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ASSESSMENT OF FATIGUE CRACK GROWTH IN PIPES SUBJECTED TO VARIABLE LOADING

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ABSTRACT
Computer simulation of fatigue crack growth in pipes subjected to variable amplitude loading is considered in this work. The fatigue crack, detected during the inspection, significantly reduces remaining fatigue life. The fatigue life is calculated by solving the equation of fatigue crack growth rate step-by-step from initial to final crack size by Runge-Kutta method. The computer program, based on this procedure, is used for the fatigue crack growth simulation.
Keywords: variable amplitude loading, steel pipes, crack-like deffect, assessment of remainig fatigue life

1. INTRODUCTION
Fatigue is one of the most frequent form of the failures of the structural details, machine elements, pressure vessels and piping systems. According to ref. [1,2,3] 50 -90 percent of all mechanical failures are fatigue failures. This study focuses on the assessment of fatigue crack growth in pipes subjected to variable loading. The fracture mechanics approach was utilized, average material properties were assumed.

2. ANALYSIS OF CRACK PROPAGATION
2.1. Crack propagation model
The crack propagation lives were calculated with the Paris equation [4]:

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where

da/dN = crack growth rate,
ΔK = range of stress intensity factor,
C and m = material constants.

2.2. Stress intensity factor
The stress intensity factor was calculated by using Raju and Newman solution [5] for internal surface longitudinal cracks in pipes (Fig.1) Eq.(2):

$$K_I = \frac{pR}{t}\sqrt{\pi\frac{a}{Q}}F_i\left(\frac{a}{c},\frac{a}{t},\frac{t}{R},\phi\right) \tag{2}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{3}$$

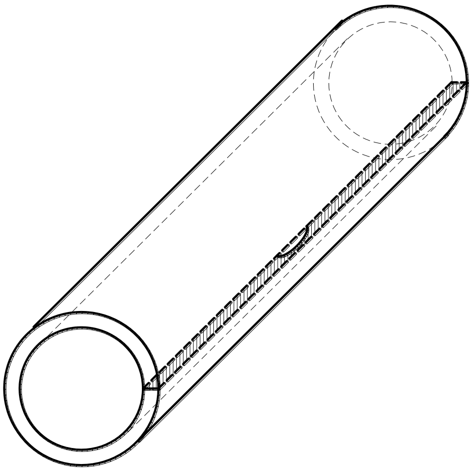


Figure 1. Pipe with longitudinal internal surface crack

$$F_i = \frac{t}{R}\left(\frac{R_0^2}{R_0^2 - R^2}\right)\left[2G_0 - 2\left(\frac{a}{R}\right)G_1 + 3\left(\frac{a}{R}\right)^2 G_2 - 4\left(\frac{a}{R}\right)^3 G_3\right] \tag{4}$$

where a = depth of surface crack, $2c$ = surface crack length, t = cylinder wall thickness, the shape factor for an elliptical crack, Q is the square of the complete elliptic integral of the second kind and is approximated by Eq.(3), p = internal pressure in cylinder, R , R_0 = inner and outer radii of cylinder. Influence coefficient for *j*th stress distribution on crack surface, G_j , was obtained from the appropriate finite element solution and given in tables [5] for the particular velues of t/R , a/c , a/t, . G_j values for another a/t values was determined in this work by using regresion analysis:

$$\begin{aligned} G_0 &= 0.90933 + 0.64333\left(\frac{a}{t}\right) \\ G_1 &= 0.60133 + 0.27\left(\frac{a}{t}\right) \\ G_2 &= 0.48133 + 0.15667\left(\frac{a}{t}\right) \\ G_3 &= 0.409 + 0.11667\left(\frac{a}{t}\right) \end{aligned} \tag{5}$$

3. PREDICTION OF THE REMAINING FATIGUE LIFE

Average material properties, for steel, were assumed: m = 3, C = $4.9 \cdot 10^{-12}$, with ΔK in units of MPa$\sqrt{\text{m}}$ and da/dN in units of m/cycle, threshold stress intensity factor ΔK_{th} = 4 MPa√m, the fracture toughness K_c=55 MPa$\sqrt{\text{m}}$. Geometrical parameters are: t/R = 0.25, t =10 mm, R = 40 mm, R_0 =50 mm, a/c =0.4, initial crack size a_{in}=0.4 mm. The pipes are subjected to variable internal pressure (Table 1 and 2). The remaing fatigue life (crack propagation life) N_p is obtained by solving Eq.(1) using the Runge-Kutta [6] method. Obtained results are shown by Δp vs N diagram in Fig.2 and compared with Miner's and Haibach's results.

