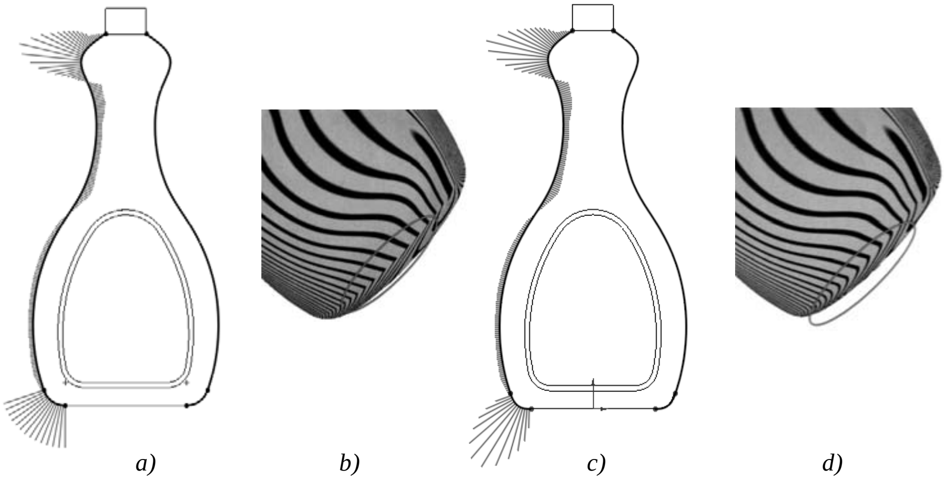
In Figure 3, the *Zebra Stripes* in example A do not match across the edge labeled A at all. This is clearly the non-tangent, contact-only case. Example B shows that the stripes match in position going across the indicated edge, but they change direction immediately. This is the tangent case. Example C shows the stripes flowing smoothly across the edge. This is the curvature continuous case.

Another geometrical analysis tool that is used to visualize the quality of transitions between faces across edges is *Display curvature.* It displays the faces of the model rendered in different colours according to their local curvature values (Figure 4). Displaying the curvature in such a way can be system resource intensive.

Figure 4. Display curvature

4. GEOMETRY EVALUATION OF A BOTTLE DESIGN

Figure 4. shows a CAD model of a bottle with the curvature of the surfaces presented in different colours. It is easy to note that there is the dramatic change in colour from the body of the bottle to the fillet around the bottom. This indicates that although the fillet is tangential to the body, it is not curvature continuous. This means the faces do not have the same curvature at the edge where they meet.
In order to analyse geometry with *Curvature combs*, a reference plane was used and two intersection curves were generated. Circular cross sections of the curvature combs at the bottom of the bottle, as well as the different lengths of the comb at the point where fillet and side of the bottle met, indicated G1 continuity between them (Figure 5a).
The same conclusion was reached with *Zebra stripes* technique – stripes changed direction immediately indicating G1 continuity (Figure 5b).
When the design of the bottle was changed *Curvature combs* and *Zebra stripes* looked as in the Figures 5c, 5d. The unequal lengths of the curvature comb, the last comb element on the body and the first element on the fillet are of the same length, stripes flowing smoothly across the edge; all indicate that the fillet is curvature continuous with the body of the bottle.



a) b) c) d)

Figure 6.CAD model of a bottle; a) initial model with Curvature combs, b) Zebra stripes on the initial model, b) redesigned model with Curvature combs, d)Zebra stripes on the redesigned model

5. CONCLUSION

In the modern world, complex shapes are not just beneficial for consumer product design, they are essential. In order to design products with complex shapes, besides skills of the conventional modelling techniques designers must know how to evaluate created geometry. What looks good in screen might not look good in reality, particularly when the continuity is visible on a reflective and very large surface, as in automotive design. This work explained the necessary information about geometry continuity as well as evaluation techniques, and can be a good foundation for all designers who plan to design complex shape products.

6. REFERENCES

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