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SIMULATION OF EXPERIMENTAL DAMPING DETERMINATION OF ELASTIC SYSTEMS

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ABSTRACT
Vibrations analysis is important part of construction design and possible failure detection. Vibration highly depends on damping characteristics. Damping characteristics must be considered or determined experimentally, which is commonly done by analysis of vibration records. Determination of damping is important due that internal damping of elastic systems sometimes could be controlled to reduce amplitude of vibrations and increase a life of construction.
In this paper, simulation of experimental damping determination of elastic systems is presented. Vibrations are simulated by numerical solution of mathematical model of damped vibrations of two d.o.f. system. Waveform obtained by simulation is imported in software for experimental data analysis and analysed as a real waveform. Damping analysis is performed in time domain and tested for different initial condition.
Keywords: damping, experimental determination, simulation

1. INTRODUCTION

Damping is material or system property that is characterized by energy dissipation in a vibration. Analysis of mechanical vibrations without damping is a mathematical fiction, which takes place in the introductory considerations in the theory of vibrations [1]. Any realistic analysis of system vibrations requires consideration of the damping effect. However, the damping is the effect that is still less researched and which modelling is the difficult. Practically, experimental determination of a damping is the most used method [2][3]. In case of the system with one degree of freedom, damping could be easily determined from the analysis of a amplitude decay in time domain [1]. But, in analysis of damped vibration of system with more degrees of freedom, it is not explicitly known relation between amplitude decay and damping in the system.
In this paper simulation of damping in elastic systems with two degrees of freedom is presented. Results obtained by simulation are imported in software for experimental data analysis and determination of damping is implemented. Original damping factor is recovered combining analysis of vibration records and derived differential equation based of system eigenvibration analysis. A comparison of obtained results and expected results that we predicted in damping simulation shows that presented method could be very efficiently used in real experimental damping determination.

2. GOVERNING EQUATIONS

Analysis is performed on system with two degrees of freedom, which is shown on Fig.1. Two rods of length l with two masses $m_1 = m_2 = m$ are connected with two angular springs of stiffness $k_1 = k_2 = k$ and damping c. Equations of motion could be written in matrix form [3]:

$$[\mathbf{M}]\{\ddot{\boldsymbol{\theta}}\}+[\mathbf{C}]\{\dot{\boldsymbol{\theta}}\}+[\mathbf{K}]\{\boldsymbol{\theta}\}=0\,, \qquad (1)$$

where $[\mathbf{M}]$ is mass matrix, $[\mathbf{K}]$ is stiffness matrix, $[\mathbf{C}]$ is damping matrix and $\{\boldsymbol{\theta}\}$is displacement vector given by

$$[\mathbf{M}]=ml^2\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix},[\mathbf{K}]=\begin{bmatrix}2k & -k\\ -k & k\end{bmatrix},[\mathbf{C}]=\begin{bmatrix}2c & -c\\ -c & c\end{bmatrix},\{\boldsymbol{\theta}\}=\begin{Bmatrix}\theta_1\\ \theta_2\end{Bmatrix}. \qquad (2)$$

Equation (1) is system of two ordinary differential equation, which could be solved in a closed form, for prescribed initial condition and system properties, but position and role of damping factor c in a solution could not be easily found.

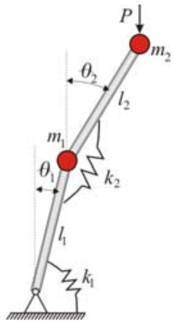

Figure 1. Analysed two d.o.f. system

2.1 Damping factor calculation

If we assume that system on Fig. 1 oscilate in the form of one eigenmode $\{\boldsymbol{\theta}_0\}$, displacement vector $\{\boldsymbol{\theta}\}$ could be written as

