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INFLUENCE OF NUMERICAL MESH PROPERTIES ON DISCRETIZATION ERROR

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ABSTRACT
When a finite volume method is used to solve equations of mathematical physics there are several factors which could influence the discretization error. The standard techniques for determining the order of a discretization schemes assume that numerical meshes have some idealized properties. However, meshes typically used in numerical simulations are characterized with a significant aspect ratios and non-orthogonality of discretization elements. The paper presents a methodology which could be used to analyze the influence of certain mesh properties on the order of a finite volume discretization. The methodology is used to study the influence of mesh uniformity and orthogonality on the accuracy of the gradient approximations based on Gauss method, Gauss method with corrections and a least square method.
Keywords: Finite volume method; Discretization error; Numerical mesh

1. INTRODUCTION
Finite Volume Method (FVM) use integral form of transport equations, where such form exists, as a starting point in problem solving. Those equations often contain terms in which gradients of dependent variable are present. An example is the general convection-diffusion equation.

$$\underbrace{\frac{\partial}{\partial \tau}\int_V \rho\psi dV}_{\substack{Rate\ of \\ change}} + \underbrace{\int_S \rho\psi\vec{v}\cdot\vec{n}dS}_{Convection} = \underbrace{\int_S \Gamma grad\psi\cdot\vec{n}dS}_{Diffusion} + \underbrace{\int_V q_\psi dV}_{Source} \qquad (1)$$

where V is a control volume surrounded with surface S, $\vec{n}$ is the unit vector normal to surface S, ψ is transported variable (e.g. energy, entropy, momentum), Γ diffusion coefficient, ρ is continuum density, $\vec{v}$ is velocity and q_ψ is source (sink) of variable ψ.
FVM typically combines three types of approximations: interpolation, differentiation and integration. These approximations are characterized by the order of their accuracy, one of the most important attributes of a certain discretization scheme.

Gradient can be approximated using different schemes. Standard methods for formal ascertaining scheme's order do not considers influence of various parameters of numerical grid (e.g. uniformity, orthogonality or influence of topology of discretization elements), assuming that numerical grids are Cartesian.
The main objective of this paper was to define the methodology for estimation of the order of gradient approximations in relation to the various parameters of the numerical grid. To simplify the analysis and interpretation of results, research was based on a 2D analysis, but all acquired results and conclusions are also valid for 3D problems. A cell centered finite volume method, similar to the one described in [2], is in focus of this study.

2. METHODOLOGY FOR EVALUATION OF ORDER OF ACCURACY

The characteristics of a numerical grid, which influence the discretization order and which are analyzed in this work, are grid uniformity and orthogonality. The methodology developed and presented in this paper requires a set of grid configurations, each emphasizing one grid characteristics which is to be analysed. The set of grid configurations which are used in this work is made of four grids shown in Figures 1 and 2. The grids are made of control volumes, where each volume is closed by a number of edges, and each edge is defined with two vertices. The computational points P are placed in centers of control volumes.

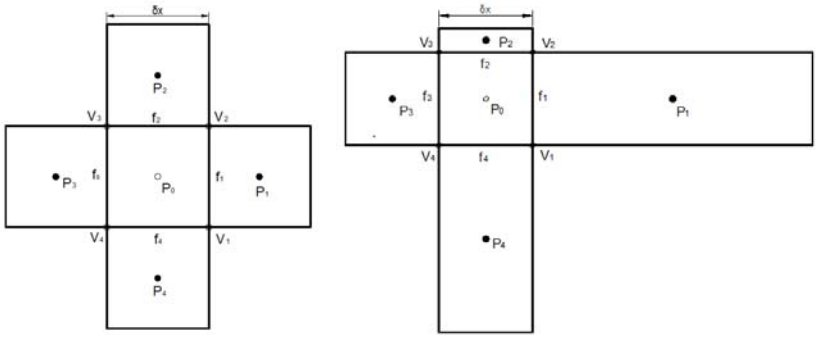


Figure 1. Cartesian (left) and orthogonal non-uniform numerical grid (right)

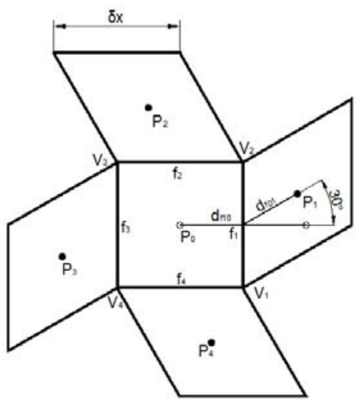


Figure 2. Non-orthogonal numerical grid

The spatial variation of dependent variable is defined using polynomials of degrees 1,2 and 3. The exact value of dependent variable are calculated and stored at computational points. They are used to estimate the gradient at the computational point P_0 using Gauss Method, Gauss Method with corrections [4] and Least Square Method [7]. The discretisation error is calculated comparing the estimated and exact values of gradients.

