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INFLUENCE OF ELASTIC RECOVERY ON DIMENSIONAL ACCURACY OF COLD BULK-FORMED PARTS

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ABSTRACT
Elastic recovery of workpiece is a common phenomenon of each forming operation affecting directly the part's final dimensions. Therefore, to predict final dimension as well to increase part accuracy one must have a good understanding of the elastic recovery phenomenon and factors affecting it. The size of elastic recovery is predominately determined by material properties and the amount of stresses accumulated in workpiece during the process of plastic deformation. Since these are variable parameters, the prediction of part's final dimension is a very difficult task.
In this paper theoretical approach to calculate elastic strain and the size of elastic recovery is given. Developed solution is applied to the process of free upsetting and verified by FEM analysis.
Key words: elastic recovery, accuracy, free upsetting

1. INTRODUCTION
Dimensional accuracy of the formed parts is influenced by many factors and phenomena [1,2,3,4]. To meet permanent demands for higher dimensional accuracy of formed part the influential factors and their effects (individual and interrelated) on part geometry should be well known and taken into account when design forming process [1].
Elastic spring back or material elastic recovery is a common phenomenon that occurs in forming processes after tool unloading, and why there is a change in the volume and dimensions of part. Unlike to design of sheet metal process where elastic springback of part is fully considered, in bulk metal forming this phenomenon is generally ignored. However, recent investigations [2,4] have shown that elastic recovery has a significant impact on dimensional accuracy of parts obtained by cold bulk forming too. This is especially true in cases of manufacturing components with close tolerances such are net shape parts.
Driving force for material elastic recovery comes from the stresses generated in the workpiece during forming process. The stress level is a function of material's properties and force applied in process, and determines the amount of elastic recovery final part will experience [5]. It means that any change of the stress level will results in variability of elastic recovery, i.e., will affects the variability of part's final dimensions. Therefore, to calculate elastic recovery of plastically deformed material it is essential to establish an analytical correlation between stresses in workpiece at the very end of forming process and elastic strains occurring after tool removing.
In this paper, based on relationship between engineering strains and associated true stresses, a general theoretical solution that enables prediction of material elastic recovery and part's final dimensions after forming is given. Developed theoretical solution is illustrated in case of free upsetting of cylindrical billet and validated by FEM analysis.

2. THEORETICAL APROACH
Relation between deformations and stresses in elastic range is governed by Hooke's low [5,6]. Generated Hooke's low (1a) can be also applied for estimation of elastic recovery of part after forming

process, but under condition of substituting engineering stresses that exist in the original expressions with true stresses.

$$\begin{aligned}\varepsilon_1 &= \frac{N_1}{E} - \frac{\nu}{E}\left(N_2 + N_3\right) \\ \varepsilon_2 &= \frac{N_2}{E} - \frac{\nu}{E}\left(N_3 + N_1\right) \\ \varepsilon_3 &= \frac{N_3}{E} - \frac{\nu}{E}\left(N_1 + N_2\right)\end{aligned} \qquad (1a) \qquad \begin{aligned}\varepsilon_1 &= \frac{\sigma_1}{E} - \frac{\nu}{E}\left(\sigma_2 + \sigma_3\right) \\ \varepsilon_2 &= \frac{\sigma_2}{E} - \frac{\nu}{E}\left(\sigma_3 + \sigma_1\right) \\ \varepsilon_3 &= \frac{\sigma_3}{E} - \frac{\nu}{E}\left(\sigma_1 + \sigma_2\right)\end{aligned} \qquad (1b)$$

where are:

ε_1, ε_2, ε_3 –engineering (elastic) strains in directions of principal axes 1, 2 and 3

E – Young's (elastic) modulus

ν - Poisson's ratio

$N_i = \dfrac{F}{A_{o,i}}$ - engineering stresses in directions of principal axes (i=1, 2 and 3)

$\sigma_i = \dfrac{F}{A_i}$ - true stresses in directions of principal axes (i=1, 2 and 3)

F – instantaneous force applied in forming process
$A_{o,i}$ – cross-sectional area before deformation
A_i - instantaneous cross-sectional area

