17th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2013, Istanbul, Turkey, 10-11 September 2013

HYDRAULIC PUMP POWER AS FUNCTION OF PISTON ROD VELOCITY AND LOAD

Prof.dr Dečan Ivanović
Faculty of Mechanical Engineering
Bul. Džordža Vašingtona bb, 81000 Podgorica, Montenegro

Prof.dr Marina Mijanović Markuš
Faculty of Mechanical Engineering
Bul. Džordža Vašingtona bb, 81000 Podgorica, Montenegro

ABSTRACT
In this paper is given the formula for determining the hydraulic pump power as function of piston rod velocity and load, for the case when the oil flow in working and returning pipe is laminar. For known values of pipes lengths and their diameters, and all the fluid energy losses in pipes, the minimum hydraulic cylinder diameter is obtained, also for laminar flow in hydraulic system.

Keywords: Hydraulic cylinder, hydraulic pumps, pump power

1. INTRODUCTION
Hydraulic systems are applications of fluid power, and use relatively incompressible liquid media such as oil. Hydraulics applications commonly use from 6.9 to 34 MPa (69-340 bar), but specialized applications may exceed 69 MPa (690 bar). The advantages of hydraulics are [1,4,5,6]: 1) Liquid does not absorb any of the supplied energy; 2) Capable of moving much higher loads and providing much higher forces due to the incompressibility; 3) The hydraulic working fluid is basically incompressible, leading to a minimum of spring action. In hydraulic systems there are hydraulic cylinder, pump, working and returning pipes, valves, pipe bends, and so on.

A hydraulic cylinder, also called a linear hydraulic motor is a mechanical actuator that is used to give a unidirectional force through a unidirectional stroke, Figure 1. Hydraulic cylinders get their power from pressurized hydraulic fluid, which is typically oil. The hydraulic cylinder consists of a cylinder barrel, in which a piston connected to a piston rod moves back and forth. The barrel is closed on each end by the cylinder bottom (also called the cap end) and by the cylinder head where the piston rod comes out of the cylinder. Hydraulic pump types: Gear pumps with external teeth and fixed displacement, Figure 2, are simple and economical pumps. The swept volume or displacement of gear pumps for hydraulics will be between about 1 cm^3 (0.001 liter) and 200 cm^3 (0.2 liter).

Figure 1. Hydraulic cylinder showing the internal components.

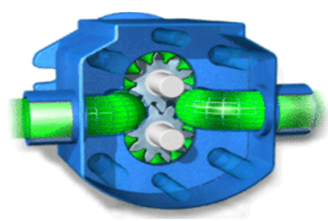

Figure 2. External gear pump.

2. HYDROULIC PUMP POWER

For hydraulic system shown in Figure 3, based on the continuity equation [2,3], there can be obtained the following expressions:

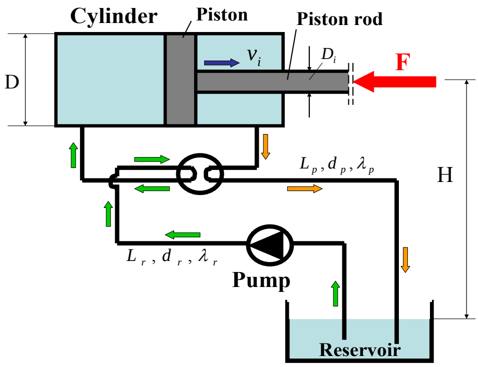

Figure3. Hydraulic system.

$$v_r = v_i D^2 / d_r^{\,2} \;, \quad (1) \qquad v_p = v_i (D - D_i)^2 / d_p^{\,2} \quad (2)$$

where are: v_i (m/s) – piston rod velocity; v_r (m/s) – oil flow velocity in working pipe; v_p (m/s) - oil flow velocity in returning pipe; d_r (m) – working pipe diameter; d_p (m) - returning pipe diameter; D (m) – cylinder diameter; D_i (m) – piston rod diameter.

By raising energy equation for oil flow through the working pipe, starting with the oil level in the reservoir above which is the atmospheric pressure, to the front of the piston [2,3], we get the following equation for head provided to the oil by the pumps H_P:

$$H_P = \frac{v_i^2}{2g} + \frac{p_r}{\rho g} + H + \frac{v_r^2}{2g}\left(\xi_{u_r} + \sum_{i=1}^{n_r} \xi_{k_{r_i}} + \xi_{R_r} + \sum_{j=1}^{m_r} \xi_{v_{r_j}} + 1\right) + \lambda_r \frac{L_r}{d_r} \frac{v_r^2}{2g} \quad (3)$$

while the energy equation for flow through the returning pipe from the section behind the piston to the oil level in the reservoir, provides:

$$\frac{v_i^2}{2g} + \frac{p_p}{\rho g} + H = \frac{v_p^2}{2g}\left(\xi_{u_p} + \sum_{i=1}^{n_r} \xi_{k_{p_i}} + \xi_{R_p} + \sum_{j=1}^{m_r} \xi_{v_{p_j}} + 1\right) + \lambda_p \frac{L_p}{d_p} \frac{v_p^2}{2g} \quad (4)$$

where are: $p_r (N/m^2)$, $p_p (N/m^2)$ - oil pressure in front section behind the piston; H (m) - vertical distance of the piston from the oil level in the reservoir;

