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MATHEMATICAL MODELING OF THERMAL FLOW AND TEMPERATURE IN FLAT WALL LAYERS DURING NON-STATIONARY THERMAL TRANSMISSION

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ABSTRACT
The paper consists in the extraction and formulation of mathematical model of thermal flux and temperature across the flat wall layers during non-stationary thermal transmission by the walls and heat stationary convection on both sides of the analyzed wall. Through Laplace transformations and simulations achieved reflection of the dynamics of thermal flux and the temperature and time for constant temperature and thermal fluxes in flat wall layers of the wall.
Key words: thermal flux, temperature, non-stationary thermal transmission, wall multi-layers, Laplace transformations.

1. INTRODUCTION

During non–stationary thermal conduct through the wall, the change of temperature depends not only from point to the point but also from time in the same drop of inspected body. The analytic solution of problems by non–stationary thermal conduct at numerous is complicated and impresses the use of mathematical models and using of Laplace transformations.

2. MATHEMATICAL MODELING OF SPECIFIC THERMAL FLUX AND TEMPERATURES THROUGH THE FLAT WALL LAYERS AREA

Thermal specific flux, at non–stationary thermal conduct through the wall, is changeable variable. The change depends on time τ and distance x of border surface in the wall. Then it is analyzed the made wall of homogeneous material where on the left from the wall is heated environment air by constant temperature t_b, while on the right of the wall is the environment air by constant temperature t_j (fig.1).

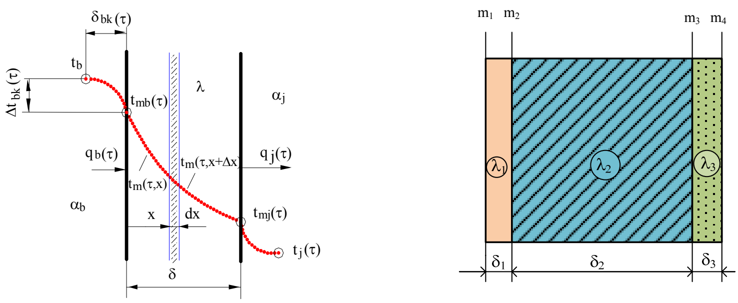

Figure1. Non – stationary thermal conduct through flat wall and wall with three layers

So, by given an elementary coat of flat wall with density *dx*, the thermal specific flux is presented by equalization [1]:

$$-\frac{\partial \dot{q}(\tau,x)}{\partial x}=c_m\cdot\rho_m\cdot\frac{\partial t_m(\tau,x)}{\partial x} \tag{1}$$

Thermal specific flux for observed elementary coat of wall, according law of Fourier on thermal conduct, is:

$$\dot{q}(\tau,x)=-\lambda\frac{\partial t_m(\tau,x)}{\partial x} \tag{2}$$

Equations (1) and (2) added equations that describe exchange of heating by convection in the left side surface (interior) and in the right side surface (out) of wall:

$$\dot{q}_b=\alpha_b\left(t_b-t_{mb}\right)\text{ and }\dot{q}_j=\alpha_j\left(t_{mj}-t_j\right) \tag{3}$$

Where are:
$t_m(\tau, x)$, ^0C - temperature in wall layer area, c_m, J/(kgK)- specific heat, and ρ_m, kg/m^3 -- density of wall; α_b , W/(m^{2}K), α_j W/(m^{2}K) - heat convection coefficients (from entrails environment in wall and from wall in out environment); t_b, ^0C, t_j, ^0C, - temperatures of environment, inside and outside of wall.

Using Laplace Transformation and after some reciprocal exchange of equations (1), (2) and (3) achieved to the wall layer area temperature equation [2]:

$$t_m(\tau,s)=t_{m1}(s)\cdot ch(k\sqrt{s}x)-\frac{\dot{Q}_{m1}(s)}{\lambda k\sqrt{s}}\cdot sh(k\sqrt{s}x) \tag{4}$$

Where: $k^2=\frac{\rho_{mi}c_{mi}}{\lambda_{mi}}$, s and $T=\frac{\rho_{mi}c_{mi}x^2}{2\lambda_{mi}}$, s

After the unification of the Laplace inverse [3] reached in the wall layer temperature:

$$t_m(\tau,x)=\frac{1}{1-e^{-\tau/T}}t_{m1}-\left(\frac{x}{\lambda}+\frac{x}{3}\frac{1}{e^{\tau/T}-1}\right)\dot{q}_{m1}=A\cdot t_{m1}-B\cdot\dot{q}_{m1} \tag{5}$$

From the above expressions can be obtained the surface temperatures on the wall layers:

$$\begin{aligned}
&\text{By } x=0:\ t_m(\tau,0)=t_{m1}(\tau)=t_b-\dot{q}_{m1}(\tau)/\alpha_b\\
&x=\delta_1:\ t_{m2}(\tau,\delta_1)=A_1t_{m1}-B_1\dot{q}_{m1}\\
&x=\delta_2:\ t_{m3}(\tau,\delta_2)=A_2t_{m2}-B_2\dot{q}_{m2}\\
&\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\
&x=\delta_i:\ t_{mi+1}(\tau,\delta_i)=A_it_{mi}-B_i\dot{q}_{mi}
\end{aligned} \tag{6}$$