However, if one needs to predict elastic recovery of plastically deformed part more accurately a proper relationship between the engineering stresses and the true stresses at the elastic region of deformation must be known. Taking into account the change in the volume due to elastic recovery and Poisson effect [5], the required relationship is derived by simple mathematical transformations (for more details see [4]). The final expressions have the next form:

$$N_1 = \sigma_1\left(1+\varepsilon_2\right)\left(1+\varepsilon_3\right); \quad N_2 = \sigma_2\left(1+\varepsilon_3\right)\left(1+\varepsilon_1\right); \quad N_3 = \sigma_3\left(1+\varepsilon_1\right)\left(1+\varepsilon_2\right) \qquad (2)$$

After substituting (2) into the system (1a) and simplifying the following system of equations relating the elastic strains with the true stresses in elastic range is obtained:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E}\left\{\sigma_1\left(1+\varepsilon_2\right)\left(1+\varepsilon_3\right) - \nu\left(1+\varepsilon_1\right)\left[\sigma_2\left(1+\varepsilon_3\right) + \sigma_3\left(1+\varepsilon_2\right)\right]\right\} \\ \varepsilon_2 &= \frac{1}{E}\left\{\sigma_2\left(1+\varepsilon_3\right)\left(1+\varepsilon_1\right) - \nu\left(1+\varepsilon_2\right)\left[\sigma_3\left(1+\varepsilon_1\right) + \sigma_1\left(1+\varepsilon_3\right)\right]\right\} \\ \varepsilon_3 &= \frac{1}{E}\left\{\sigma_3\left(1+\varepsilon_1\right)\left(1+\varepsilon_2\right) - \nu\left(1+\varepsilon_3\right)\left[\sigma_1\left(1+\varepsilon_2\right) + \sigma_2\left(1+\varepsilon_1\right)\right]\right\}\end{aligned} \qquad (3)$$

By solving the system (3) for the engineering strains, and considering the basic equation that defines engineering strain (4), it is possible to determine the final dimension of a workpiece.

$$l_{o,i} = \frac{l_i}{\varepsilon_i + 1} \qquad (4)$$

In equation (4) l_i is the dimension when workpiece is in a state of stress (function of forming operation) and $l_{o,i}$ is the dimension when the stress is no longer applied to the material (final dimension of part).

Unfortunately, the system (3) has no solution for engineering strains in closed (analytical) form so that must be solved numerically, except for few cases (unaxial stress state, plane strain state etc.). Regardless to this, analyzing the equations (3) some general insight into the sources and nature of the elastic recovery phenomenon in cold bulk forming operations can be gained. Since the true stresses in plastically deformed material depend mostly on yield stress (k) which in turn is a function of plastic material properties and the amount of effective (true) strain ($\varphi_{ef.}$), from (3) it follows that any rise of yield stress increase the values of engineering strains. At the same time, increase of the elastic modulus has opposite effect. In well controlled forming processes the effective strain is likely to be same as for most metals Young's modulus is constant indicating that the variability in the engineering strain and associated elastic recovery come only from the variability in the plastic material properties.

3. CASE STUDY – FREE UPSETTING

Engineering strains in case of free upsetting of cylindrical billet (axially-symmetric stress state where $\sigma_1=\sigma_z$, $\sigma_2=\sigma_3=\sigma_r$, $\varepsilon_1=\varepsilon_z$ and $\varepsilon_r=\varepsilon_\theta$) could be calculated by inserting the analytical solutions for contact stresses (5) at die-workpiece surface into the system (3).

$$\sigma_z = k \cdot e^{\frac{2\cdot\mu}{h}(R-r)} \; ; \qquad \sigma_r = k \cdot \left[e^{\frac{2\cdot\mu}{h}(R-r)} - 1 \right] \tag{5}$$

When engineering strains (ε_z and ε_r) are known, elastic recovery in axial and radial direction are easily determined by using the following expressions:

$$\begin{aligned} \Delta z &= \varepsilon_z \cdot h_{kr} \\ \Delta r &= \varepsilon_z \cdot r_{kr} \end{aligned} \tag{6}$$

where *h* and *r* denote the height and radius of the workpiece at the very end of forming process. In this case the analytical solution of the system (3) exists, but it is very complex and hence not suitable for practical use. However, from the equation (5) and previous analysis it can be concluded that engineering strains will vary along the radial direction. Also, the greater ratio *R/h* and intensive friction cause the greater elastic recovery of material after unloading.