$\rho (kg/m^3)$ - oil density; $g (m/s^2)$ – gravity acceleration; ξ_{u_r}, ξ_{u_p} – loss coefficient in entrance of working and returning pipe; ξ_{R_r}, ξ_{R_p} – loss coefficient of working and returning pipe through the distributor; $\sum_{i=1}^{n_r} \xi_{k_{r_i}}$, $\sum_{i=1}^{n_p} \xi_{k_{p_i}}$ – loss coefficient of bends, n_r in working pipe and n_p in returning pipe; $\sum_{j=1}^{m_r} \xi_{v_{r_j}}$, $\sum_{j=1}^{m_r} \xi_{v_{p_j}}$ – loss coefficient of valves, m_r in working pipe and m_p in returning pipe; $L_r (m)$, $L_p (m)$ – lengths of working and returning pipe; λ_r, λ_p – friction factors of working and returning pipe.

If K_r and K_p expressions percentages of local losses compared to losses due to friction in working pipe and returning pipe, then can be writen: $\left(\xi_{u_r} + \sum_{i=1}^{n_r} \xi_{k_{r_i}} + \xi_{R_r} + \sum_{j=1}^{m_r} \xi_{v_{r_j}} + 1\right) = K_r \lambda_r \frac{L_r}{d_r}$ (5)

$$\left(\xi_{u_r} + \sum_{i=1}^{n_r} \xi_{k_{r_i}} + \xi_{R_r} + \sum_{j=1}^{m_r} \xi_{v_{r_j}} + 1\right) = K_p \lambda_p \frac{L_p}{d_p} \quad (6)$$

Balance equation for cylinder piston, gives: $p_r D^2 \pi / 4 = F + p_p (D^2 - D_i^2) \pi / 4$ (7)

where the load on piston rod is noted by F.

Using (3),(4),(5) and (6), from (7) can be obtained for pump head H_P the following equation:

$$H_P = \frac{4F}{\rho g \pi D^2} + \left(\frac{v_i^2}{2g} + H\right)\left(\frac{D_i}{D}\right)^2 + \frac{v_r^2}{2g}(1 + K_r)\lambda_r \frac{L_r}{d_r} + \frac{v_p^2}{2g}(1 + K_p)\lambda_p \frac{L_p}{d_p}\left[1 - \left(\frac{D_i}{D}\right)^2\right] \quad (8)$$

Introducing the Reynolds numbers Re_r and Re_p for working and returning pipe, using (1) and (2), they become: $\mathrm{Re}_r = \frac{v_r d_r}{\nu} = v_i \frac{D^2}{\nu d_r}$; $\mathrm{Re}_p = \frac{v_p d_p}{\nu} = v_i \frac{(D^2 - D_i^2)}{\nu d_p}$ (9)

where is $\nu (m^2/s) -$ kinematic viscosity of oil.

The laminar flow in hydraulic system pipes is for Re < 2320, and for Re > 2320 the flow is turbulent. Based on these facts, and based on the relation (9), with the required piston velocity v_i and the adopted diameter of working pipe d_r for known oil kinematic viscosity ν, it appears that the flow in working pipe will always be laminar if for cylinder diameter D adopt a value::

$$D < \left(2320 \nu d_r / v_i\right)^{1/2} \quad (10)$$

In the same way, in the returning pipe will be also laminar flow, if, for known values v_i, d_p, D_i and ν, for cylinder diameter D adopt a values:

$$D < \left(D_i^2 + 2320 \nu d_p / v_i\right)^{1/2} \quad (11)$$

If the adopted cylinder diameter D is greater than $\left(2320 \nu d_r / v_i\right)^{1/2}$ then in working pipe the flow is turbulent, and in returning pipe the flow is also turbulent, if diameter D is larger than $\left(D_i^2 + 2320 \nu d_p / v_i\right)^{1/2}$. The present laminar ot turbulent flow in pipes has especially considerng, because in the cases will appear a different values of energy losses, which has an impact on the hydraulic pump power selection. It may appear two more cases. The first case is when in working pipe is laminar flow and turbulent flow in returning pipe. This means that diameter D satisfies the relation (10) and the relation (11) will not. In the second case is conversely, ie. In working pipe is turbulent and in returning pipe is laminar flow. It means that the diameter D will not satisfy the relation (10) and will the relation (11). However, this whole analysis can be carried out, when for known v_i, ν and adopted diameter D, we must adopted d_r and d_p according to the relations:

$$d_r > v_i \frac{D^2}{2320 \nu}; \; d_p > v_i \frac{(D^2 - D_i^2)}{2320 \nu} \quad (12)$$

Namely, if the values d_r and d_p satisfy the relation (12), then in the pipes there are laminar flow, and if not in the pipes the flow will be turbulent. Owing to the complexity of calculation, only laminar flow in hydraulic system pipes will be considered.

