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STRUCTURAL INTEGRITY ANALYSIS OF A CRACKED CYLINDRICAL PRESSURE VESSEL J INTEGRAL

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ABSTRACT
The basic aim of this paper is to contribute to the assessment of structural integrity of a cracked combustion chamber, i.e. cylindrical pressure vessel. Pressure vessels are leak proof containers. They may be of any shape and range from beverage bottles to the sophisticated ones encountered in engineering structures. The ever-increasing use of vessels for storage, industrial processing and power generation under unusual conditions of pressure, temperature, and environment has given special emphasis to analytical and experimental methods for determining their operating stresses.
Keywords: pressure vessel, non-destructive evaluation, elasto-plastic fracture

1. PRESSURE VESSELS

Pressure vessels are leak proof containers. They may be of any shape and range from beverage bottles to the sophisticated ones encountered in engineering structures. The ever-increasing use of vessels for storage, industrial processing and power generation under unusual conditions of pressure, temperature, and environment has given special emphasis to analytical and experimental methods for determining their operating stresses. Of equal importance is appraising the meaning or significance of these stresses. But, as a matter of fact, besides high yield strength, material for pressure vessel must be tough and ductile, or in other word its resistance to crack growth should be as high as possible.The force applied to a pressure vessel or its structural attachments are referred to as load and, as in any mechanical design, the first requirement in vessel design is to determine the actual values of the stress which the vessel will be subjected in operation. These are determined on the basis of past experience , design codes, calculation, or testing.

1.1. Stress state in cylindrical pressure vessel

To demonstrate the application of fracture mechanics to practical structures, such as cylindrical pressure vessel, one has to know its stress state. To accomplish this, a calculation is first made to determine the relation between the load and the maximum stress that exists in the structure. The maximum stress so determined is then compared with the materials strength. An acceptable design and safe operating pressure are achieved when the maximum stress is less than the strength of the material, suitably reduced by a safety factor. For the hoop stress, consider the pressure vessel section by planes sectioned by planes a, b, and c for Figure 1. A free body diagram of a half segment along with the pressurized working fluid is shown in Fig. 2

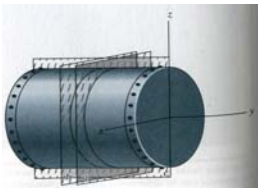

Figure 1. Cylindrical Thin-Walled Pressure Vessel Showing Coordinate Axes and Cutting Planes (a, b, and c)

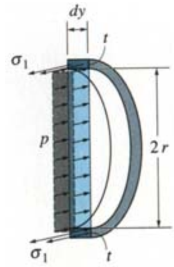

Figure 2. Free-Body Diagram of Segment of Cylindrical Thin-Walled Pressure Vessel Showing Pressure and Internal Hoop Stresses

2. RESULTS AND DISCUSION

Input data are:

- geometry $R_i = 500\,\text{mm}$ $R_0 = 600\,\text{mm}$ $t = 100\,\text{mm}$ $a = 25\,\text{mm}$
- material

$\sigma_0 = 414\,\text{MPa}$ $\alpha = 1{,}12$ $\varepsilon_0 = 0{,}002$ $n = 10$ Limite value for J_{Ic} is about $350\,kN/m$

$$K_{Jc} = \sqrt{\frac{J_{Ic}E}{1-\nu^2}} = 282\,\text{MPa}\sqrt{m} \tag{1}$$

$$\sigma_{flow} = \frac{\sigma_0}{2}\left[1 + \frac{(N/\varepsilon_0)^N}{\exp(N)}\right] \tag{2}$$

We use steel A533B ($\sigma_0 = 414\,MPa$, $N = 0{,}1$) and got $\sigma_{flow} = 484\,MPa$. Now, we can use FAD (fracture analysis diagram) with coordinate $K_r = K_I / K_{Ic}$ and $S_r = \sigma / \sigma_c$, σ_c indicates that a critical stress fracture can be determined on the basis of Hil's pressure solution needed tu fully plastically deformed cyilinder:

$$P_L^{(0)} = \frac{2}{\sqrt{3}}\sigma_{flow}\ln\left(\frac{R_0}{R_i}\right) = 102\,MPa \text{ in the case of the cylinder without crack} \tag{3}$$

$$P_L^{(1)} = \frac{2}{\sqrt{3}}\sigma_{flow}\ln\left(\frac{R_0}{R_i + a_0}\right) = 74{,}6\,MPa \text{ in the case of the cylinder with crack} \tag{4}$$

$P_L^{(1)}$ is used to approximately determine the critical stress fracture σ_c. On the basis of dimensionless variable S_r becomes $S_r = P / 74{,}6$