$$\{\boldsymbol{\theta}\}=\theta\{\boldsymbol{\theta}_0\}\,. \qquad (3)$$

Multiplyng equation (1) by $\{\boldsymbol{\theta}_0\}^{\mathrm{T}}$ and $\{\boldsymbol{\theta}_0\}$, the following single differential equation is obtained

$$\{\boldsymbol{\theta}_0\}^{\mathrm{T}}[\mathbf{M}]\{\boldsymbol{\theta}_0\}\ddot{\theta}+\{\boldsymbol{\theta}_0\}^{\mathrm{T}}[\mathbf{C}]\{\boldsymbol{\theta}_0\}\dot{\theta}+\{\boldsymbol{\theta}_0\}^{\mathrm{T}}[\mathbf{K}]\{\boldsymbol{\theta}_0\}\dot{\theta}=0 \qquad (4)$$

From equation (4) relative damping factor δ is

$$\delta=\{\boldsymbol{\theta}_0\}^{\mathrm{T}}[\mathbf{C}]\{\boldsymbol{\theta}_0\}/(2\{\boldsymbol{\theta}_0\}^{\mathrm{T}}[\mathbf{M}]\{\boldsymbol{\theta}_0\}) \qquad (5)$$

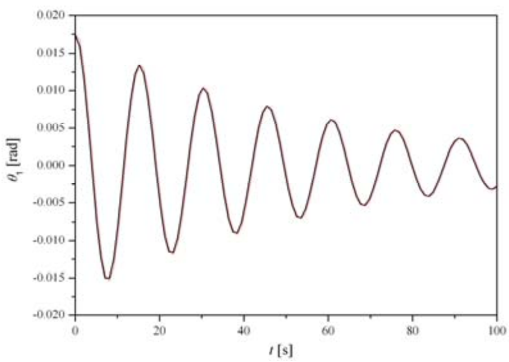

Figure 2. Thime change of θ_1 obtained by numerical solution of (1) and by (4)

In order to calculate damping factor from recorded amplitude change, maximum values of amplitudes are extracted and interpolated by exponential decay function of the form $A\exp(-\delta t)$, from which damping factor c is determined.

3. SIMULATION OF EXPERIMENT

Differential equations of motion for given model are solved by Runge-Kutta method. For experiment, different initial conditions are used. In Table 1. combinations of initial data used for experiment simulation are given. Time step used for Runge-Kutta solution is $\Delta t = 0{,}05$ seconds. Initial data 1 and 2 corresponds to first vibration eigenmode.

Table 1. Initial data used for experiment simulation

	θ_1[°]	θ_2[°]	l_1[m]	l_2[m]	m_1[kg]	m_2[kg]	k_1[N/°]	k_2[N/°]	c [Ns/°]
1	1	1.41	1	1	1	1	1	1	0,1
2	1	1.41	1	1	1	1	1	1	0,2
3	1	2	1	1	1	1	1	1	0,1
4	1	-1	1	1	1	1	1	1	0,1
5	1	2	1	1	1	1	2	4	0,2
6	1	-1	1	1	1	1	2	4	0,2

After gained results by Runge-Kutta method, waveforms obtained by simulation are imported in software for recording and analysis of experimental data ORIGIN 7.5. Obtained waveforms for third and fourth combination of initial data are shown on Fig.3. and Fig.4.

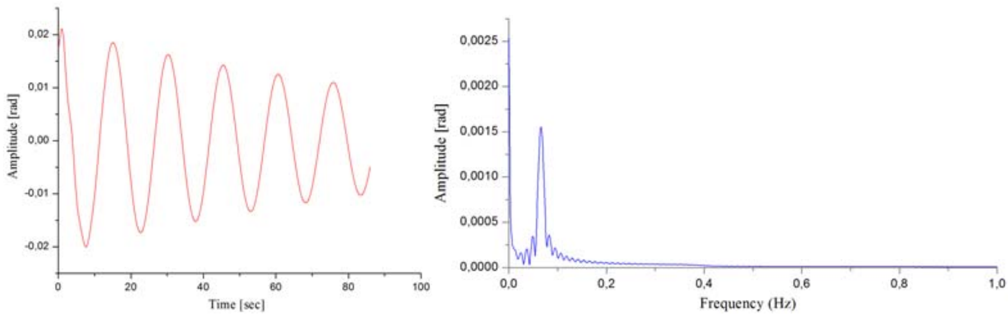

Figure 3. Waveform and frequency spectrum obtained from simulated data for third combination