Table 1. Heavy spectrum of internal pressure ranges

Block number. i	1	2	3	4	5	6	7	8	9	10
Frequency γ_i	0,003	0,007	0,008	0,014	0,024	0.044	0.078	0.138	0.248	0.436
Normalized pressure range $\Delta p_i / \Delta p_1$	1	0,944	0,927	0,906	0,850	0.800	0.770	0.735	0.680	0.630

Table 2. Medium spectrum of internal pressure ranges

Block number. i	1	2	3	4	5	6	7	8	9	10
Frequency γ_i	0,003	0,007	0,008	0,014	0,024	0.044	0.078	0.138	0.248	0.436
Normalized pressure range $\Delta p_i / \Delta p_1$	1	0,890	0,852	0,814	0,767	0.690	0.600	0.510	0.420	0.340

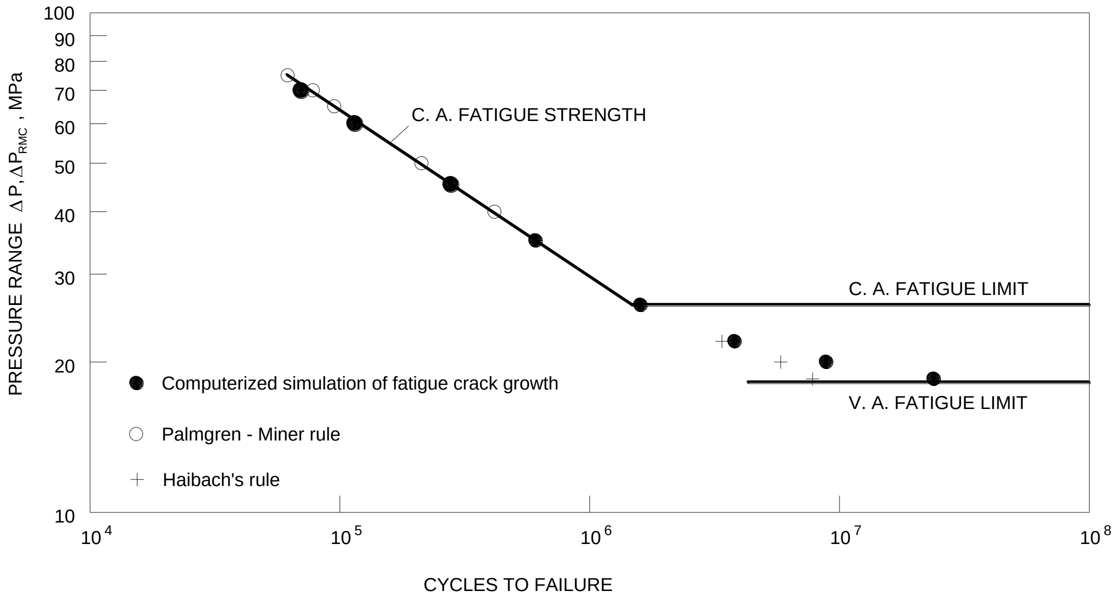


Figure 2. Comparison of predicted fatigue lives with Miner's and Haibach's results for heavy spectrum

Table 3. Predicted fatigue lives for variable-amplitude fatigue N_p, cycles

	Heavy spectrum			Medium spectrum		
Equivalent pressure range Δp_{RMC}, MPa	Miner rule or Haibach's rule	Computer simulation	Difference, %	Miner rule or Haibach's rule	Computer simulation	Difference, %
70.0	76467	70003	+9.2	77876	60003	+29.8
60.0	121507	115003	+5.7	123732	103003	+20.1
45.0	288372	279003	+3.4	293321	267003	+9.9
35.0	613559	618003	-0.7	633397	600003	+5.6
26.0	1637898	1600003	+2.4	1722933	1668003	+3.3
22.0	3381058	3850316	-13.9	3654867	3104003	+17.7
20.0	5818038	8930003	-53.5	5465287	4513003	+21.1
18.5	7793835	23945003	-3.07 times	7099957	6290003	+12.9
16.0				11296608	12239100	-8.3
14.0				21010953	30700003	-46.1