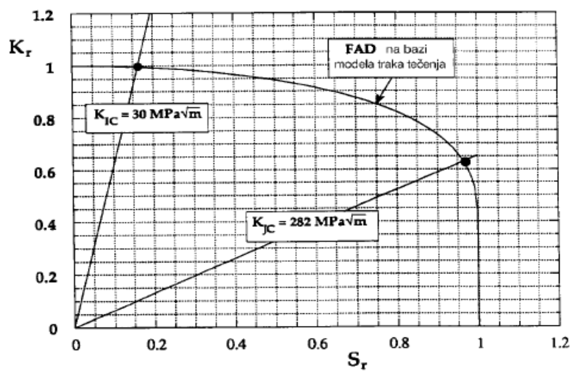

Figure 3. Fracture analysis diagram (FAD)

$$K_I = \frac{2PR_0^2}{R_0^2 - R_i^2}\sqrt{\pi a}F(\frac{a}{t}) = 1{,}1 + A\left[4{,}951(\frac{a}{t})^2 + 1{,}092(\frac{a}{t})^4\right] \quad (5)$$

And $A = \left(0{,}125\frac{R_i}{t} - 0{,}25\right)^{1/4} = 0{,}78254$ (6)

The initial depth of crack $a/t = 0{,}25$ is obtained

$$F(0{,}25) = 1{,}345486,\ K_I(0{,}25,P) = 2{,}468P,\ K_r = 2{,}468P/K_{Ic} \quad (7)$$

$P_f^L = 12{,}10MPa$ for lower plateau toughness

$P_f^U = 71{,}97MPa$ for upper plateau toughness (8)

Compared to values in equations (7) there is a certain safety factor and we apply EPRI procedure. In this case, we are using equation (5) for K factor, respectively elastic J integral is calculated as:

$$J_{el} = \frac{K_I^2(1-\nu^2)}{E} = \frac{(0{,}91)(4\pi)t}{207000MPa}\cdot\frac{XP^2}{\left[1-(R_i^2/R_0^2)\right]^2}\cdot F^2(X)\Big|_{X=0{,}25} = 0{,}02678P^2 \quad (9)$$

where $F(X)$ is defined as

$$F(X) = 1{,}1 + A\left[4{,}951X^2 + 1{,}092X^4\right],\quad X = a/t \quad (10)$$

Other extreme is the fully plastic zone. In this case the value of the plastic J integral is given in the form

$$J_{pl} = \alpha\varepsilon_0\sigma_0\frac{ba}{t}h_1\left(\frac{a}{t}\right)\left(\frac{P}{P_0}\right)^{n+1} \quad (11)$$

Taking into account the input data and values and auxillary functions $h_1(a/t)$ for $n = 10$ $h_1(0{,}25) = 9{,}45$ for J integral is obtained

$$J_{pl}\Big|_{a=25mm} = 164{,}3166\left(\frac{P}{P_0}\right)^{11} \quad (12)$$

Total J integral is sum of elastic and plastic components.

$\left|J_{el} + J_{pl}\right|_{a=25mm} = J_{Ic}$ or (13a)

$$0,02678P^2 + 164,317\left(\frac{P}{68,292}\right)^{11} = 350 \tag{13b}$$

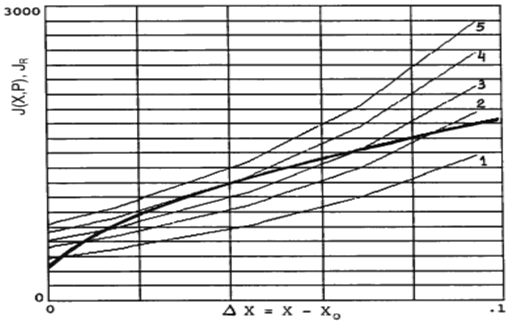

Figure 4. Crack growth force as a function of crack length, $X = a/t$ *1)* $72MPa$, 2) $74MPa$, 3) $75MPa$, 4) $76MPa$, 5) $77MPa$

3. CONCLUSION

Appraisal entails means of determining the value and extent of the stresses and strains, establishing the behaviour of the material involved, and evaluating the compatibility of these to factors in the media or environment to which they are subjected. Acknowledge of material behaviour is required not only to avoid failures , but also to enable maximum economy of material choice and amount used. Analytical formulas for the evaluation of stresses are usually based on elastic theory and elastic behaviour of the material, i.e.; material which obeys to Hooke's law and it may at first be thought that materials which follow this behaviour right up to the breaking point would be most desirable for use.

4. REFERENCES

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