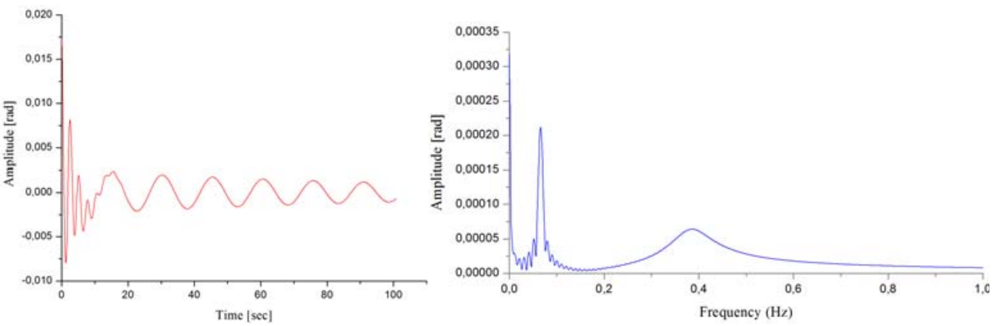

Figure 4. Waveform and frequency spectrum obtained from simulated data for fourth combination

As it could be seen, if initial conditions are equal or closed to vibration eigenmode, solution has similar for as in the one case if freedom

3.1 Analysis of damping

Amplitudes of simulated waveforms are interpolated by exponential function, from which damping ratio is determined using equation (5).

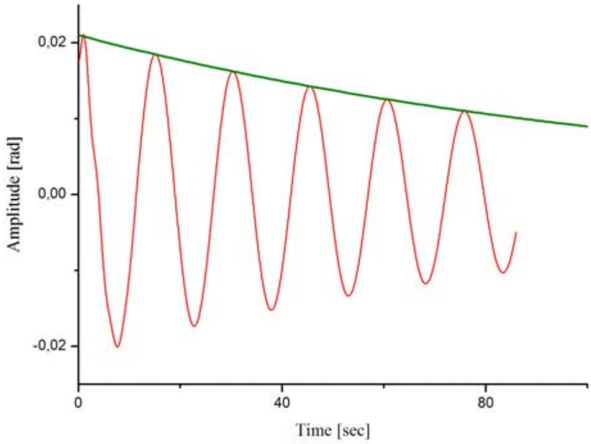

Figure 5. Amplitude interpolated by exponential decay function

For initial condition which produces solution where both natural frequencies are present, digital filter was used to eliminate higher one.
Damping coefficients used for simulation and coefficients obtained by analysis are compared are shown and compared in Table 2.

Table 2. Damping coefficient used for simulation, and coefficients obtained by analysis

	Damping coefficient c [Ns/m]		Error [%]
	Simulation	Analysis	
1	0,1	0,0991	0,90
2	0,2	0,1993	0,35
3	0,1	0,10258	2,58
4	0,1	0,10141	1,41
5	0,2	0,19817	0,91
6	0,2	0,20982	4,91

4. CONCLUSION

In this paper, simulation of experimental damping determination, in the case of damped vibration of the two d.o.f. system, is shown. Data obtained by simulation are imported in software for experimental data analysis and relative damping coefficient is determined. Original damping coefficient used for simulation is recovered by expression derived on the basics of eigenvibration analysis. For all used initial conditions, there is small difference between damping coefficient used for simulation and damping coefficient obtained by analysis and it is under 5%. This error is the less if initial conditions are closed to vibration eigenmode.

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