In order to calculate the order of approximations it was necessary to generate a set of systematically refined meshes. These meshes are created by scaling the initial coarsest mesh with factors 0.5, 0.25, 0.125, 0.0625, 0.03125, as it is shown on the Figure 3.

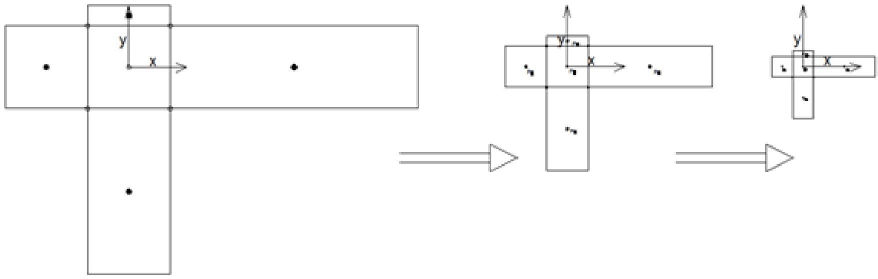
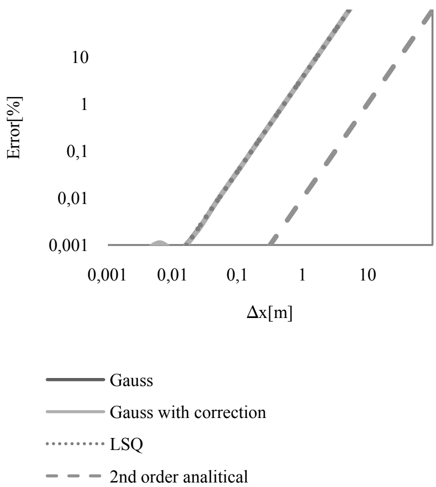


Figure 3. Grid refinement

The way how finer grids are created guaranteed that grid characteristics remain preserved on all grids. The rate at which the dicretization error was reduced with grid refinement was used to calculate the order of approximation for each methods on a given grid configuration.

3. RESULTS AND CONCLUSION

As one would expect, all three gradient approximation are second-order accurate on the Cartesian grid. Figure 4 shows us that the discretization error reduces by a factor of 4 with each grid refinement (grid scaling), what is a characteristic of a second-order scheme. Grid non-uniformity, the dominant characteristic of our second grid configuration, reduces all three methods for gradient calculation to first-order approximations. This can be concluded from Figure 5, where the discretization errors are halved by each systematic grid refinement.



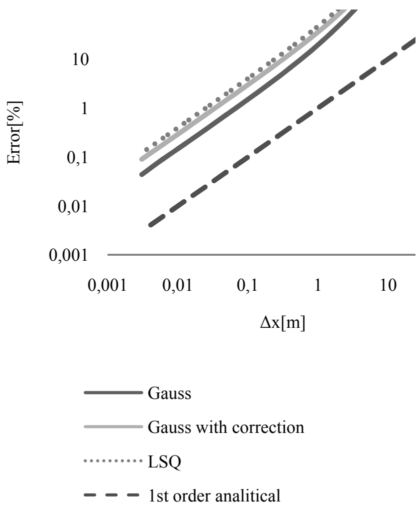


Figure 4. Discretization error on Cartesian grid *Figure 5: Discretization error on non-uniform grid*

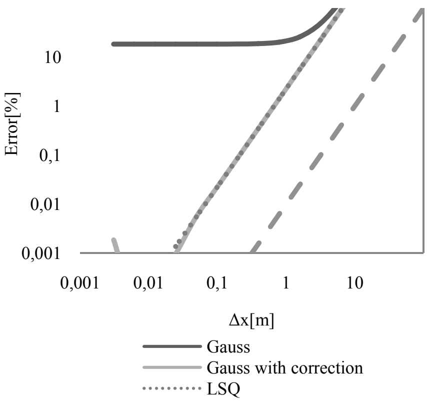


Figure 6. Discretization error on non-orthogonal grid

Non-orthogonality of our third grid configuration makes the Gauss method for the calculation of gradients zero-order accurate. The method calculates the face value of dependent variable using a simple interpolation between the two cells which share the face. Non-orthogonality causes that the interpolated value is not in the center of the face and consequently approximation of surface integrals becomes zero-order accurate. Gauss method with corrections iteratively improves the estimate of the face center value and is consequently a second-order approximation. Least square method does not require integration over the cell faces, and is also a second-order approximation on this grid.

The computer program which we developed allows us to study the interaction between different types of grid and different numerical methods for approximation of gradients. The idea behind the presented methodology is not limited to gradient estimations only, but it can also be applied to any discretization scheme. A representative matrix of different grid configurations and discretization schemes should help us to have a better understanding how some representative grid characteristics interact with a specific discretization technique. This could help us in selecting an appropriate discretization for a given meshing strategy, or in defining grid optimization criteria which respects the ways how grid interacts with selected discretizations. This will be a subject of our future work.

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