In the same way achieved to the wall layer area thermal flux equation:

$$\dot{Q}_m(\tau,s)=-\lambda t_{m1}(s)k\sqrt{s}\cdot sh(k\sqrt{s}x)+\dot{Q}_{m1}(s)\cdot ch(k\sqrt{s}x) \tag{7}$$

After the unification of the Laplace inverse reached in the wall layer thermal flux:

$$\dot{q}_m(\tau,x)=\frac{1}{1-e^{-\tau/T}}\dot{q}_{m1}+\frac{c_{mi}\rho_{mi}x}{T\left(e^{\tau/T}-1\right)}t_{m1}=A\cdot\dot{q}_{m1}+C\cdot t_{m1} \tag{8}$$

And:

$$\begin{aligned}
&\text{By } x=0:\ \dot{q}_m(\tau,0)=\dot{q}_{m1}(\tau)=\alpha_b\left(t_b-t_{m1}\right)\\
&x=\delta_1:\ \dot{q}_{m2}(\tau,\delta_1)=A_1\cdot\dot{q}_{m1}+C_1t_{m1}\\
&x=\delta_2:\ \dot{q}_{m3}(\tau,\delta_2)=A_2\cdot\dot{q}_{m2}+C_2t_{m2}\\
&\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\
&x=\delta_i:\ \dot{q}_{mi+1}(\tau,\delta_i)=A_i\dot{q}_{mi}+C_it_{mi}
\end{aligned} \tag{9}$$

For wall of three layers the thermal flux is: $\dot{q}_{m4} = A_3 \cdot \dot{q}_{m3} + C_3 t_{m3} = \alpha_j \left(t_{m4} - t_j\right)$

Where: $A = \frac{1}{1-e^{-\tau/T}}$, B$= \frac{x}{\lambda} + \frac{x}{3}\frac{1}{e^{\tau/T}-1}$ and $C = \frac{c_{mi}\rho_{mi}x}{T\left(e^{\tau/T}-1\right)}$

3. ANALYSIS OF SPECIFIC THERMAL FLUXES AND TEMPERATURES ON THE FLAT WALL LAYERS AREA

For analysis is used wall of three layers with characteristics of materials as in table 1.

Table 1.

	Material layer		
Thermal-physical parameters	***1- Lime***	***2- Hollow brick***	***3- Styrofoam***
Density ρ, kg/m3	1600	1600	25
Specific heat c, J/(kgK)	1050	920	1260
Coeff. heat conduction λ, W/(mK)	0.81	0.64	0.041

For analyses it is inspected the case for a flat wall by characteristics: δ_{m1}=0.005m; δ_{m2}=0.3m; δ_{m3}=0.007m; α_b=8W/(m^{2}K); α_j=24W/(m^{2}K); t_b =20^{0}C and t_j-10. In figure 2 is given Changing of temperature through wall layers area in function of time τ and in fig. 3 Changing of thermal flux through wall layers area in function of time τ.

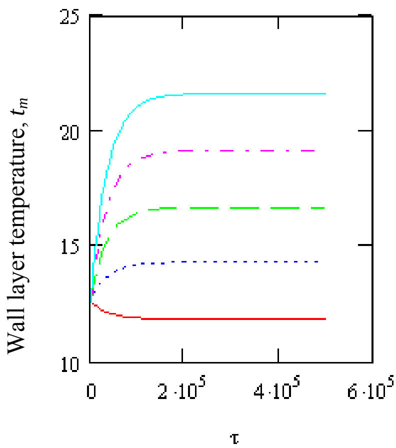

Figure 2. Changing of temperature through wall layer area in function of time τ

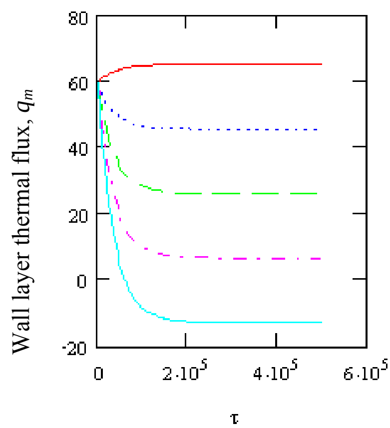

Figure 3. Changing of thermal flux through wall layer area in function of time τ

4. CONLUSIONS

Based on numerous practical problems associated with non-stationary transmission and Laplace transformations, here are exposed the mathematical expressions which have described the dynamics of specific thermal flux and the temperature on three wall layers. Importance of the paper is that the obtained expressions may also be used for walls with different number of layers and analyzed the other parameters for example the heat accumulation, thermal capacity of walls etc.

5. REFERENCES

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