In continue the process of free upsetting of cylinder with initial dimensions of 20x25mm under condition of low friction (μ=0.11) is considered. Engineering strains and associated elastic recovery in axial (z) and radial (r) directions are calculated for three different values of the true strains (φ=0.4, 0.8 and 1.2). As workpiece material a low carbon steel C15E is chosen with following flow curve $k = 309{,}15 + 409{,}57 \cdot \varphi^{0.518}$ $[MPa]$, as Poisson's ratio and Young's modulus are ν=0.3 and E=210000MPa, respectively.

In order to verify the derived theoretical solution for engineering strains, the upsetting process is also analyzed by Finite element method (FEM). For that purpose a commercial software package Simufact.Forming.11 was used. In the simulation, dies were modeled as rigid bodies.

4. RESULTS AND CONCLUDING REMARKS

Distribution of engineering strains ε_z and ε_r after upsetting process along workpiece radius obtained by theory and FEM simulation are depicted in Fig.1. Both solutions indicate that the size of the engineering strains depend very much on achieved degree of plastic deformation (φ), i.e., that they are sensitive to yield stress (k) and *R/h* ratio too. The larger true strain corresponds to the higher engineering strains (elastic recovery). Positive values of ε_z, suggest that the final height of workpiece will be greater than before tool releasing. At the same time, negative value of the ε_r deneote reduction of workpiece diameter. Compared to results of FEM analysis theoretical solution predicts slightly higher values for engineering strains in both directions as well as greater differences between maximum (r=0) and minimum (r=R) values.

In Fig. 2 maximum amplitudes of elastic recovery of workpiece in axial direction are shown and related with an International Tolerance grade (IT) with goal to display the change of the dimensions more efficiently. As it can be seen, the error in the workpiece height due to elastic recovery corresponds to IT9-IT10 grades for all three values of true strains, which may be considered as a significant loss of accuracy.

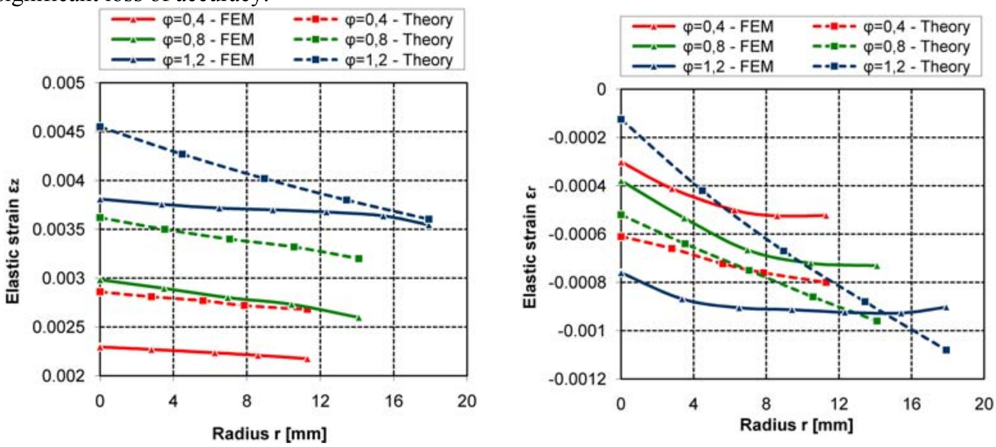

Figure 1. Engineering (elastic) strains in axial (left) and radial (right) direction

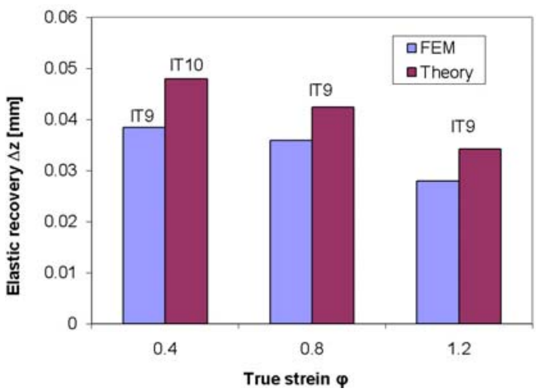

Figure 2. The values of maximum elastic recovery in axial direction

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