The folowing formula were used: Miner's [7], if all pressure ranges are above fatigue limit

$$N = \frac{N_1}{\sum_{i=1}^{10} \gamma_i \left(\frac{\Delta p_i}{\Delta p_1} \right)^m} \tag{6}$$

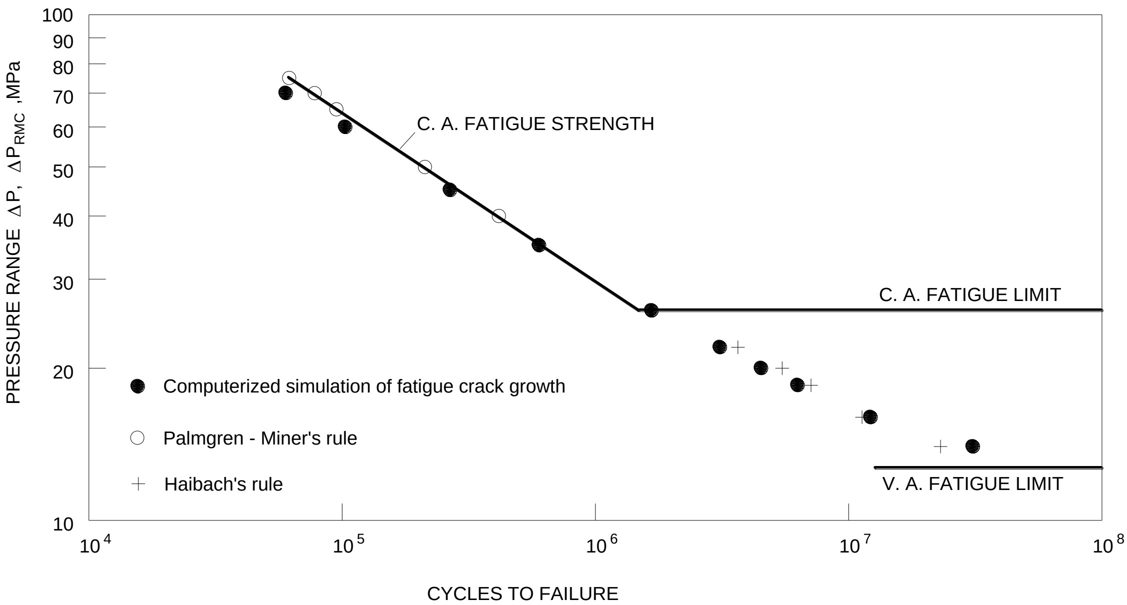
and Haibach's [8], which acounts for the damaging effects of pressure ranges below the constant amplitude (CA) fatigue limit

$$N = \frac{N_1}{\sum_{i=1}^{k} \gamma_i \left(\frac{\Delta p_i}{\Delta p_1} \right)^m + \left(\frac{\Delta p_1}{\Delta p_{CAFL}} \right)^{m-1} \sum_{i=k+1}^{n} \gamma_i \left(\frac{\Delta p_i}{\Delta p_1} \right)^{2m-1}} \tag{7}$$

where equivalent pressure range is

$$\Delta p_{RMC} = \left[\sum_{i=1}^{n} \gamma_i \left(\frac{\Delta p_i}{\Delta p_1} \right)^m \right]^{\frac{1}{m}} \Delta p_1 \tag{8}$$

The similar procedure was performed for medium spectrum:



4. CONCLUSIONS

When all pressure ranges are above CA fatigue limit results of computerized simulation are identical to those obtained by Miner rule (difference is less than 1%) as derived analiticaly by Maddox [9]. This is valid only if interval a_i to a_f is the same for VA and CA load. If the final crack size is determined by K_c than Miner's results becam noncoservative: 9% for heavy spectrum and 30% for medium spectrum. When some of the pressure ranges are below CA fatigue limit Haibach's results are conservative for heavy spectrum and nonconservative for medium spectrum.

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