For laminar flow in pipes, the friction factors, using (1) and (2) are given as:

$$\lambda_r = \frac{64}{\mathrm{Re}_r} = \frac{64\nu}{v_r d_r} = \frac{64\nu d_r}{v_i D^2} \quad (13) \qquad \lambda_p = \frac{64}{\mathrm{Re}_p} = \frac{64\nu}{v_p d_p} = \frac{64\nu d_p}{v_i (D^2 - D_i^2)} \quad (14)$$

so the equation (8) becomes: $H_{P_{ll}}(D) = a_{ll} / D^2 + b_{ll} D^2 + c_{ll}$ (15)

where are:

$$a_{ll} = \frac{4F}{\rho g \pi} + D_i^2 \left(\frac{v_i^2}{2g} + H\right) + \frac{64 v_i (1+K_p)\nu L_p D_i^4}{2 g d_p^4}; \; b_{ll} = \frac{64 v_i (1+K_r)\nu L_r}{2 g d_r^4} + \frac{64 v_i (1+K_p)\nu L_p}{2 g d_p^4}; \; c_{ll} = -\frac{64 v_i (1+K_p)\nu L_p D_i^2}{g d_p^4}$$

Thus, the pump head $H_{P_{ll}}(D)$ is given as a function of cylinder diameter D, for known coefficients a_{ll}, b_{ll} and c_{ll}. To determine the cylinder diameter D, for which the pump head will be the smallest, it is necessary to find such $D_{H_{P_{ll}\min}}$ that satisfies the relation $dH_{P_{ll}}(D)/dD = 0$, so after using (15), one

can obtained: $$\frac{dH_{P_{ll}}(D)}{dD} = 0 \;\Rightarrow\; D_{H_{P_{ll}\min}} = \left(b_{ll} / a_{ll}\right)^{1/2} \quad (16)$$

and the smallest pump head becomes: $$H_{P_{ll\min}} = \frac{a_{ll}^{\,2}}{b_{ll}} + \frac{b_{ll}^{\,2}}{a_{ll}} + c_{ll} \quad (17)$$

Hydraulic pump power is $P = \rho g Q H_P / \eta_P$, where the coefficient of pump efficiency is noted by η_P. Since the flow through the pipes Q is given as $Q = v_r d_r^2 \pi / 4 = v_i D^2 \pi / 4$, based on relations (1),

hydraulic pump power becomes: $$P_{ll} = \frac{\rho g \pi v_i D^2}{4\eta_P} H_{P_{ll}} \quad (18)$$

After putting (15) in (18) we get that the hydraulic pump power, dependence of cylinder diameter D and for laminar flow in both pipes, is given as: $P_{ll}(D) = a_{ll}^{*} D^4 + b_{ll}^{*} D^2 + c_{ll}^{*}$ (19)

where are:

$$a_{ll}^{*} = \frac{\rho g \pi v_i}{4\eta_P}\left[\frac{64 v_i (1+K_r)\nu L_r}{2 g d_r^4} + \frac{64 v_i (1+K_p)\nu L_p}{2 g d_p^4}\right]; \; b_{ll}^{*} = -\frac{64 \rho g \pi v_i^2 (1+K_p)\nu L_p D_i^2}{4\eta_P g d_p^4} \quad c_{ll}^{*} = \frac{\rho g \pi v_i}{4\eta_P}\left[\frac{4F}{\rho g \pi} + D_i^2\left(\frac{v_i^2}{2g} + H\right) + \frac{64 v_i (1+K_p)\nu L_p D_i^4}{2 g d_p^4}\right] \quad (20)$$

For known parameters all expressions in (20), i.e. a_{ll}^{*}, b_{ll}^{*}, c_{ll}^{*} become constants, so the hydraulic pump power for selected cylinder diameter D, can be obtained by (19). In further research it would be useful to examine in detail the all numerical possibilities of determing the hydraulic pump power for the adopted diameter D, given with the dependence:

$$P_{ll}(D) = a_{ll}^{*}(\rho, \nu, v_i, L_r, L_p, d_r, d_p, K_r, K_p) D^4 + b_{ll}^{*}(\rho, \nu, v_i, D_i, L_p, d_p, K_p) D^2 + + c_{ll}^{*}(\rho, \nu, F, v_i, D_i, H, L_p, d_p, K_p) \quad (21)$$

which is very complex.

3. CONCLUSION

There is given the equation for determining the hydraulic pump power as function of piston rod velocity and load, for the case when the oil flow in working and returning pipe is laminar. In further research it would be useful to examine in detail the all numerical possibilities of determing the hydraulic pump power for the adopted diameter D, and for: 1) laminar flow in working pipe and turbulent flow in returning pipe 2) turbulent flow in working pipe and laminar flow in returning pipe and 3) turbulent flows in working and returning pipe.

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